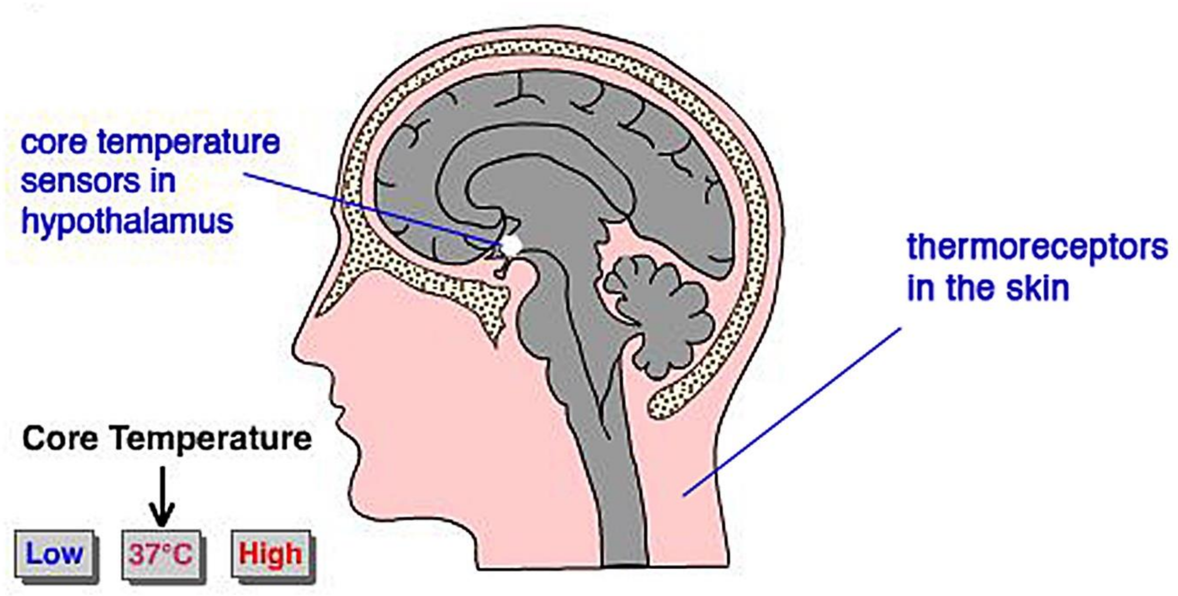
Introduction to modeling and control

What is a control system?

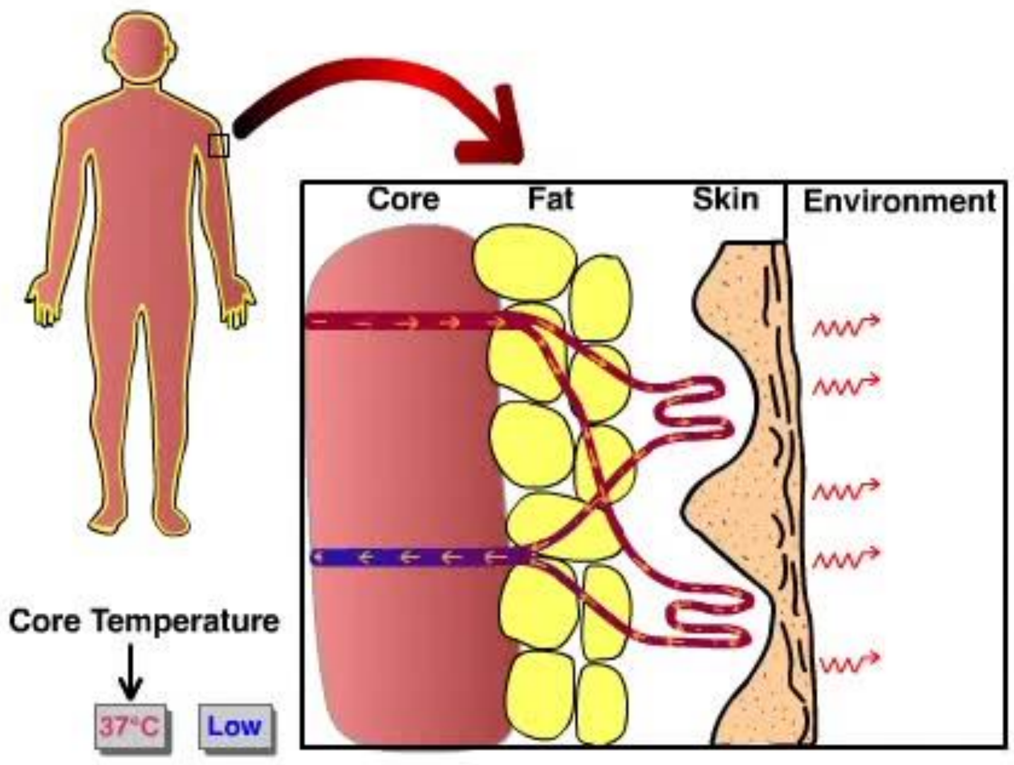
- In humans, the body temperature is controlled by the thermoregulatory center in the hypothalamus



Regulation of Body Temperature

<u>Exposure to cold</u>

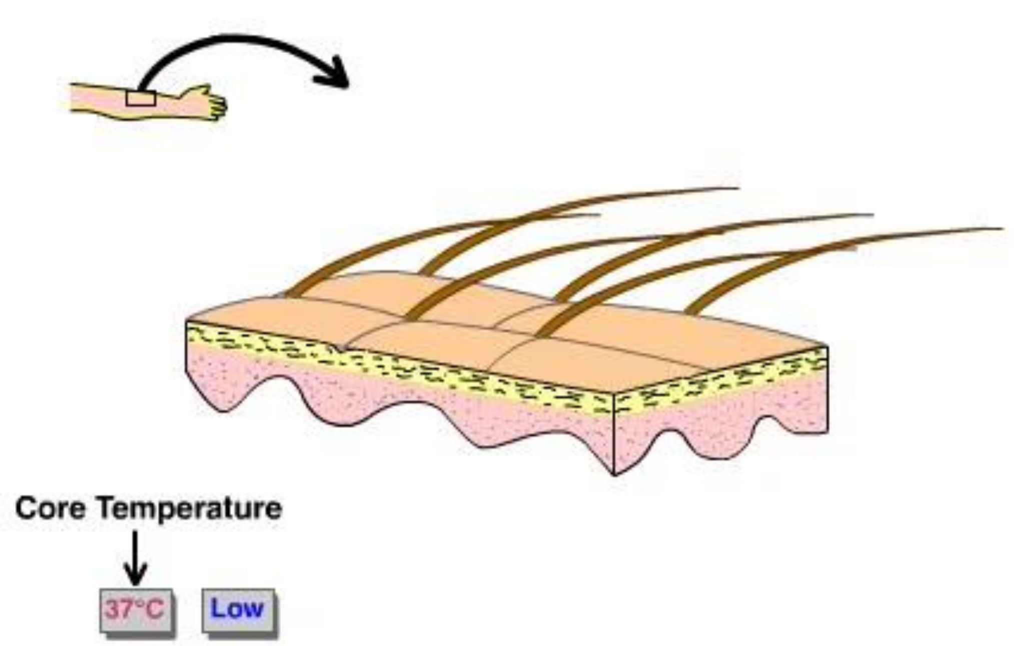
- Shivering
- Decreased blood flow to skin



Regulation of Body Temperature

<u>Exposure to cold</u>

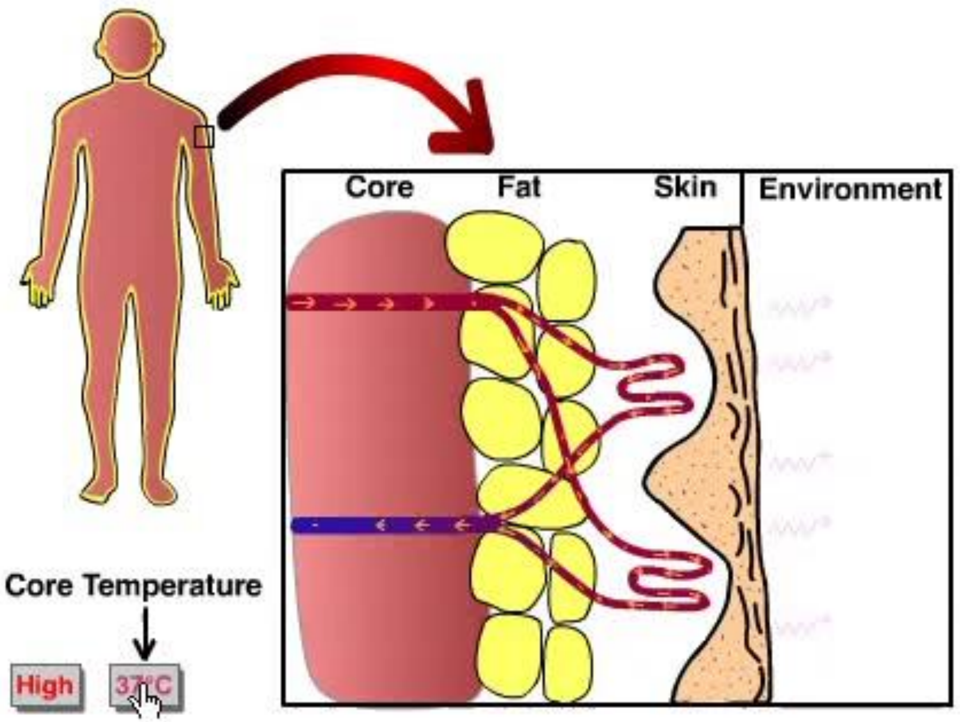
- Shivering
- Decreased blood flow to skin
- Goose bumps
- Behavioral changes



