Exercise 1(Points: 7)

A vertical plane wall of thickness $t = 0.1m$ and height $L = 3m$ has a thermal conductivity $k = 25W/mK$. Within the whole plate there is a uniform volumetric heat generation $\dot{q} = 0.3MW/m^3$. On one side, the plate is perfectly insulated. On the other side, water at $T_\infty = 92°C$ flows parallel to the wall with a constant velocity $u_\infty = 63mm/s$.

a) Sketch the temperature profile across the entire system as accurately as you can based on the available information.

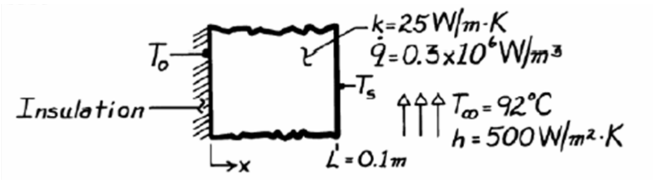
b) Calculate the convection coefficient. Estimate the needed physical properties of water at $T_f = 127°C = 400K$.

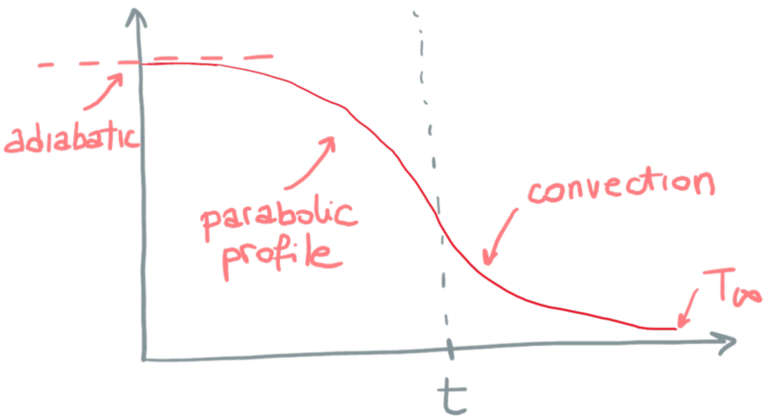
c) Determine the temperature of the surface exposed to the fluid

d) Determine the maximum temperature in the system. In doing so, for the plane wall explicitly state the boundary conditions and their mathematical expressions.

Solution

a) **1.5 pt** : 0.5 for the parabolic profile in the wall, 0.5 for the horizontal derivative at the insulation layer, 0.25 for the maximum temperature at the insulation layer, 0.25 for the minimum temperature in the fluid.





b) **2.5 pt** : 0.25 for the right Table; 0.25 for the right T_f; 0.25 for rho and 0.25 for mu (correct units); 0.25 for Re and 0.25 for turbulent; 0.25 for the right Nu expression; 0.25 for the Nu value; 0.25 for the h expression; 0.25 for h value.

For estimating the convection coefficient we need to calculate Re and $\overline{Nu}$. From Table $A.6$, for a temperature of $400K$, we get the following physical properties of water:

- $\rho = 1/0.001067 kg/m^3 = 937.207 kg/m^3$
- $\mu = 217 \cdot 10^{-4} Ns/m^2$
- $k = 0.688 W/mK$
- $Pr = 1.34$

With these we obtain: $Re = \frac{\rho u_\infty L}{\mu} = 816277 > 5 \cdot 10^5$, i.e. turbulent flow.

Therefore:

$$\overline{Nu_L} = 0.037 Re^{4/5} Pr^{1/3} = 2187.96 \approx 2188$$

And finally:

$$h = \frac{\overline{Nu_L} k}{L} = 501.8 W/m^2K \approx 502 W/m^2K$$

c) **1 pt** : 0.5 for the energy balance (0.25 for each half), 0.25 for the right expression of T_s, 0.25 for the right value of T_s

The amount of heat removed by convection must equal the amount of heat generated in the wall.

$$q''_{conv} = h(T_s - T_\infty) = \dot{q}t$$

$$T_s = T_\infty + \frac{\dot{q}t}{h} = 92 + \frac{30000}{502} = 151.7°C \approx 152°C$$

d) **2 pt** : 0.25 for the right T profile formula; 0.25 for the adiabatic BC; 0.25 for the convection/temperature BC; 0.25 for C_1; 0.25 for C_2; 0.25 for the right x for the maximum T; 0.25 for the right expression of T_{max}; 0.25 for the value of T_{max}.

