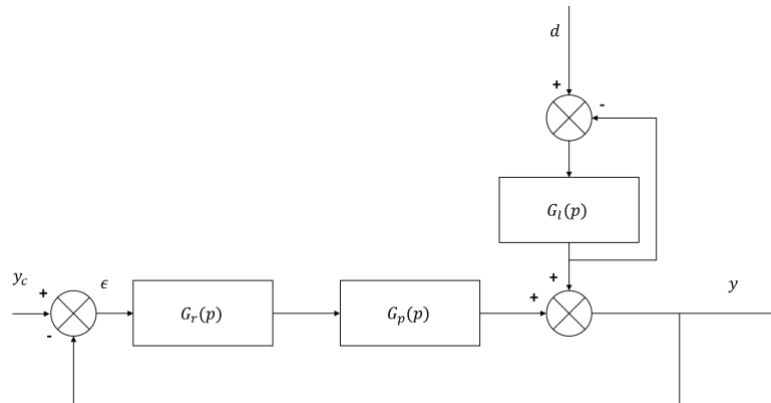


1. **Regulation and Control (35 points)** Consider the control system shown below. Given the transfer functions $G_l(s) = \frac{1}{s^2+2s-1}$ and $G_p(s) = \frac{1}{(s+2)}$

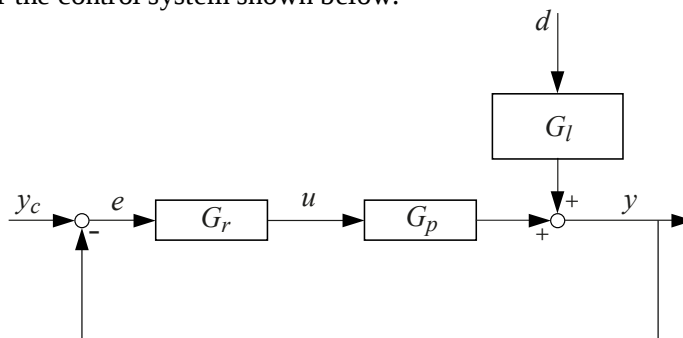


- Calculate the transfer function $H_c = \frac{Y}{Y_c}$.
 - Design a PID controller that eliminates the permanent error between the reference signal $y_c(t)$ and the output $y(t)$. Find an equation on the parameters of your controller so that the poles of your closed-loop system have the *fastest aperiodic* response.
 - Calculate the transfer function $H_d = \frac{Y}{D}$.
 - Does the controller designed in a) eliminates the influence of step disturbance $d(t) = D_0 e(t)$ at $t \rightarrow \infty$?
 - We want the perturbation error at infinity to respect a margin: $\frac{y_\infty - D_0}{D_0} \leq 0.1$. Derive a condition on the PID parameters to verify this.
 - Additionally, physical constraints impose $K = 10$. It is more expensive to reduce τ_I , as it requires more accurate instruments. Choose the cheapest τ_I respecting e), and from the constraint derived in b), determine τ_D for having a controller with the *fastest aperiodic* response.
2. **(20 points)** Consider the following dynamical system

$$y^{(iv)}(t) + 2y^{(iii)}(t) + 2y^{(ii)}(t) + 4y^{(i)}(t) + 4y(t) = 2u^{(i)}(t) + u(t),$$

with $y(t)$ being the output, $u(t)$ the input of the system, and $\cdot^{(k)}$ denotes the k -th derivative.

- Calculate the static gain and the transfer function of the system.
 - Determine the stability of the system. If the system is unstable, determine how many of its poles have a positive real part.
 - The closed-loop system is controlled by a P controller. Find the range of the controller's parameter that ensures the stability of the closed-loop system.
3. **20 points)** Consider the control system shown below.



The behavior of a closed-loop system is specified by the transfer function $G_{BF}(s) = \frac{Y(s)}{Y_c(s)} = K_d \frac{\omega_0^2}{s^2 + 2\xi\omega_0 s + \omega_0^2}$ with the damping factor $\xi = 0.7$, and the natural frequency $\omega_0 = 1 \text{ rad/s}$.

- Choose K_d so that the steady-state error e between the reference signal $y_c(t)$ and the output $y(t)$ is equal to zero.
- Given $G_p(s) = \frac{K}{(s+1)(s+1.4)}$, calculate a controller to satisfy $G_{BF}(s)$. Express the parameters of the obtained controller knowing that it is in a PID form (K_R or K_R, τ_D or K_R, τ_I or K_R, τ_I, τ_D).
- For $G_l(s) = \frac{1}{s^2(s+1)}$, does the controller obtained in b) eliminate the influence of disturbance $d(t)$ at $t \rightarrow \infty$?

4. **(20 points)** A dynamical system is given by its step response:

$$y(t) = 0.25 - 0.25e^{-t} \cos \sqrt{3} t + 0.25\sqrt{3}e^{-t} \sin \sqrt{3} t, \quad t \geq 0$$

- Calculate the transfer function of the system.
 - Calculate the static gain, the natural frequency, the damping factor and the time constant(s) of the system.
 - Calculate and sketch the impulse response of this system.
5. **(20 points)** On considère le réacteur batch de la figure 1. On souhaite garder la température du réacteur T constante à l'aide d'une commande en cascade qui manipule la température à l'entrée du manteau $T_{m,in}$.
- Dessiner le schéma fonctionnel d'une telle commande en cascade en identifiant toutes les variables du schéma.
 - On a estimé les fonctions de transfert $\frac{T_m(s)}{T_{m,in}(s)} = \frac{0,2}{5s+1}$ et $\frac{T(s)}{T_m(s)} = \frac{0,5e^{-s}}{10s+1}$. Calculer les régulateurs d'une commande PI-P avec les propriétés suivantes : (i) la boucle secondaire a une constante de temps de 1, et (ii) le régulateur primaire est calculé selon les règles de Ziegler-Nichols.
 - Bonus (5 points):** Est-il possible de déstabiliser le système commandé en augmentant le gain du régulateur P (justification qualitative suffit) ?