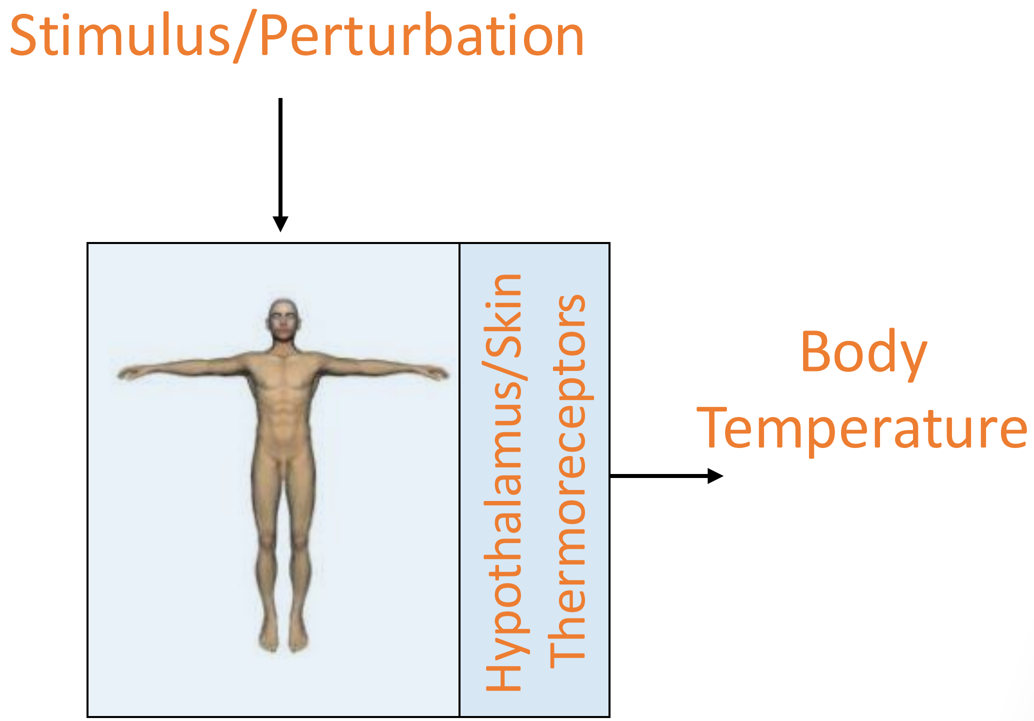
Regulation of Body Temperature

<u>Exposure to heat</u>

- Blood vessels dilate
- Sweating
- Behavioral changes



Negative feedback in body temperature regulation



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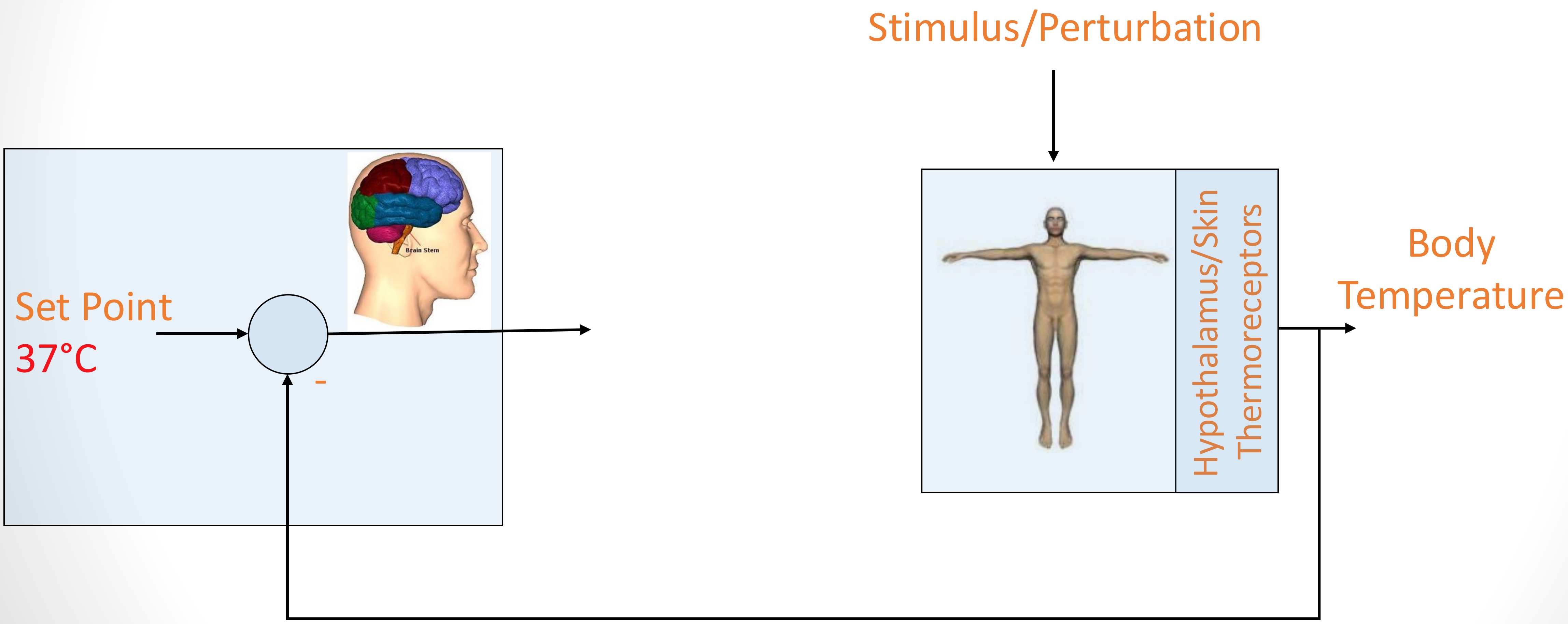

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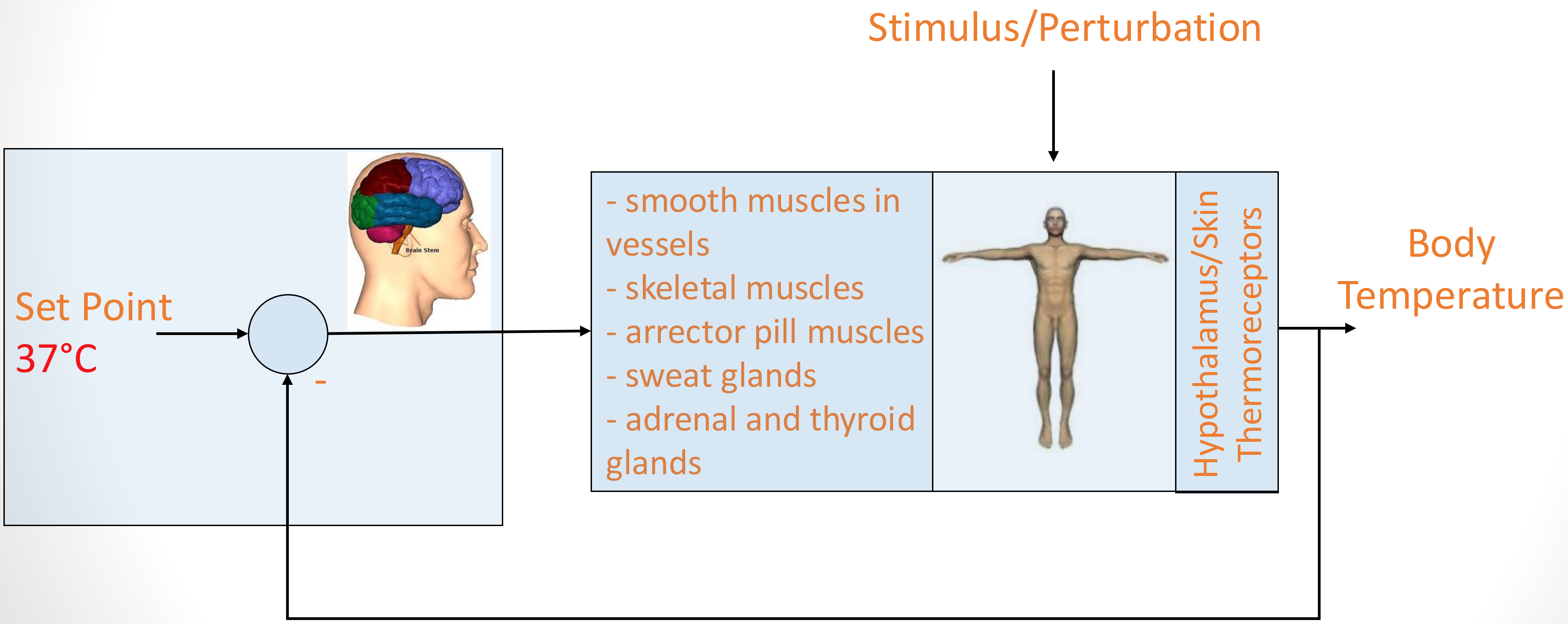

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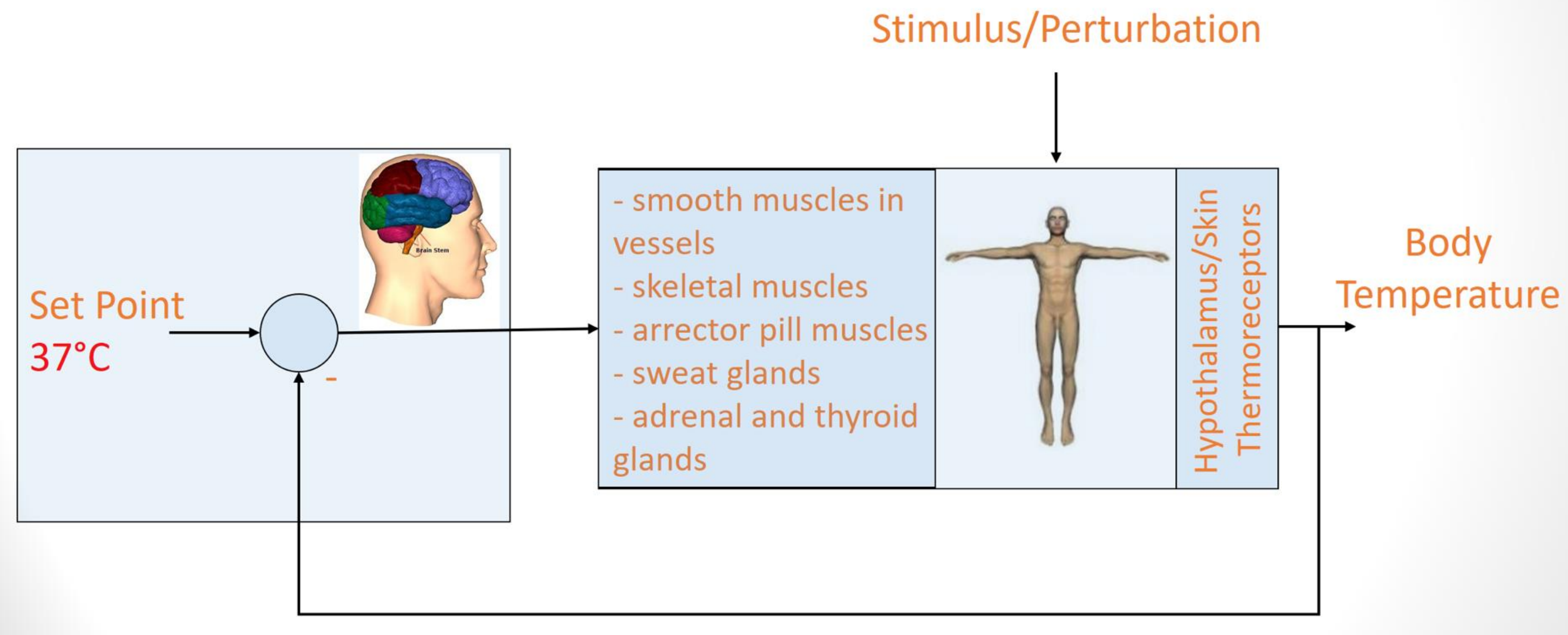
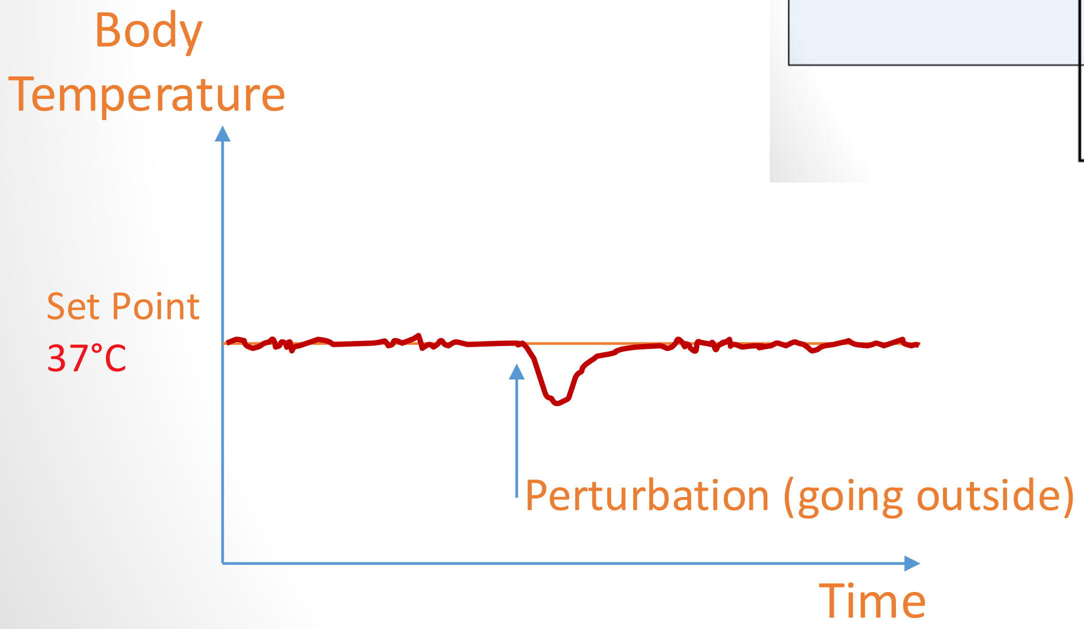
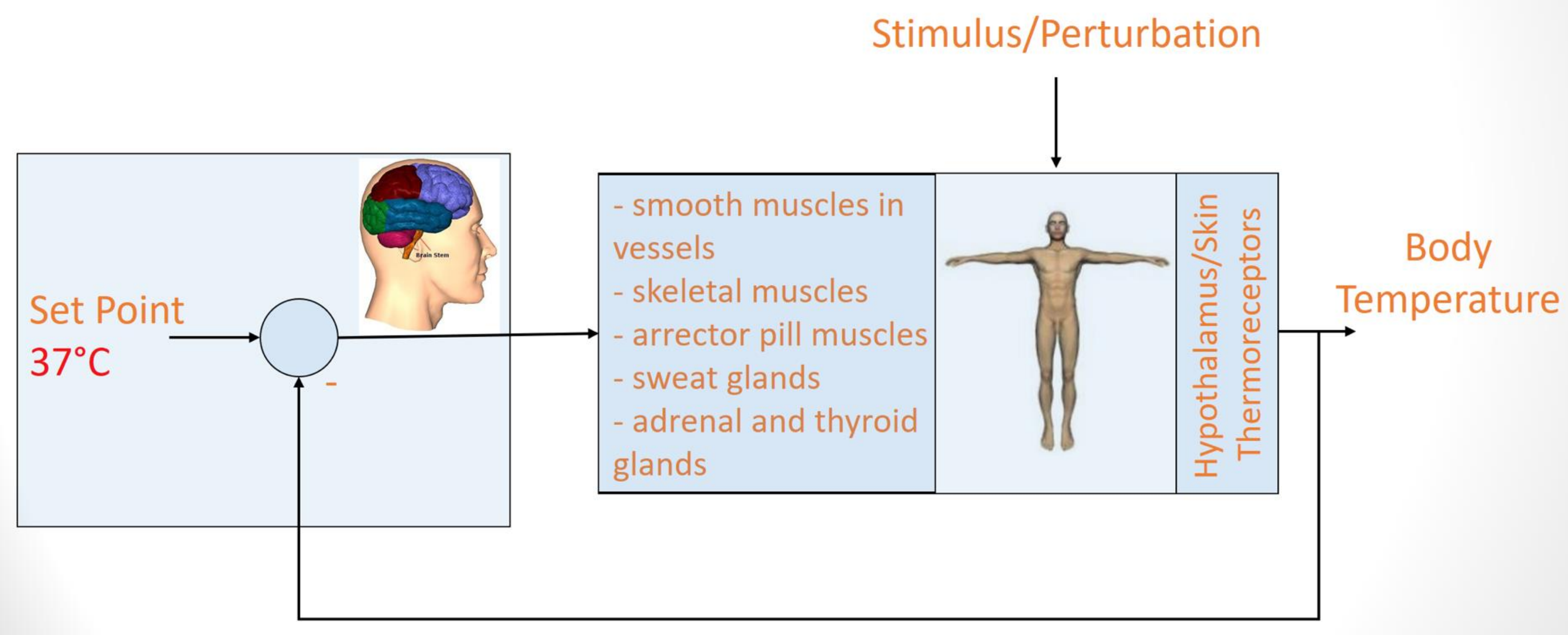
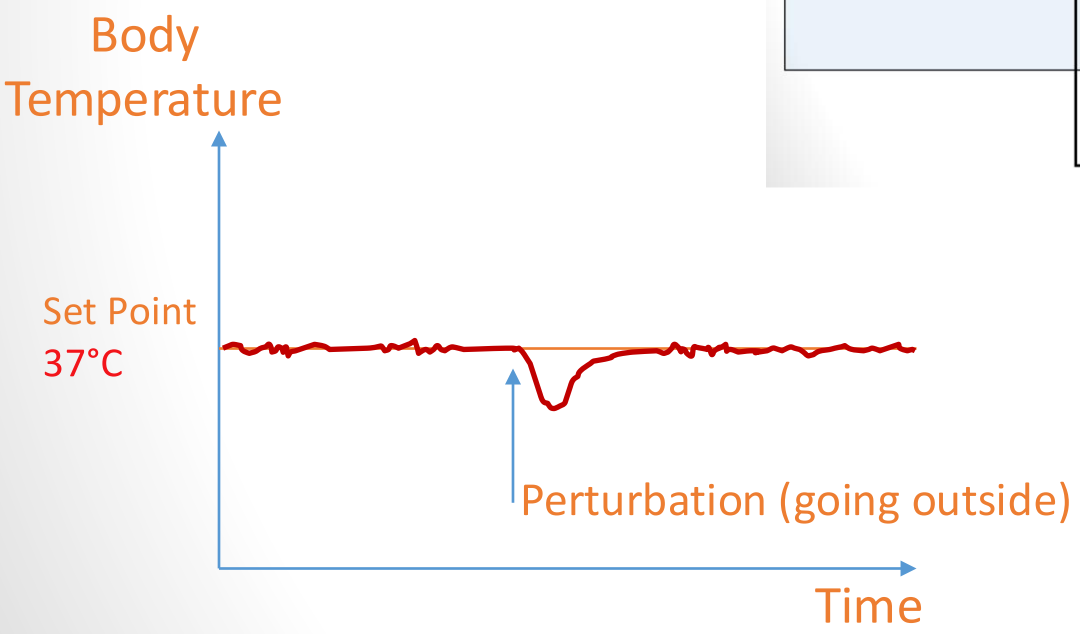