The temperature profile in the wall is parabolic. From the formula sheet we have:

$$T(x) = -\frac{\dot{q}}{2k}x^2 + C_1 x + C_2$$

The boundary conditions are:

- adiabatic boundary condition at $x = 0$: $\frac{dT}{dx} = 0$
- convection boundary condition at $x = t$: $-k\frac{dT}{dx} = h(T_s - T_\infty)$

Alternatively:

- adiabatic boundary condition at $x = 0$: $\frac{dT}{dx} = 0$
- temperature boundary condition at $x = L$: $T(x = t) = T_s$

Applying these boundary conditions the temperature profile in the planar wall we obtain the two constants:

$$-\frac{\dot{q}}{k}(x = 0) + C_1 = 0 \rightarrow C_1 = 0$$

$$T(x = t) = T_s = -\frac{\dot{q}t^2}{2k} + C_2 \rightarrow C_2 = T_s + \frac{\dot{q}t^2}{2k}$$

Hence:

$$T(x) = -\frac{\dot{q}}{2k}x^2 + \frac{\dot{q}t^2}{2k} + T_s$$

The maximum temperature is next to the insulation layer, i.e. at $x = 0$ hence we get:

$$T_{max} = T(x = 0) = \frac{\dot{q}t^2}{2k} + T_s = \frac{0.3 \cdot 10^6 \cdot 0.1^2}{2 \cdot 25} + 151.7 = 211.7°C \approx 212°C$$

Exercise 2(Points 8.5)

The evaporator section of a refrigeration unit consists of thin-walled tube with diameter $D = 10mm$ through which refrigerant passes at a temperature of $-18°C$. Air flows around the tube and is subsequently routed to the refrigerator compartment.

a) Given the negligible conduction resistance of the thin-walled tube, assume that the surface temperature of the tube is equal to the refrigerant temperature. If air at atmospheric pressure and at $T_\infty = -3°C$ is blown over the tube at a velocity $u_\infty = 8.2m/s$, what is the convection coefficient?

b) For the foregoing conditions, what is the rate of heat transfer from the air to the refrigerant per unit tube length?

c) Due to a malfunction of the system, a $2mm$ layer of ice forms on the outer surface of the tubes. If the ice is assumed to have a thermal conductivity $k_{fr} = 0.4W/mK$, what is the additional thermal resistance due to this layer?

d) For the case with ice, sketch the equivalent thermal circuit for the heat transfer from the air to the refrigerant. Using the same physical properties of air determined at the beginning of this exercise, calculate the rate of heat transfer from the air to the refrigerant per unit length under these conditions. Comment on its value compared to the design conditions.

e) (*Difficult question*) Following the formation of the ice layer, the refrigerator is disconnected. If the tubes are now left in ambient air with $T_{\infty,room} = 20°C$ and natural convection maintains a convection coefficient of $h_2 = 2W/m^2K$, how long will it take for the ice to melt? Assume that during the melting the ice temperature is constant and equal to $T_{ice} = 0°C$. The ice may be assumed to have the following physical properties: mass density $\rho_{ice} = 700kg/m^3$, latent heat of fusion $h_{sf,ice} = 334kJ/kg$.

Solution

a) **2.75 pt** : 0.25 right T_f; 0.25x4 for each value (interpolation); 0.25 right characteristic dimension; 0.25 Re value; 0.5 right correlation; 0.25 right value of Nu; 0.25 value of h

Based on the assumption that the $T_s = -18°C = 255K$, the physical properties of air must be estimated at the film temperature $T_f = \frac{T_s+T_\infty}{2} = 262.5K$.

From Table $A.4$ we get:

- $\rho = 1.336375kg/m^3$
- $\mu = 165.8610^{-7}Ns/m^2$
- $k = 0.0233W/mK$
- $Pr = 0.71675$

and hence

$$Re_D = \frac{\rho u_\infty D}{\mu} = 805.73u_\infty = 6607$$

For the external forced convection over a cylinder we use the Churchill-Bernstein correlation:

$$\overline{Nu_D} = 0.3 + \frac{0.62Re_D^{1/2}Pr^{1/3}}{[1+(0.4/Pr)^{2/3}]^{1/4}}[1+(\frac{Re_D}{282000})^{5/8}]^{4/5} = 42.94 \approx 43$$

and therefore:

$$h = \frac{\overline{Nu_D}k}{D} = \frac{43 \cdot 0.0233}{0.01} = 100.19 \approx 100W/m^2K$$

b) **0.75 pt** : 0.5 equation (right area and right T sign); 0.25 q' value;

The heat transfer rate from the airflow to the refrigerant due to the forced convection is equal to:

$$q' = h\pi D(T_\infty - T_s) = 100 \cdot \pi 0.1(270 - 255) = 47.1W/m$$

so the air transfers $47.1W/m$ to the refrigerant.

c) **0.75 pt**: 0.25 use of cylindrical resistance; 0.25 expression of resistance per unit length; 0.25 value of R'.