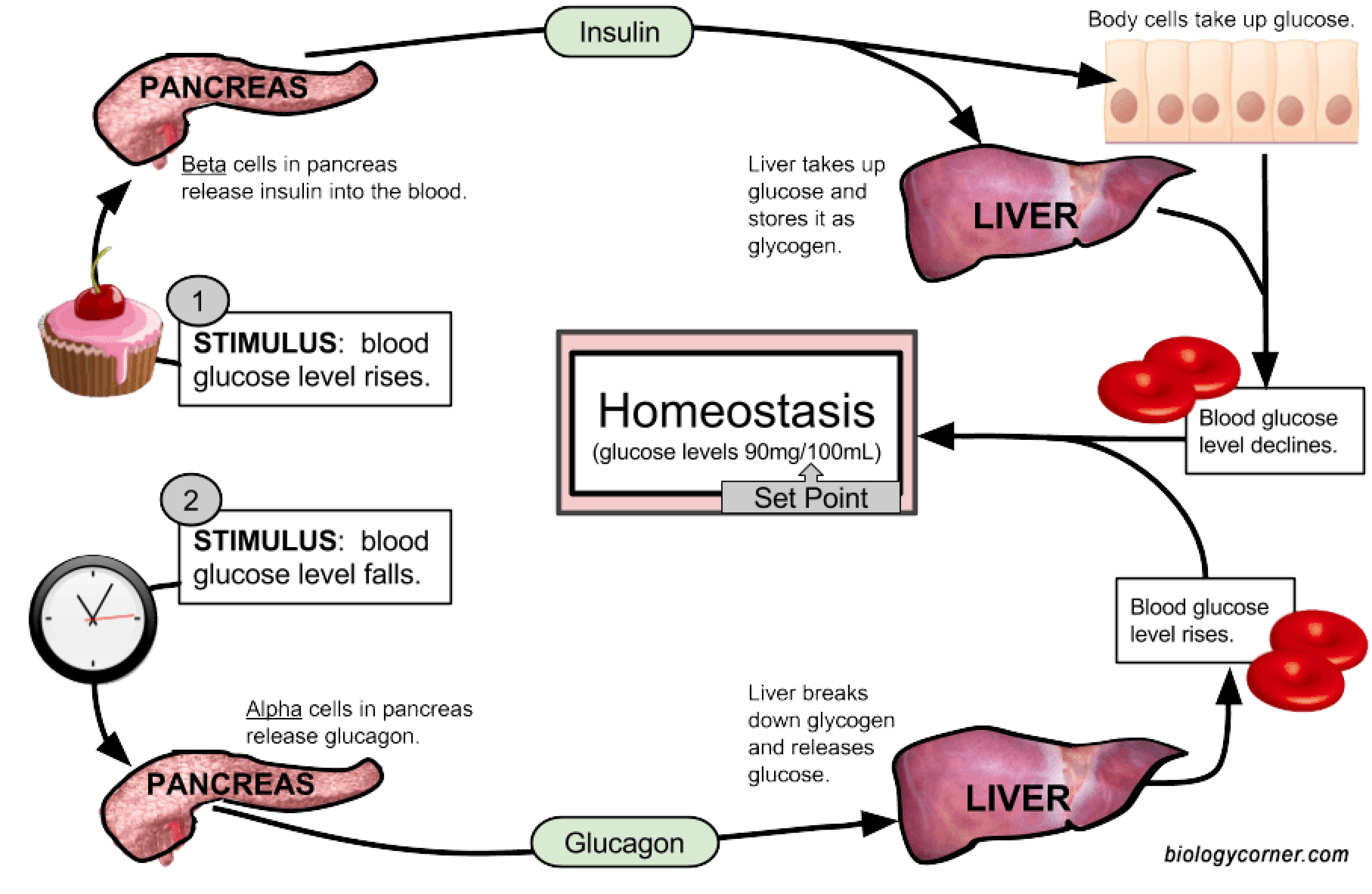
Negative feedback in body temperature regulation



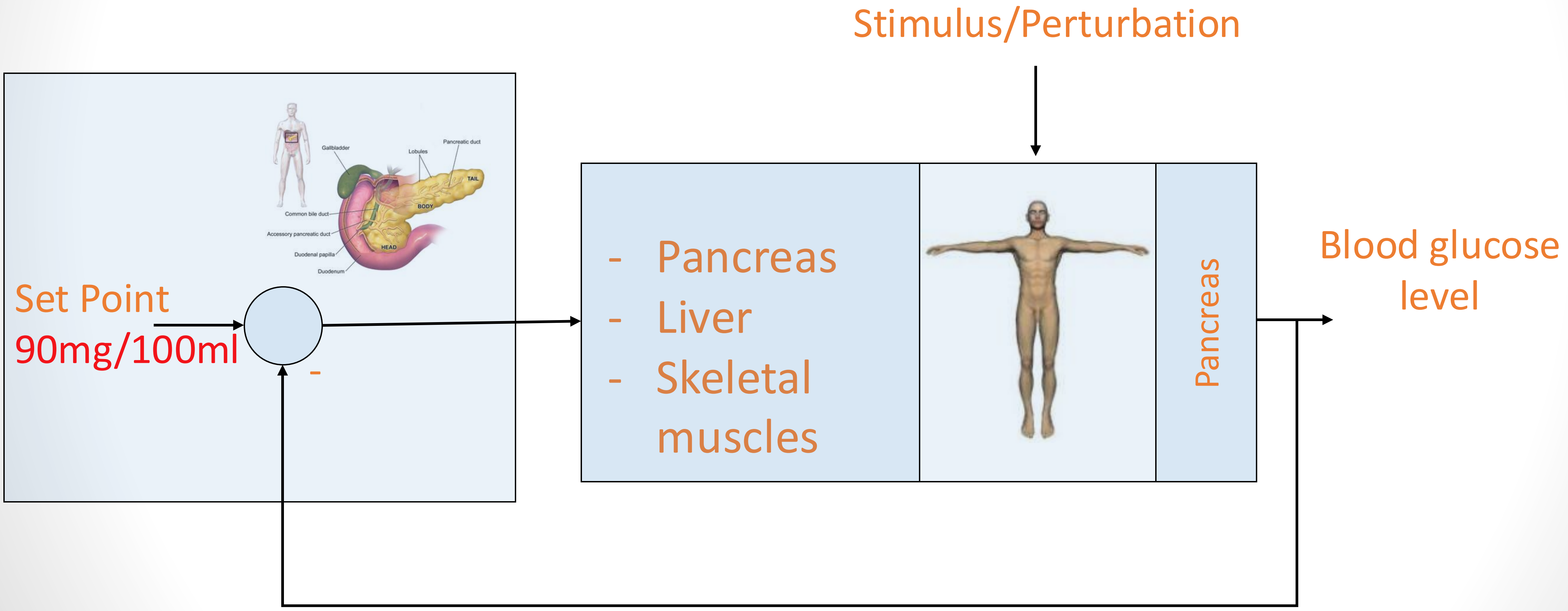


Negative feedback counteracts the change and brings the system back to the set point!

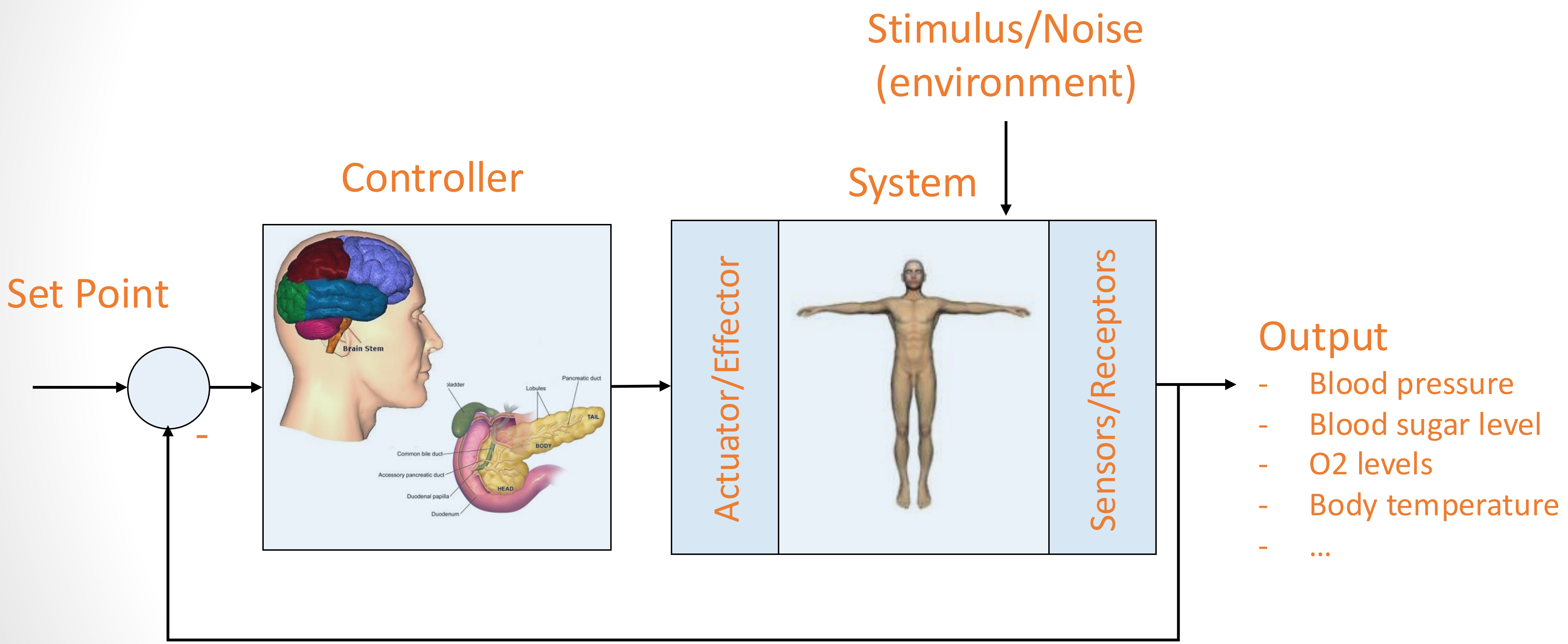
Negative feedback



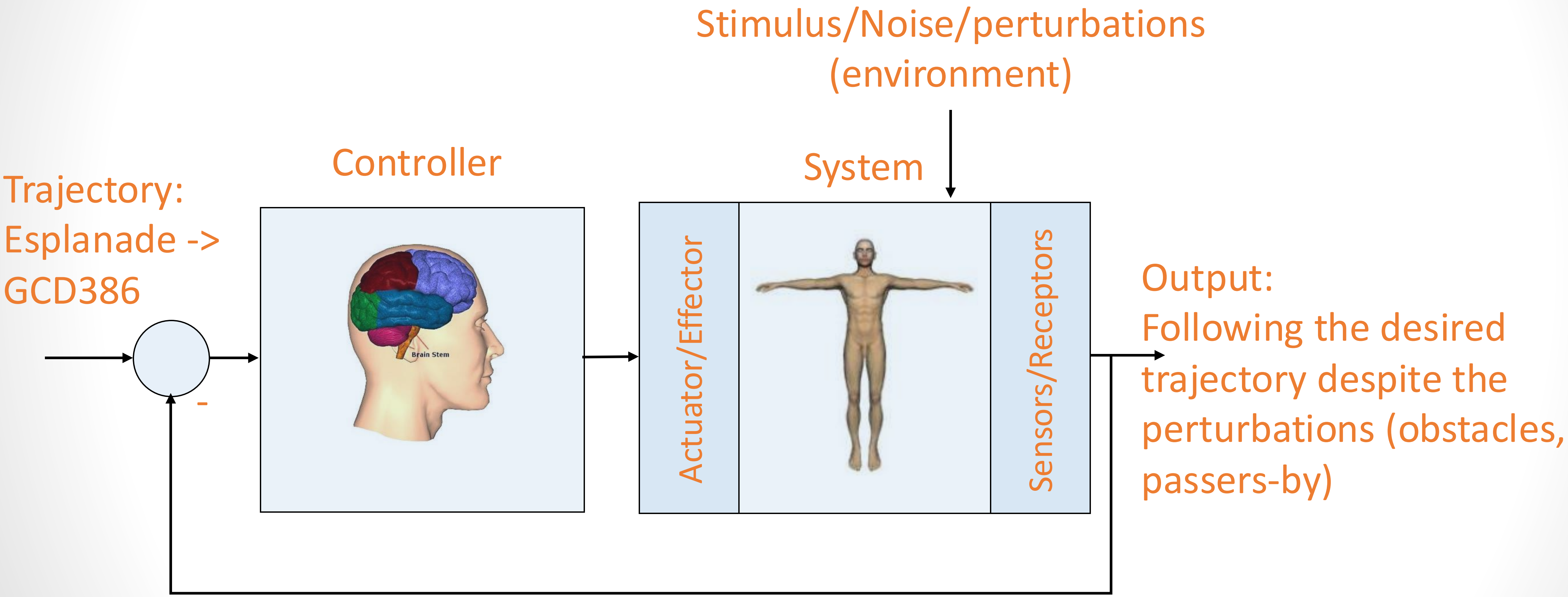
Negative feedback in glucose level regulation



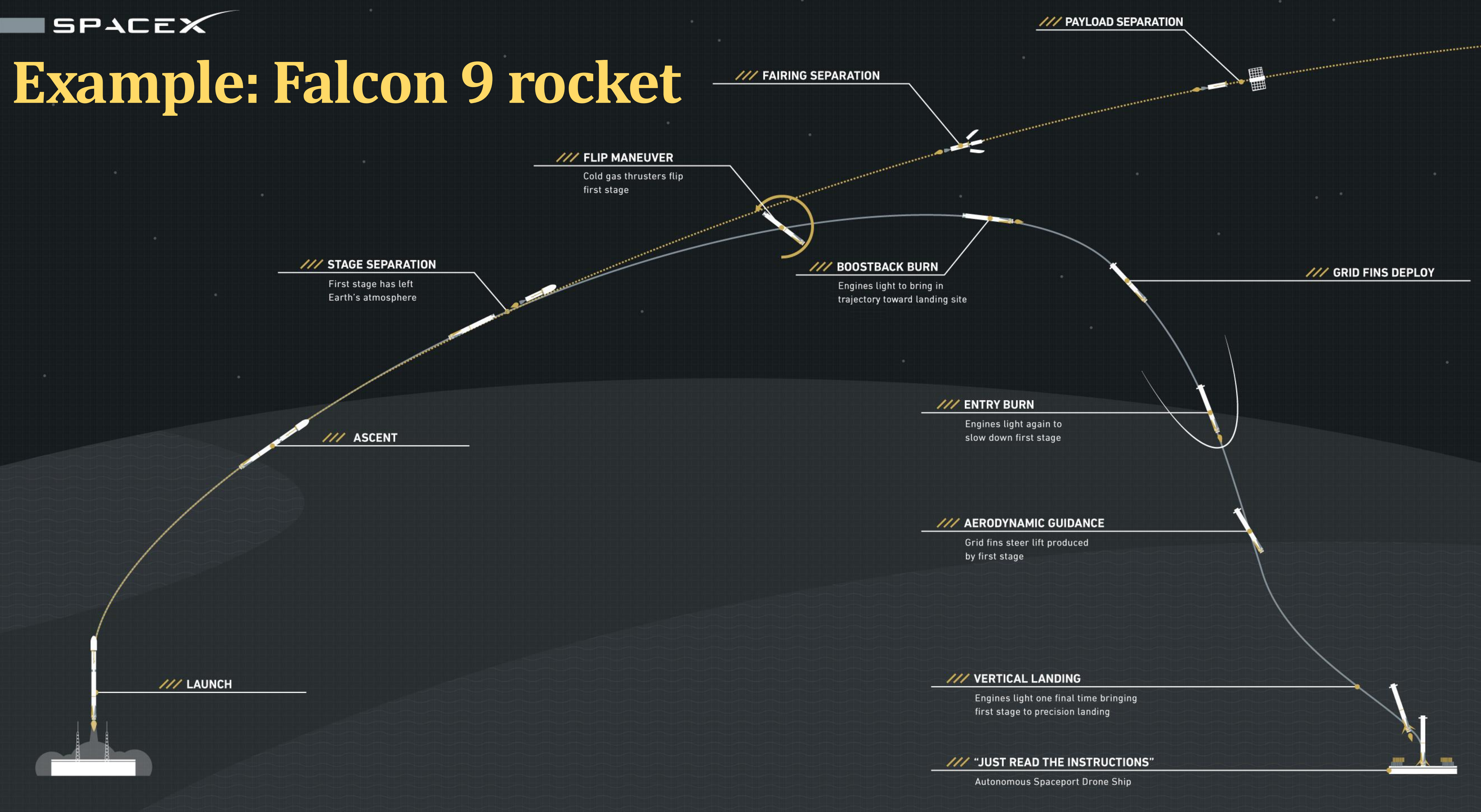
Negative feedback



Negative feedback in tracking a trajectory



Example: Falcon 9 rocket



Example: Space X Falcon 9 rocket

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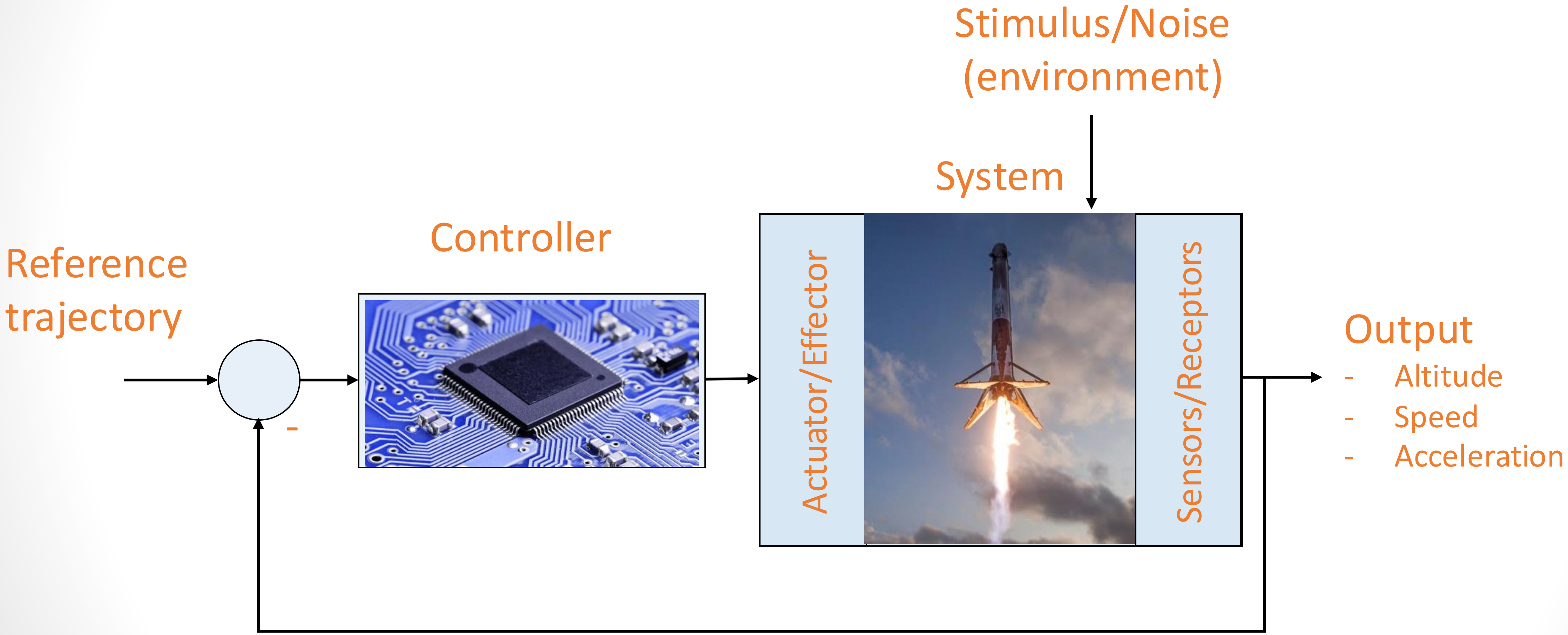
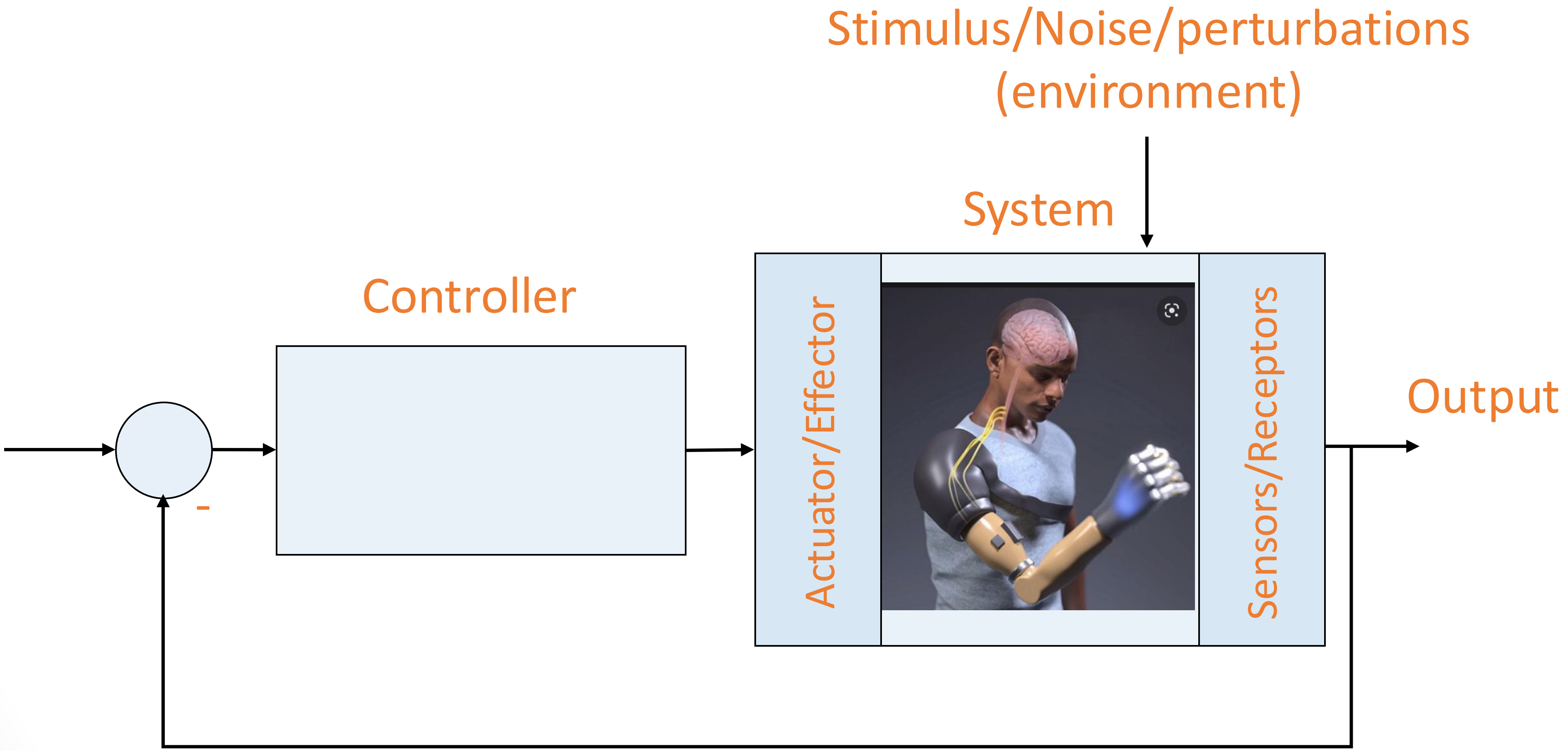
Negative feedback