The ice conduction resistance is equal to:

$$R'_{cond,ice} = R_{cond,ice}L = \frac{\ln(r_2/r_1)}{2\pi k_{ice}} = \frac{0.007/0.005}{2\pi 0.4} = 0.134Km/W$$

d) **2.25 pt** : 0.5 sketch (0.25 for convection resistance ad 0.25 for conduction resistance); 0.25 expression of convection resistance per unit length and 0.25 for value; 0.25 for total resistance expression (series) and 0.25 for value; 0.25 for total heat transfer rate expression and 0.25 for value; 0.25 for the comment.

When the frost forms, the equivalent electrical circuit is composed of the frost conduction resistance and the convection resistance. We assume that the convection resistance is unchanged. The frost conduction resistance is equal to:

$$R'_{conv} = \frac{1}{h2\pi r_2} = 0.227Km/W$$

Hence the total resistance is: $R'_{tot} = R'_{conv} + R'_{cond,fr} = 0.361Km/W$ and therefore the heat transfer per unit length of the tube from the air to the refrigerant is:

$$q' = \frac{T_\infty - T_s}{R'_{tot}} = 41.55W/m$$

This is equivalent of a $\approx 12\%$ reduction in the heat transfer from the air to the refrigerant.

e) **2 pt** : 0.25 for energy balance idea; 0.5 for left-hand-side (0.25 for $dU = hdm$ and 0.25 for $dm = \rho dV$); 0.25 for right hand side; 0.5 for integration boundaries; 0.25 for final expression of t; 0.25 for value

We can write an energy balance onto the frost layer and assume that the heat is remove by convection. Hence we get:

$$\frac{dU}{dt} = \frac{h_{sf,ice}dm}{dt} = \frac{h_{sf,ice}\rho dV}{dt} = -h(2\pi rL)(T_{\infty,room} - T_{ice})$$

$$h_{sf,ice}\rho(2\pi rL)dr = -h(2\pi rL)(T_{\infty,room} - T_{ice})dt$$

Integrating from r_2 to r_1 and from $t = 0s$ to $t = t_1$ we obtain:

$$t_1 = \frac{h_{sf,ice}\rho_{ice}(r_2 - r_1)}{h_2(T_{\infty,room} - T_{fr})} = \frac{334 \cdot 10^3 \cdot 700 \cdot (0.007 - 0.005)}{2 \cdot (293 - 273)} = 11690s \approx 3.25h$$

Exercise 3(Points: 7.5)

Circular pure copper rods of diameter $D = 1mm$ and length $L = 25mm$ are used to enhance heat transfer from a surface that is maintained at $T_{s,1} = 100°C$. One end of each rod is attached to this surface (at $x = 0$), while the other end ($x = 25mm$) is joined to a second surface, which is maintained at $T_{s,2} = 0°C$. Air flowing between the surfaces, and hence over the rods, is also at a temperature of $T_\infty = 0°C$. Due to this air flow, a convection coefficient of $h = 100W/m^2K$ is maintained.

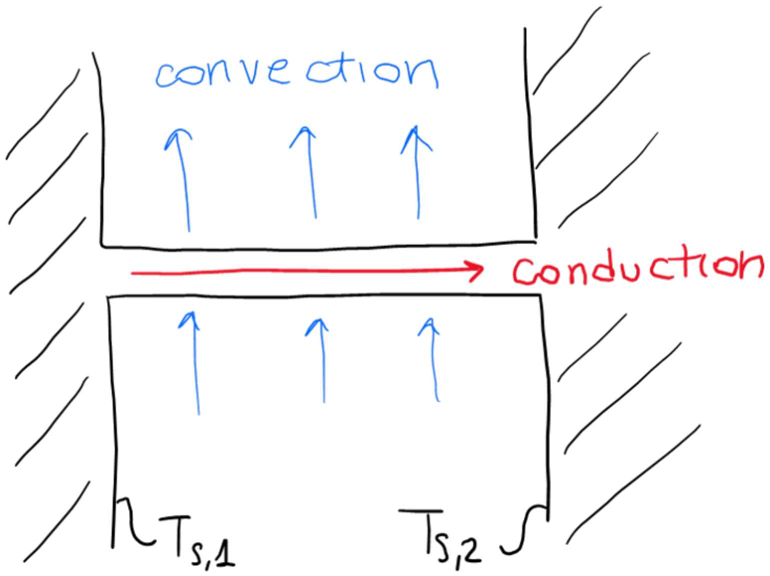
a) Consider a single copper rod, joining the two surfaces. Sketch the geometry and indicate with an arrow the directions of conduction and convection heat transfer processes. What is the total heat transfer rate from a single copper rod, Q_1?