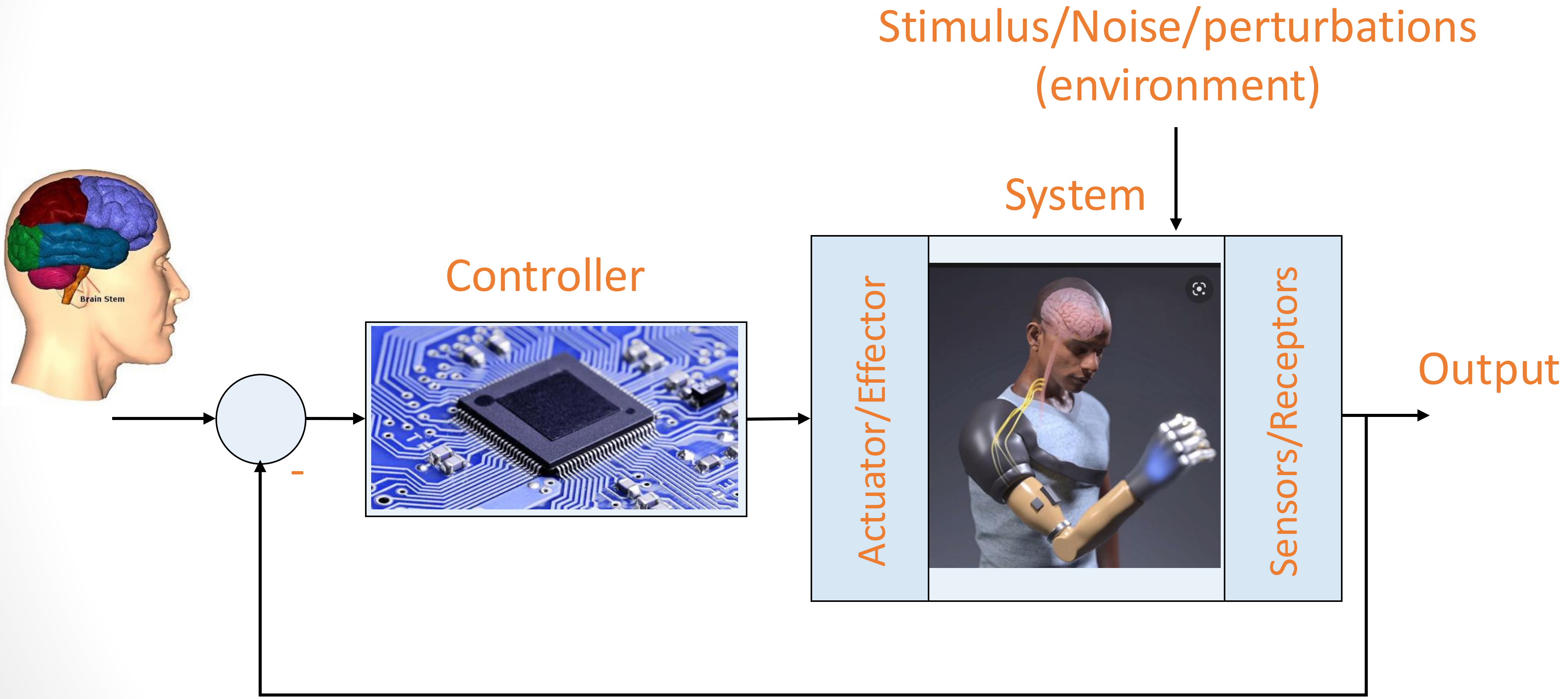
Yet Another Example

Artificial limb control

Negative feedback in artificial limb control

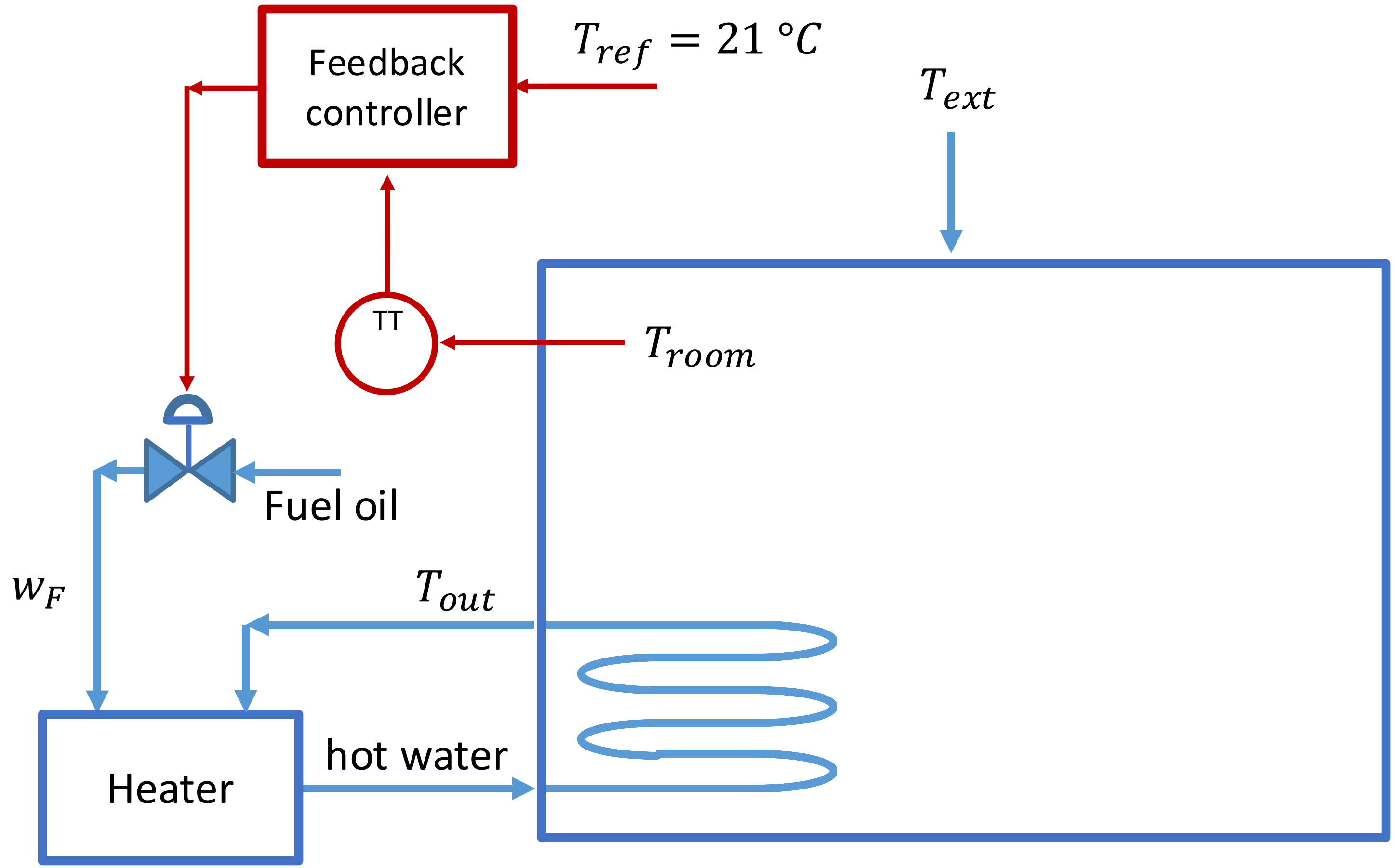


Negative feedback in artificial limb control



Temperature regulation of a room

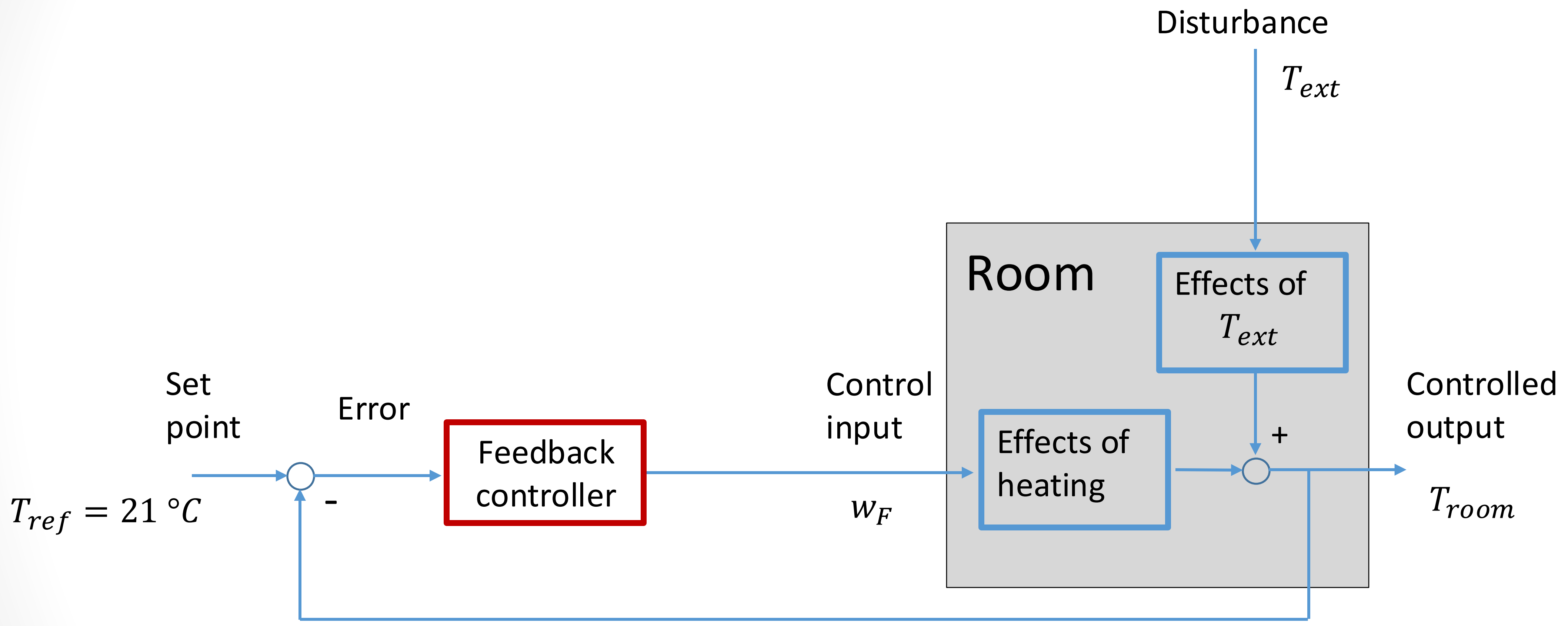
with feedback controller



- Feedback controller will correct all perturbations in the system

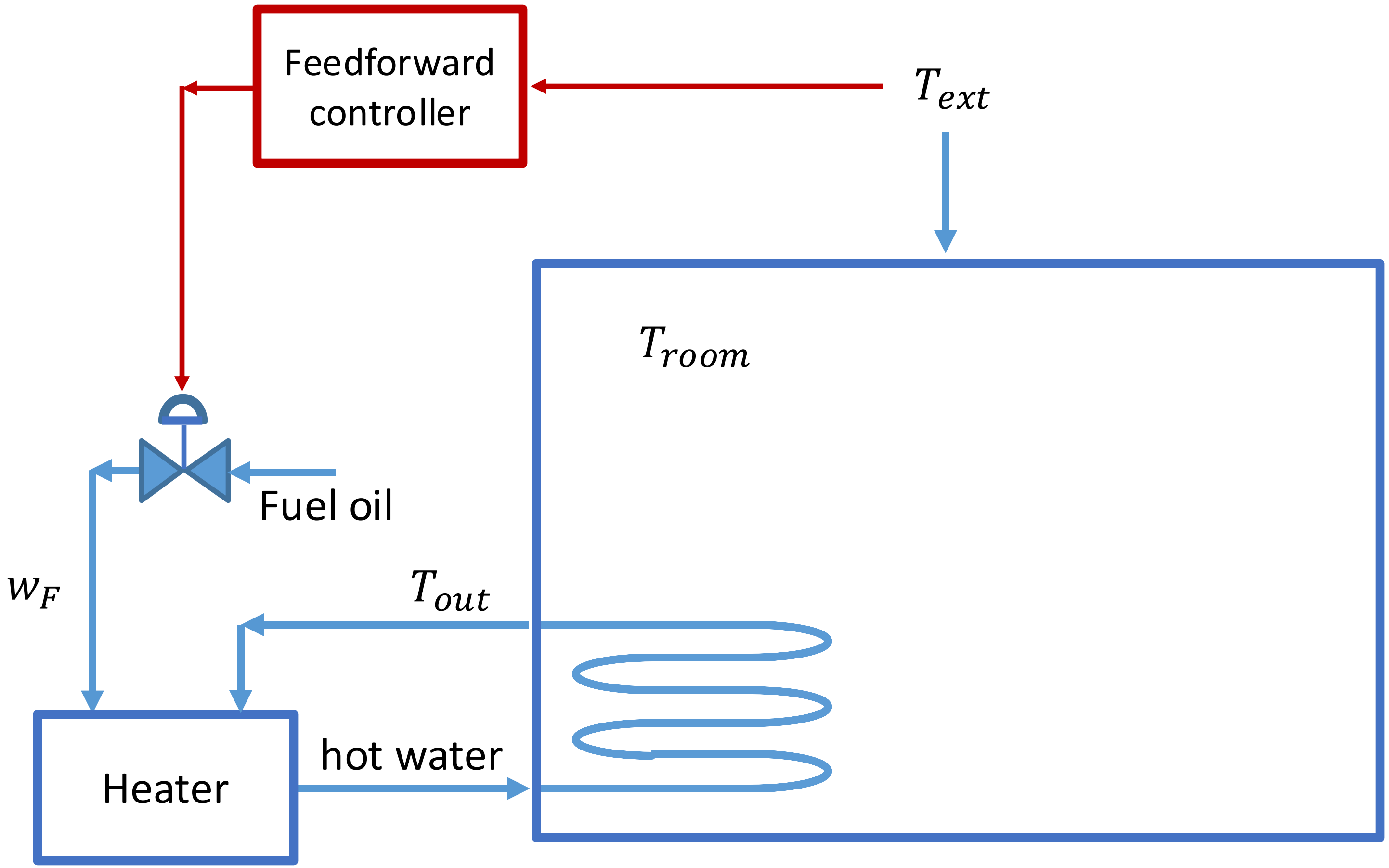
Scheme of a control system

with feedback controller



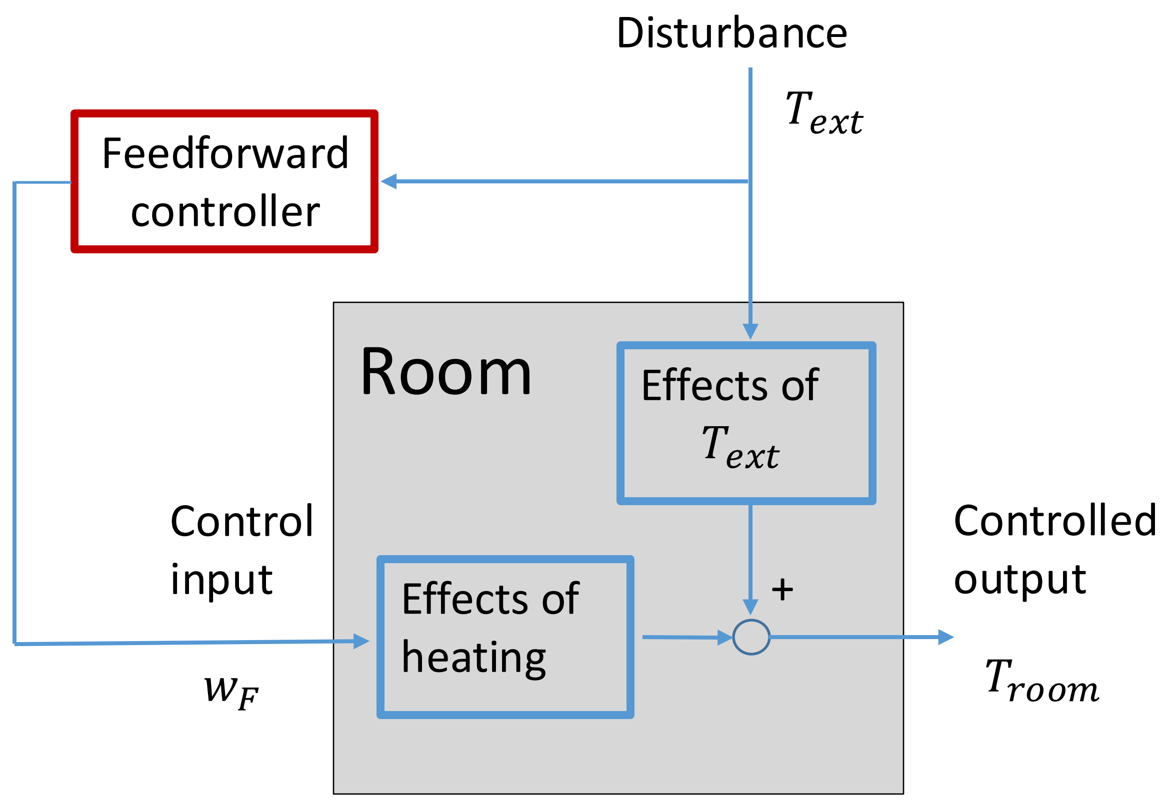
Temperature regulation of a room

with feedforward controller



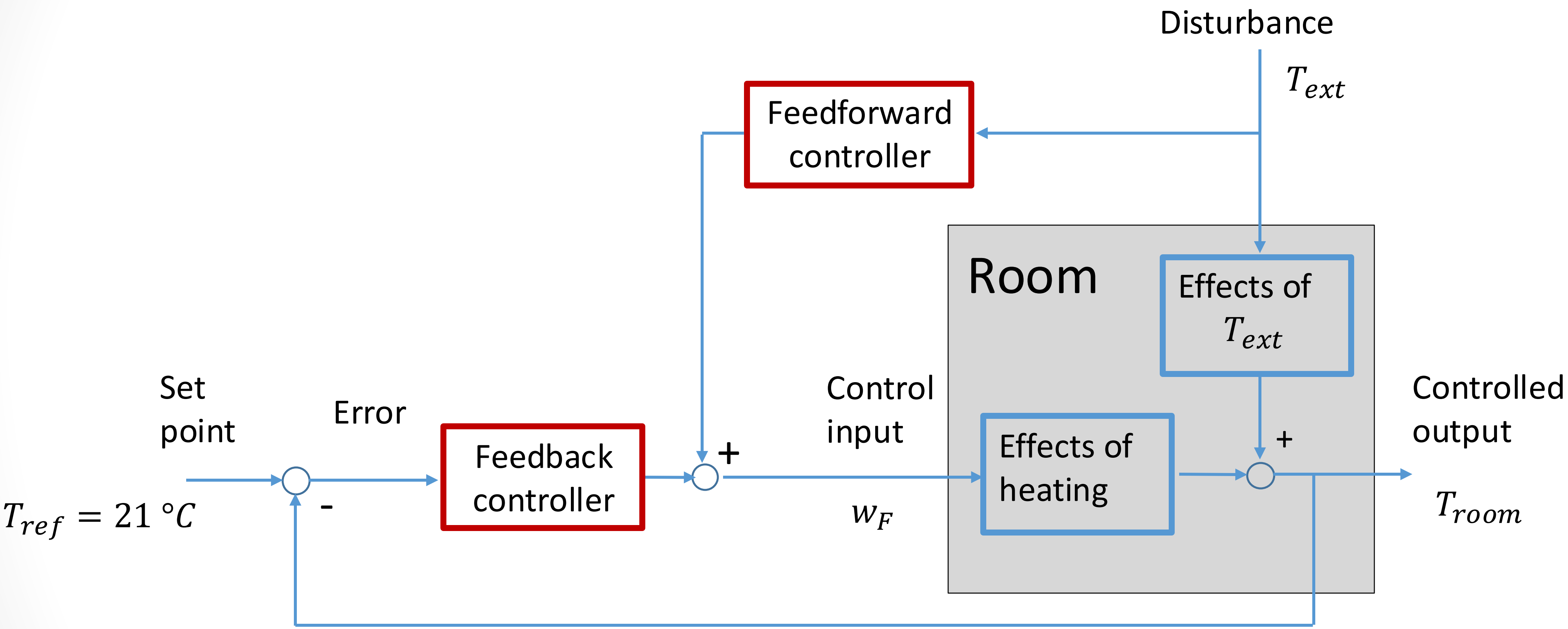
Scheme of a control system

with feedforward controller

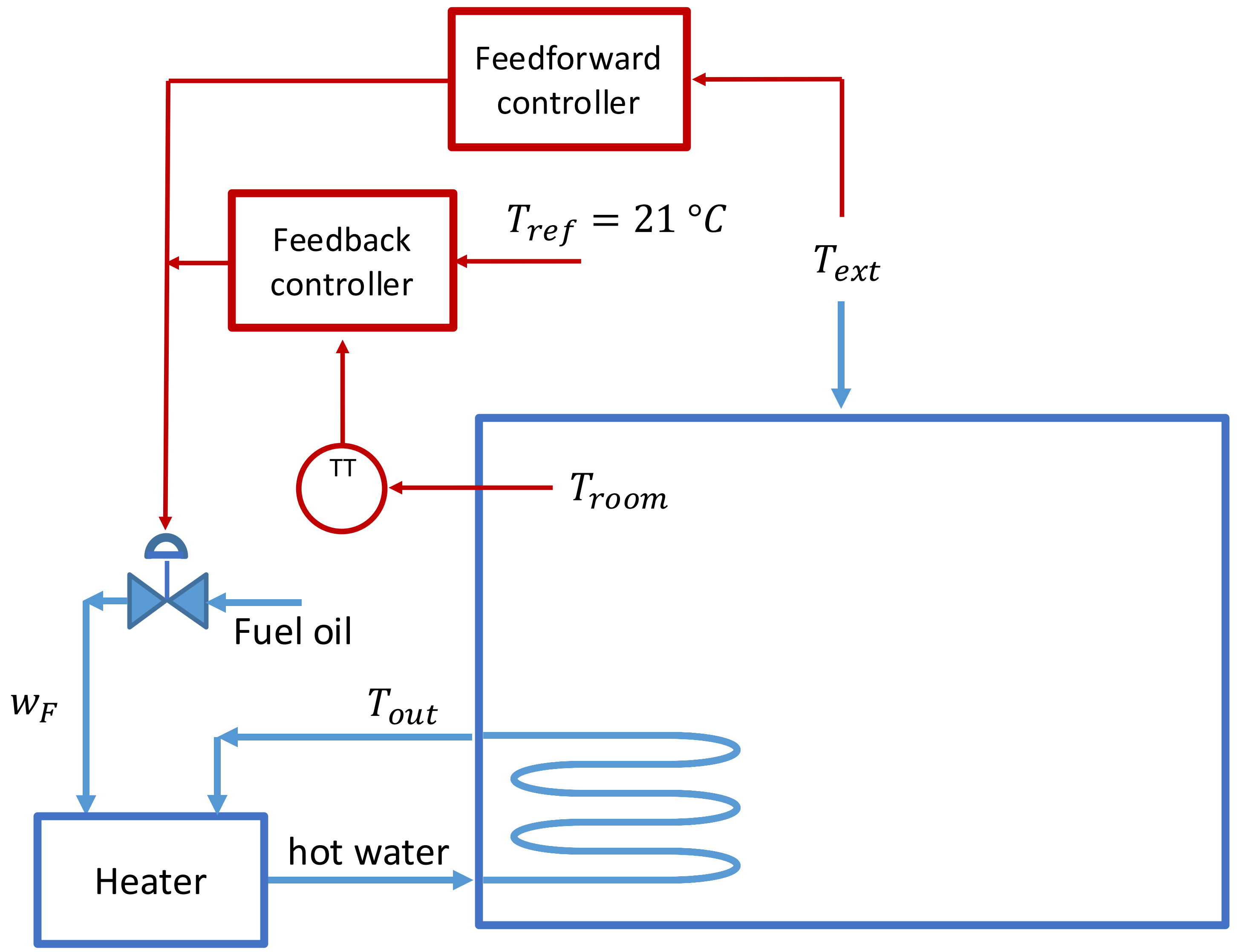


Scheme of a control system

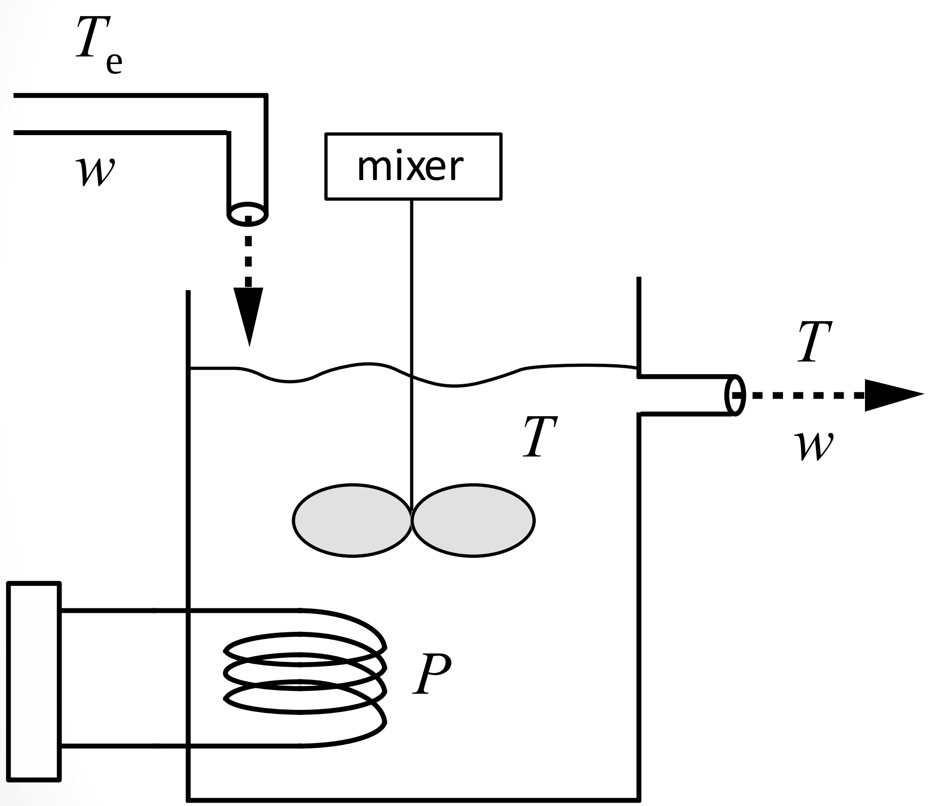