b) A bundle of the rods is installed on a $1m - by - 1m$ section of the surface at $100°C$ with $4mm$ centers and a square (aligned) arrangement. What is the total rate of heat transfer from this section of the surface at $100°C$? What is the heat transfer rate enhancement compared to the case of no rods?

c) (*Difficult Question*) The total rate of heat transfer from one copper rod calculated in point a) consists of both heat transfer to the air via convection Q_{conv} and heat transfer to the second surface via conduction $Q_{cond,2}$. Derive a mathematical expression for $Q_{cond,2}$. Then calculate the values of $Q_{cond,2}$ and Q_{conv}.

Solution

a) **3.5 pt** : 0.5 sketch (0.25 conduction and 0.25 convection); 0.25 fin concept; 0.25 temperature BC; 0.25 Q_1 expression; 0.25 P; 0.25 A_c; 0.25 θ_b; 0.25 for θ_L; 0.25 simplified Q_1 expression; 0.25 k; 0.25 m; 0.25 M; 0.25 Q_1



Each rod can be seen as a fin with uniform (circular) cross-section. Because the temperature is fixed on both surfaces, the temperature boundary condition must be applied at the tip.

From the formula sheet we obtain immediately the expression of the heat transfer rate for a fin with temperature boundary condition:

$$Q_1 = Q_f = M\frac{cosh(mL) - \theta_L/\theta_b}{sinh(mL)}$$

where:

- $M = \sqrt{hPkA_c} \cdot \theta_b$
- $m = \sqrt{\frac{hP}{kA_c}}$
- $P = \pi D = 0.00314m$ and $A_c = \pi D^2/4 = 7.854 \cdot 10^{-7} m^2$
- $\theta_b = T(x = 0) - T_\infty = 100 - 0 = 100K$
- $\theta_L = T(x = L) - T_\infty = 0 - 0 = 0K$

hence:

$$Q_f = M\frac{1}{tanh(mL)}$$

From Table $A.1$ we find that for pure copper the thermal conductivity is $k = 401W/mK$, therefore we get:

- $m = 31.58$
- $M = 0.9947$
- $Q_f = 1.51W$

b) **1.5 pt** : 0.5 for expression Q (0.25 for rods and 0.25 for convection from area); 0.25 for expression of $A_{s,1}$; 0.25 for N; 0.25 for value Q; 0.25 for enhancement.

The total heat transfer rate from the surface is now a combination of the heat transfer rate from the rods and the heat transfer rate from the remaining surface. Given the geometry information, the bundle of rods consists of $N = 250 \cdot 250 = 62500$ rods while the surface area not covered by the rods is $A_{s,1} = 1m^2 - N \cdot \pi D^2/4 = 0.951m^2$

$$Q = N \cdot Q_f + hA_{s,1}(T(x=0) - T_\infty) = 1.037 \cdot 10^5 W$$

Without rods, we would only have convection $Q_0 = h \cdot A(T(x=0) - T_\infty) = 10^4 W$, therefore the enhancement is:

$$Enhancement = \frac{1.037 \cdot 10^5 - 10^4}{10^4} = 9.37$$

We thus see that, despite their small footprint (they cover $\approx 5\%$ of the surface area), the application of the rods has a dramatic effect on the heat transfer rate, enhancing it by almost 10 times.

c) **2.5 pt** : 0.5 for Fourier law; 0.5 tip BC; 0.25 for θ expression simplification; 0.25 for right derivative; 0.25 for value of derivative; 0.25 for value of $Q_{cond,2}$; 0.25 for expression Q_{conv} and 0.25 for value Q_{conv}

Based on Fourier law we can write that the heat transfer rate by conduction from the rod into the second surface is:

$$Q_{cond,2} = -kA_c \frac{dT}{dx}\bigg|_{x=L} = -kA_c \frac{d\theta}{dx}\bigg|_{x=L}$$

From the formula sheet we retrieve the expression of the temperature profile in a fin with temperature boundary condition at the tip:

$$\theta = \theta_b \cdot \frac{\theta_L/\theta_b sinh(mx) + sinh(m(L-x))}{sinh(mL)} = \frac{\theta_b}{sinh(mL)} \cdot sinh(m(L-x))$$

where we have used the fact that in our case $\theta_L = 0$.

Hence:

$$\frac{d\theta}{dx}\bigg|_{x=L} = \frac{\theta_b}{sinh(mL)} \cdot [-m \cdot cosh(m(L-x))]\bigg|_{x=L} = -\frac{\theta_b m}{sinh(mL)} = -3613$$

$$Q_{cond,2} = -401 \cdot 7.854 \cdot 10^{-7} \cdot (-3613) = 1.137W$$

Finally, the total heat transfer rate by convection is obtained as:

$$Q_{conv} = Q_f - Q_{cond,2} = 1.51 - 1.137 = 0.373W$$

ALTERNATIVE SOLUTION for part c):

2.5 pt : 0.5 for Fourier law; 0.25 for BCs; 0.25 for C_i; 0.25 for θ expression derivation; 0.25 for right derivative; 0.25 for value of derivative; 0.25 for value of $Q_{cond,2}$; 0.25 for expression Q_{conv} and 0.25 for value Q_{conv}

Based on Fourier law we can write that the heat transfer rate by conduction from the rod into the second surface is:

$$Q_{cond,2} = -kA_c \frac{dT}{dx}\Big|_{x=L} = -kA_c \frac{d\theta}{dx}\Big|_{x=L}$$

We know that the general for of the θ function is:

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

with boundary conditions:

- $\theta(x = 0) = T(x = 0) - T_{s,2} = T_{s,1} - T_{s,2} = 100K$
- $\theta(x = L) = T(x = L) - T_{s,2} = T_{s,2} - T_{s,2} = 0K$

we get:

$$\begin{cases} C_1 + C_2 = 100 \\ C_1 e^{mL} + C_2 e^{-mL} = 0 \end{cases} \Rightarrow \begin{cases} C_1 = \frac{100}{1-e^{2mL}} \\ C_2 = -\frac{100e^{2mL}}{1-e^{2mL}} \end{cases}$$

Hence:

$$\theta(x) = \frac{100}{1 - e^{2mL}}(e^{mx} - e^{2mL-mx})$$

From this we get:

$$\frac{d\theta}{dx} = \frac{100}{1 - e^{2mL}} m(e^{mx} + e^{2mL-mx})$$

This expression can then be substituted into the Fourier equation and from there $Q_{cond,2}$ can be calculated.

$$Q_{cond,2} = -401 \cdot 7.854 \cdot 10^{-7} \cdot (-3613) = 1.137W$$

Finally, the total heat transfer rate by convection is obtained as:

$$Q_{conv} = Q_f - Q_{cond,2} = 1.51 - 1.137 = 0.373W$$

Exercise 4(Points: 10.5)

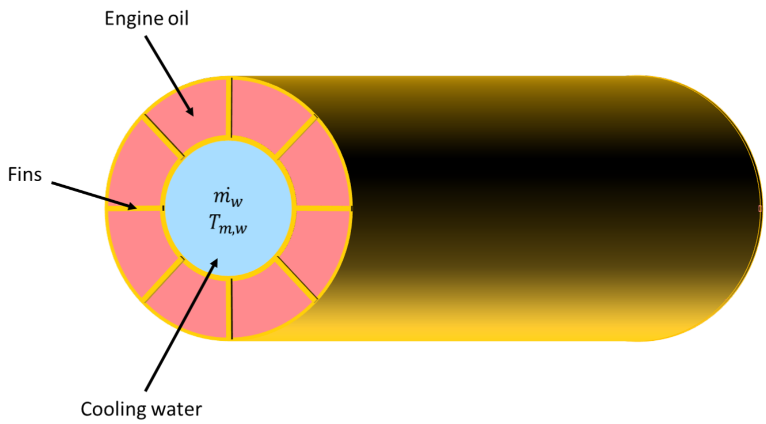
A counter-flow concentric tube heat exchanger is used for engine cooling. The cooling waters flows in an inner copper tube while hot engine oil flows in the annular space surrounding it. The total length of the heat exchanger is $L = 15.9m$.