with feedback and feedforward controller

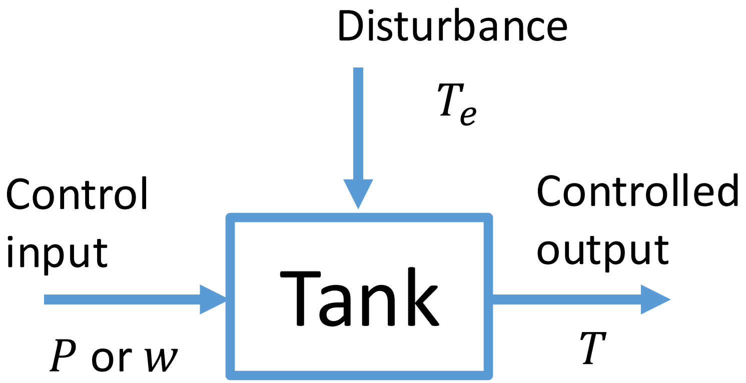


Temperature regulation of a room



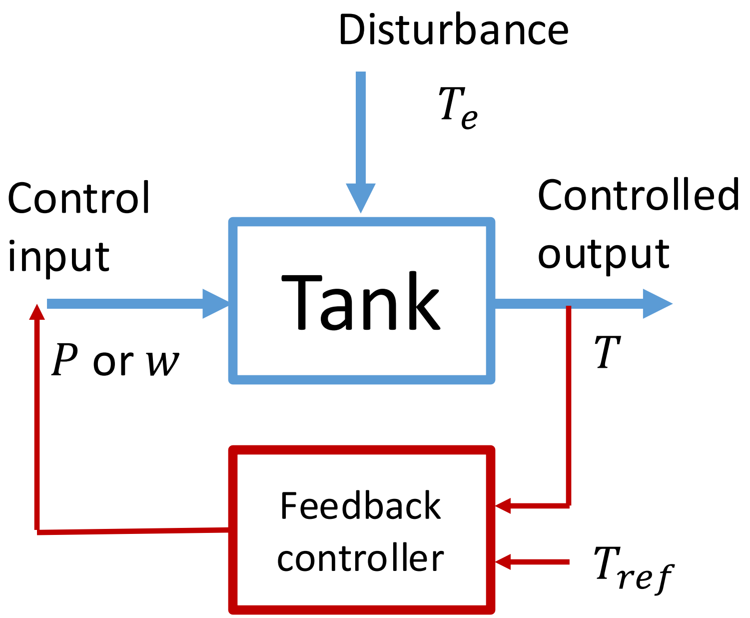
Temperature regulation of a tank



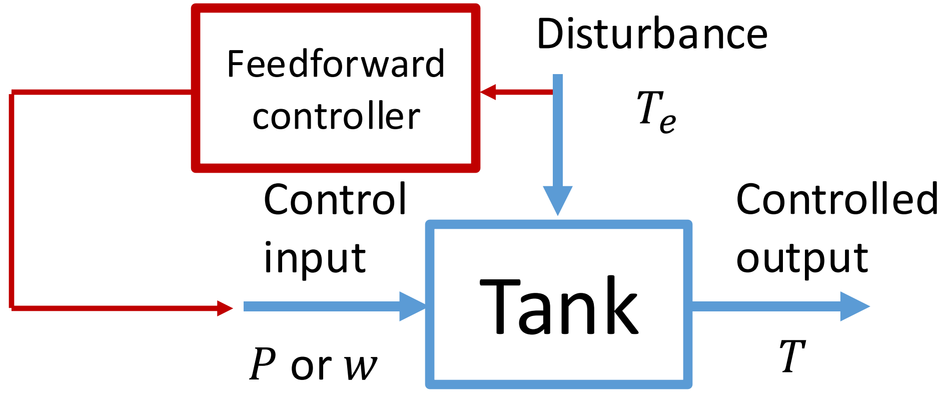


Possible control systems

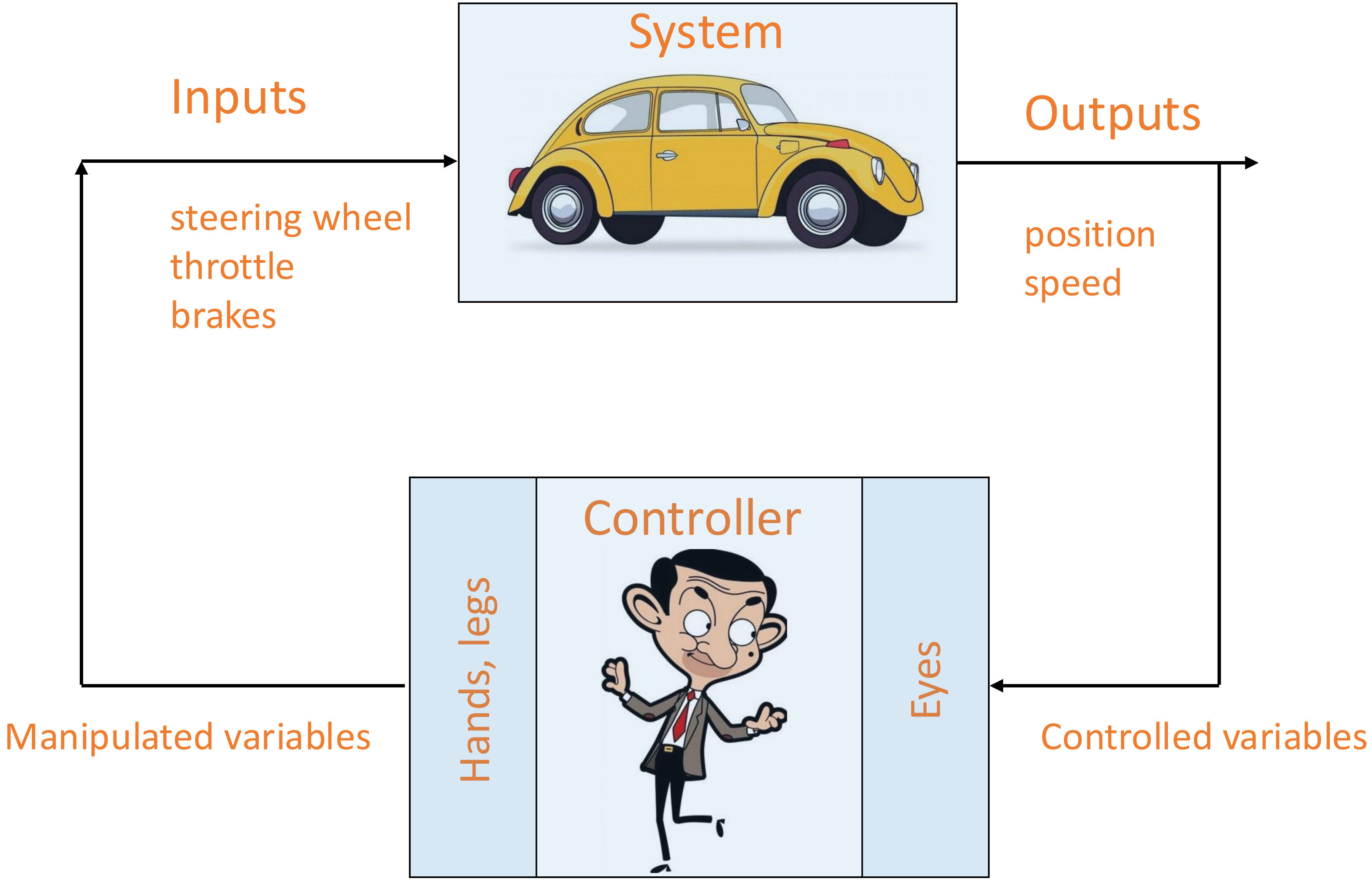
Negative feedback



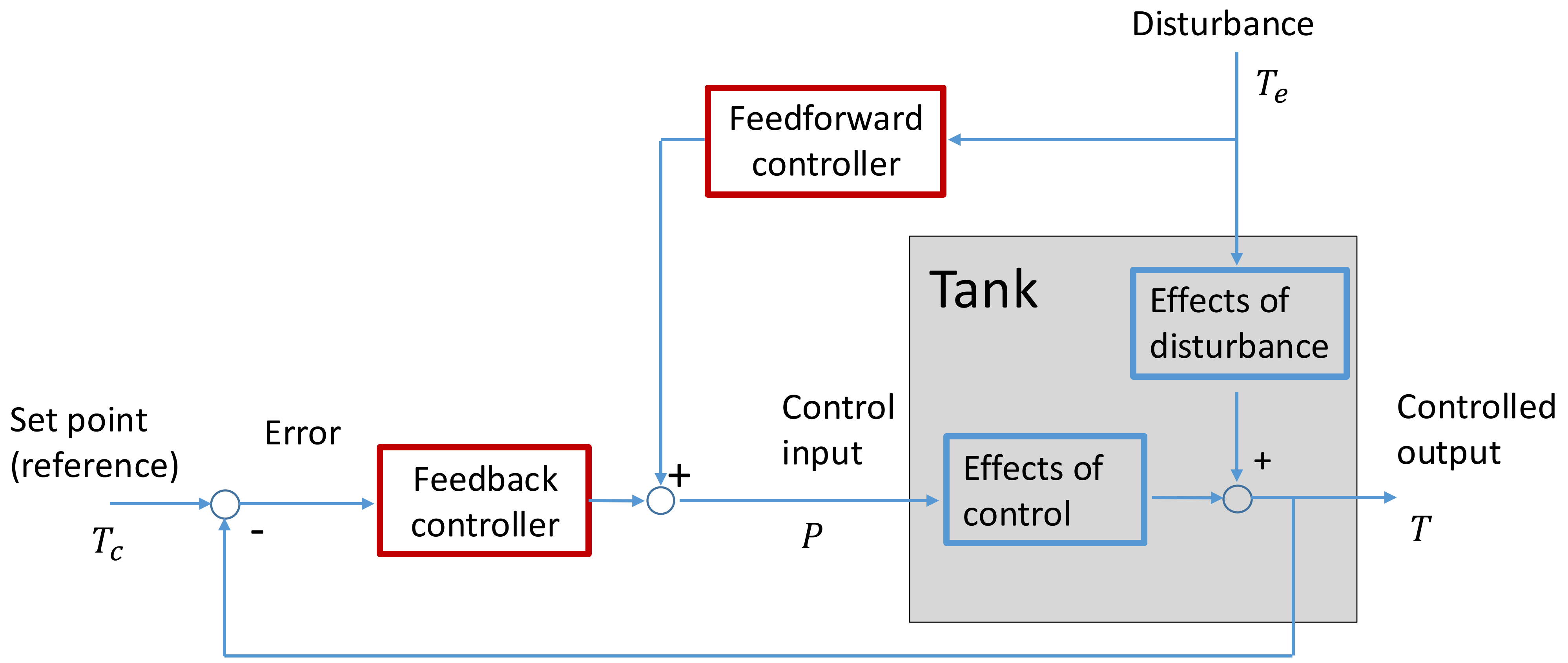
Disturbance rejection (anticipation)