The thin-walled copper tube has a diameter of $D = 10mm$. The water flow rate inside the tube is $\dot{m}_w = 0.2kg/s$ and it has an average temperature of $T_{m,w} = 32°C$. In the annular gap fins are used to enhanced the heat transfer. The fin array has an overall efficiency of $\eta_o = 0.8$ and the convection coefficient is $h_o = 65W/m^2K$.

a) Calculate the *design value* of the overall heat transfer coefficient U_d.

b) The heat exchanger has been in service for an extended period of time. Hence engineers perform a test to verify the value of the overall heat transfer coefficient. During the test, engine oil flowing at $\dot{m}_o = 0.1kg/s$ is cooled from $110°C$ to $64°C$ by water supplied at a temperature of $25°C$ and a flow rate $\dot{m}_w = 0.2kg/s$. Calculate the outlet temperature of the water then draw a sketch of the temperature profiles of the water and oil in the heat exchanger.

c) Using the results of the test, determine whether fouling has occurred during the service period. If so, calculate the fouling factor $R_f^{"}[m^2K/W]$ assuming that fouling occurs only on the water side of the heat exchanger.



Solution

a) **4.5 pt** : 2 for Rtot expr (0.25 for R_{cond}; 0.25 for A; 0.25 no fouling; 0.25 outer fins); 0.25 forced internal conv; 0.25 for right values and 0.25 for right units of water properties; 0.25 for expression of water velocity ad 0.25 for value; 0.25 for value of Re; 0.25 for right correlation; 0.25 for value; 0.25 for h_i value; 0.25 for U_d value;

Considered the characteristics of the heat exchanger the expression for the overall heat transfer coefficient can be simplified as follow:

$$\frac{1}{U_d A} = R_{tot} = \frac{1}{\eta_o h_o A} + \frac{1}{h_i A}$$

- $R_{cond} = 0$ because of the thin-walled tube
- $A_o = A_i = A$ because of the thin-walled tube
- no fouling
- no fins on the inner side
- fins on the outer side

Therefore we need to calculate h_i. This is thus internal forced convection. For water we get the physical properties from Table $A.6$ at $T_m = 305K$:

- $\rho = 1/0.001005 = 995kg/m^3$
- $\mu = 769 \cdot 10^{-6} Ns/m^2$
- $k = 0.620W/mK$
- $Pr = 5.2$

The average velocity of the water in the tube is:

$$u_m = \frac{4\dot{m}_w}{\rho \pi D^2} = \frac{4 \cdot 0.2}{995 \cdot \pi 0.1^2} = 0.0256m/s$$

Hence:

$$Re = \frac{\rho u_m D}{\mu} = \frac{995 \cdot 0.0256 \cdot 0.1}{769 \cdot 10^{-6}} = 3311 > 2300$$

thus the flow is turbulent. For this value of Re we use the correlation:

$$f = (0.790 \ln Re - 1.64)^{-2} = 0.0441$$

$$Nu = \frac{(f/8)(Re - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)} = 22.94$$

and we get:

$$h_i = \frac{Nuk}{D} = 142.2W/m^2K$$

We can then calculate:

$$\frac{1}{U_d} = R_{tot}A = \frac{1}{\eta_o h_o} + \frac{1}{h_i} = 0.02626m^2K/W$$

and finally:

$$U_d = 38.07W/m^2K$$

b) **2.5 pt** : 0.25 for energy balance approach; 0.25 for hot side expr. 0.25 for cold side expr.; 0.25 for T_m of oil; 0.25 for $c_{p,h}$ value (including units); 0.25 for $c_{p,c}$ value (including units); 0.25 for value Q_h; 0.25 for value $T_{c,out}$; 0.25 for cold fluid sketch; 0.25 for hot fluid sketch;

The oil is the hot fluid and water is the cold fluid. The outlet cold temperature is missing, hence we apply an energy balance on the hot and cold fluid, respectively.

$$Q_h = \dot{m}_h c_{p,h}(T_{h,in} - T_{h,out})$$
$$Q_c = \dot{m}_c c_{p,c}(T_{c,out} - T_{c,in})$$
$$Q_h = Q_c$$

Hence:

$$T_{c,out} = \frac{Q_h}{\dot{m}_c c_{p,c}} + T_{c,in}$$

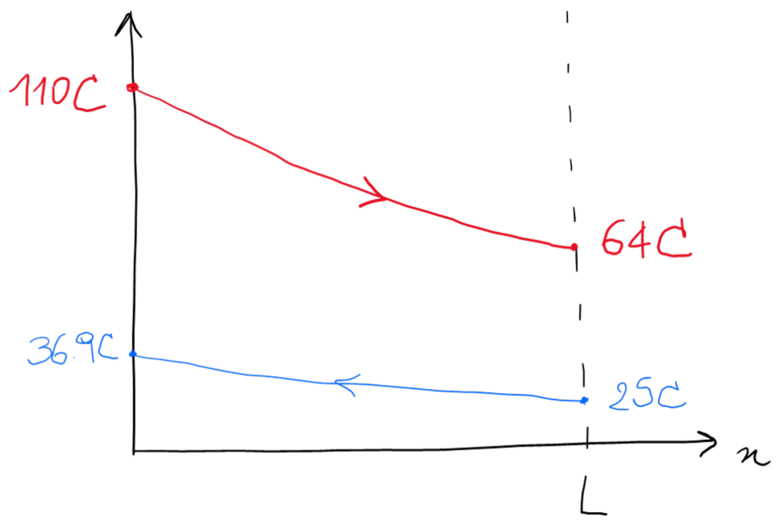
We thus need to determine the specific heat of the hot and cold fluids.

For the engine oil (hot fluid) at a $T_m = (T_{h,in} + T_{h,out})/2 = 87°C = 360K$ from Table $A.5$ we get $c_{p,h} = 2.161kJ/kgK = 2161J/kgK$.

For the water (cold fluid) at a $T_m = 32°C = 305K$ from Table $A.6$ we get $c_{p,c} = 4.178kJ/kgK = 4178J/kgK$.

Thus we have:

$$Q_h = 9940.6W$$
$$T_{c,out} = 36.9°C = 309.9K$$



c) **3.5 pt** : 0.25 for LMTD approach; 0.25 for right Q value from part b); 0.25 for ΔT_1 value and 0.25 for ΔT_2 value; 0.25 for ΔT_{lm} expr and 0.25 for value; 0.25 for expression and 0.25 for value of A; 0.25 for value of U; 0.25 for recognition of fouling; 0.25 for expression of fouling factor and 0.25 for value of fouling factor.

For a concentric heat exchanger the total heat transfer can be expressed by the LMTD equation. From the formula sheet we get:

$$Q = Q_h = Q_c = UA\Delta T_{lm} = UA\frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}$$

For this counter-flow heat exchanger we have:

$$\Delta T_1 = T_{h,in} - T_{c,out} = 110 - 36.9 = 73.1K$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 64 - 25 = 39K$$

$$\Delta T_{lm} = \frac{39 - 73.1}{\ln\left(\frac{39}{73.1}\right)} = 54.3K$$

$$A = \pi D \cdot L = \pi \cdot 0.1 \cdot 15.9 = 4.995m^2 \approx 5m^2$$

Therefore, from this test we obtain that the overall heat transfer coefficient for the heat exchanger is:

$$U = \frac{Q}{A\Delta T_{lm}} = 36.6W/m^2K \neq U_d$$

Therefore, fouling must have occurred. We know that including the fouling the overall heat transfer coefficient expression is (formula sheet):

$$\frac{1}{UA} = \frac{1}{U_dA} + \frac{R_f^{"}}{A}$$

$$R_f^{"} = \frac{1}{U} - \frac{1}{U_d} = 0.001m^2K/W$$

Exercise 5(Points: 5)

An ocean thermal energy conversion system is being proposed for electric power generation. Such a system is based on the standard power cycle for which a working fluid is evaporated, passed through a turbine and subsequently condensed. In this project, oceanic water near the surface at a temperature of $300K$ is used as a heat source to evaporate the working fluid. Instead, oceanic water at larger depths with a temperature of $280K$ is used as a heat sink for the condensation of the working fluid. You are asked to design the evaporator and you are given the following information.

The evaporator is a heat exchanger consisting of a single-shell and two tube passes. The working fluid is evaporated at its phase-change temperature of $288K$ with ocean water entering at $300K$ and leaving at $290K$. The overall heat transfer coefficient is $U = 1200W/m^2K$. The required total heat transfer rate in the heat exchanger is $Q = 66.7MW$.

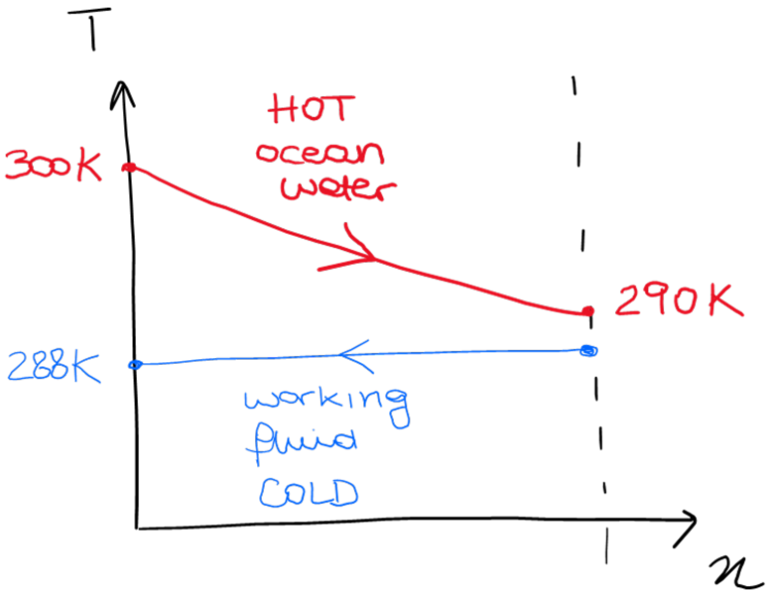
a) Sketch the temperature profiles of the hot and cold fluids in the evaporator

b) What flow rate must be maintained for the ocean water passing through the evaporator?

c) What is the the heat exchanger area required for the evaporator? If the tube length per pass is $L = 10m$ and its external diameter is $D_{out} = 10cm$, how many tubes are needed to reach the required area?

Solution

a) **0.5 pt** : 0.25 decreasing hot T profile; 0.25 constant cold T.



b) **1 pt** : 0.25 expression energy balance hot side; 0.25 T_m ; 0.25 expression $\dot{m}_h$ and 0.25 for value.

From an energy balance on the ocean water (hot fluid) side, we have:

$$Q_h = Q = \dot{m}_h c_{p,h}(T_{h,in} - T_{h,out})$$

From table $A.6$ using an average temperature of $T_m = \frac{300+290}{2} = 295K$ we get: $c_{p,h} = 4181 J/kgK$ and thus:

$$\dot{m}_h = \frac{Q}{c_{p,h}(T_{h,in} - T_{h,out})} = 1595 kg/s$$

c) **3.5 pt** : 0.5 NTU method; 0.25 C_c; 0.25 C_h; 0.25 C_r; 0.25 Q_{max}; 0.25 ϵ; 0.5 correct NTU expression; 0.25 NTU value; 0.25 A expression; 0.25 A value; 0.25 N expression; 0.25 N value.

To determine the needed surface area of a shell-and-tube heat exchanger we need to use the NTU Method. In the evaporator we have that:

- $C_c = \infty$ (phase-change)
- $C_h = c_{p,h}\dot{m}_h = 6.67 \cdot 10^6 W/K$

therefore $C_{min} = C_h$ and $C_r = C_{min}/C_{max} = 0$.

We then calculate:

$$Q_{max} = C_{min}(T_{h,in} - T_{c,in}) = 6.67 \cdot 10^6(300 - 288) = 80.04 MW$$

$$\epsilon = \frac{Q}{Q_{max}} = 0.83$$

For an evaporator we use the relationship:

$$NTU = -\ln(1 - \epsilon) = 1.77$$

Finally:

$$NTU = \frac{UA}{C_{min}}$$

$$A = \frac{NTU \cdot C_{min}}{U} = \frac{1.776.67 \cdot 10^6}{1200} = 9838.25m^2$$

The extremely large mass flow rates and surface area are due to the very small temperature differences of the fluids in the heat exchanger.

$$A = \pi D \cdot 2L \cdot N$$

$$N = \frac{A}{\pi D \cdot 2L} = \frac{9838.25}{\pi 0.1 \cdot 2 \cdot 10} = 1565.8 \approx 1566$$

WRONG SOLUTION with LMTD approach - 1.5 pt: 0.25 T_1; 0.25 T_2; 0.25 A expression; 0.25 A value; 0.25 N expression; 0.25 N value.

$$Q = UA\Delta T_{lm} = UA\frac{\Delta T_2 - \Delta T_1}{ln\frac{\Delta T_2}{\Delta T_1}}$$

where:

- $\Delta T_2 = T_{h,out} - T_{c,in} = 290 - 288 = 2K$
- $\Delta T_1 = T_{h,in} - T_{c,out} = 300 - 288 = 12K$

Hence:

$$A = \frac{Q}{U\Delta T_{lm}} = \frac{66.7 \cdot 10^6 W}{1200W/m^2K \cdot 5.58} = 9961.17m^2$$

Considered that:

$$A = \pi D \cdot 2L \cdot N$$

we get:

$$N = \frac{A}{\pi D \cdot 2L} = 1585.37 \approx 1586$$