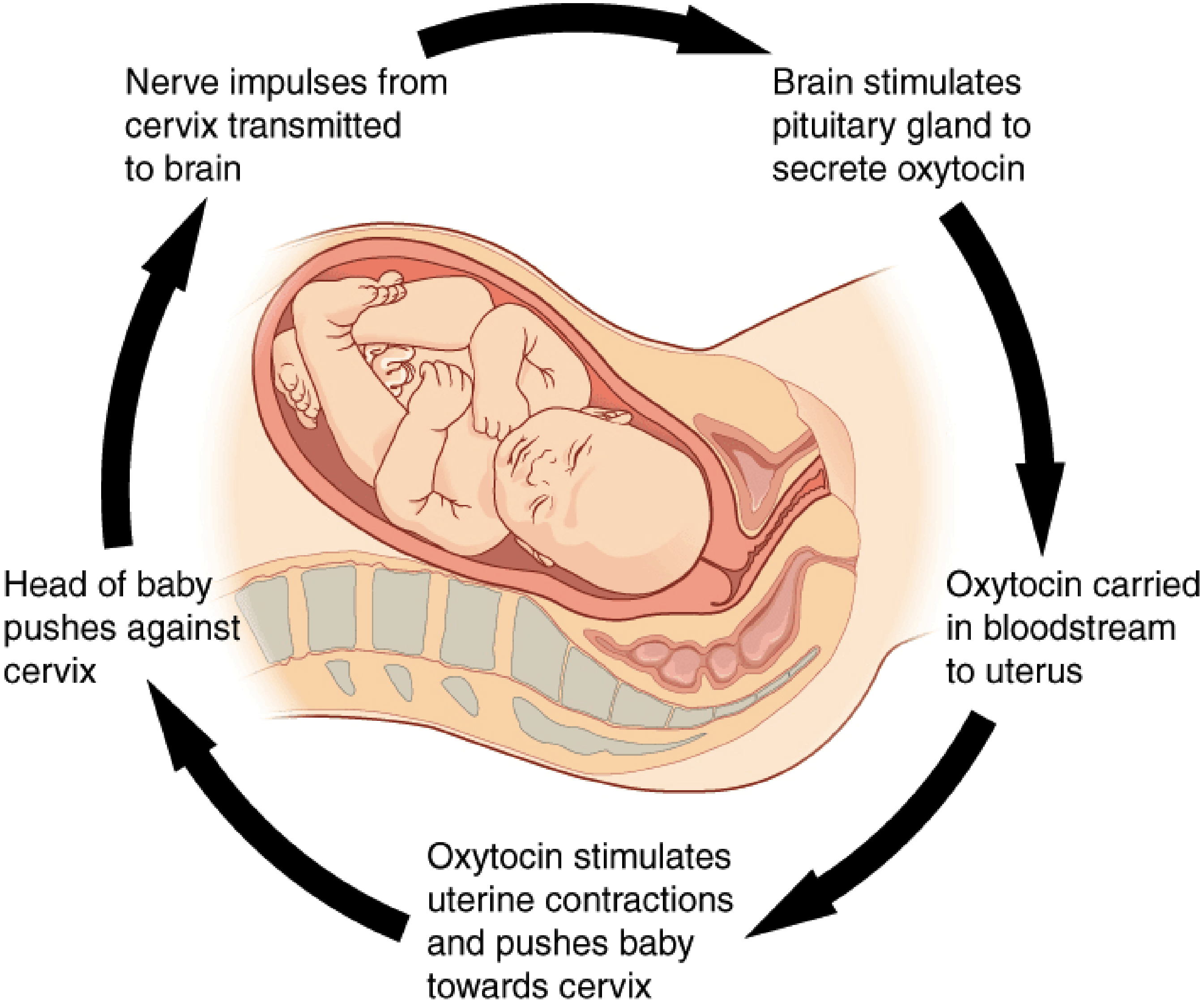
Why “Automatic” control?



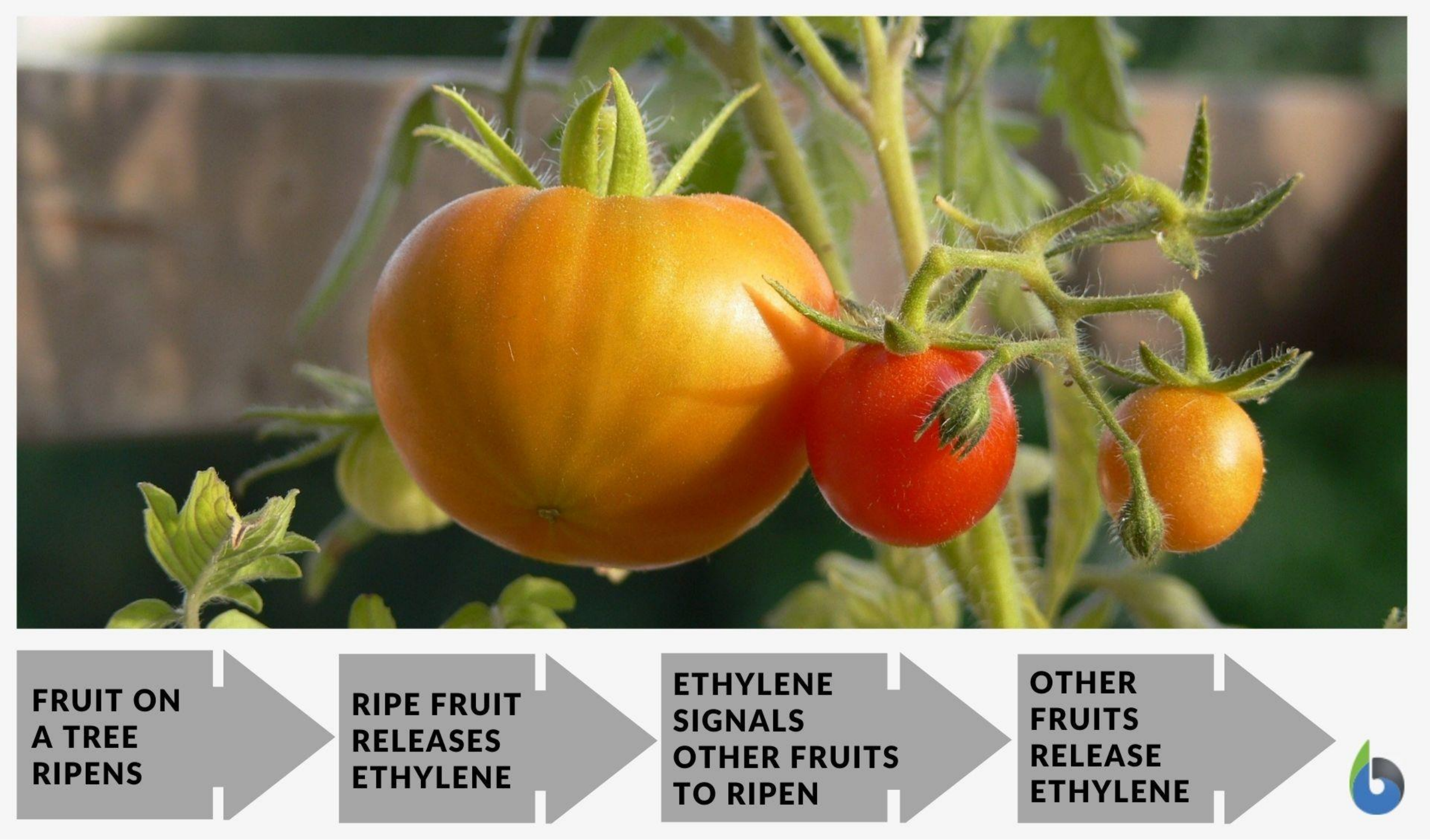
Summary: elements of a control loop



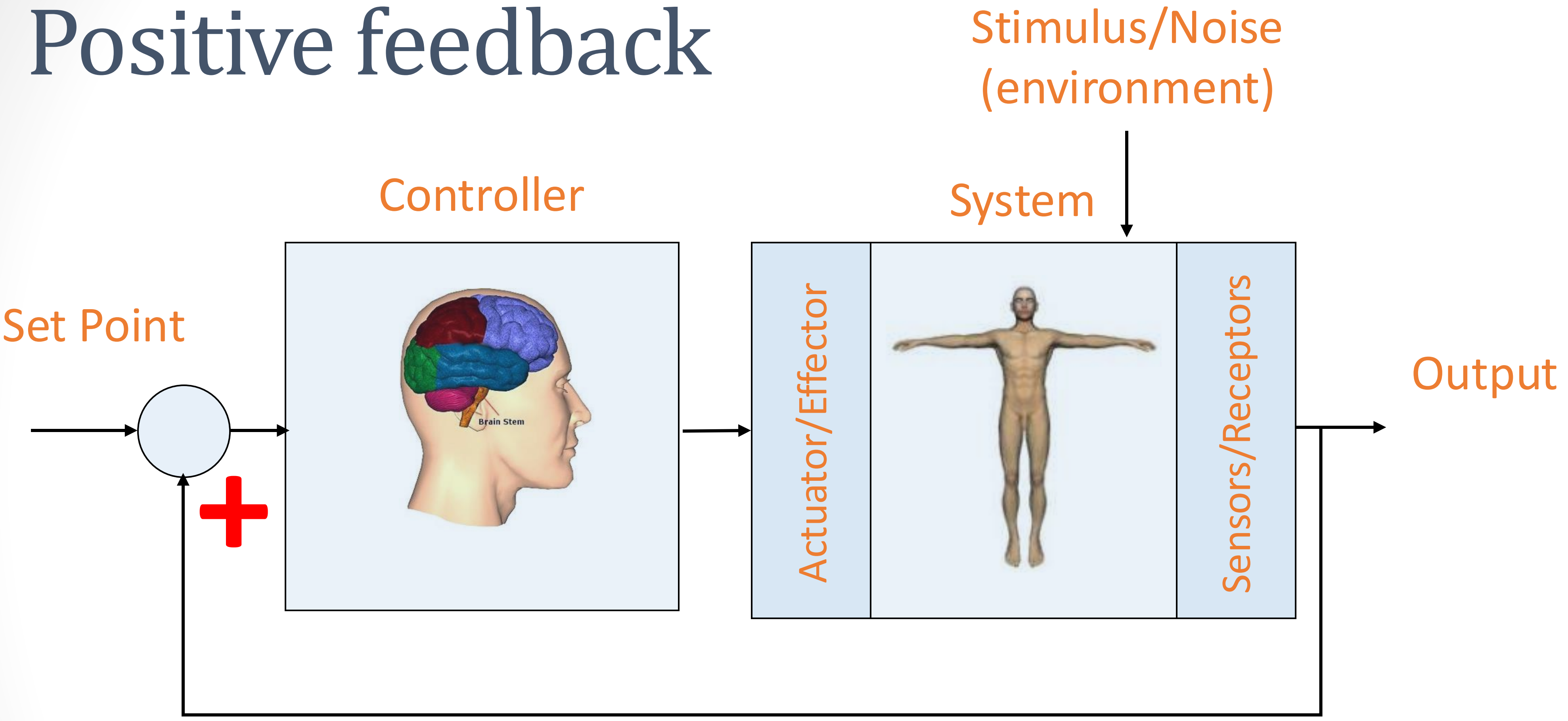
Positive feedback



Positive feedback



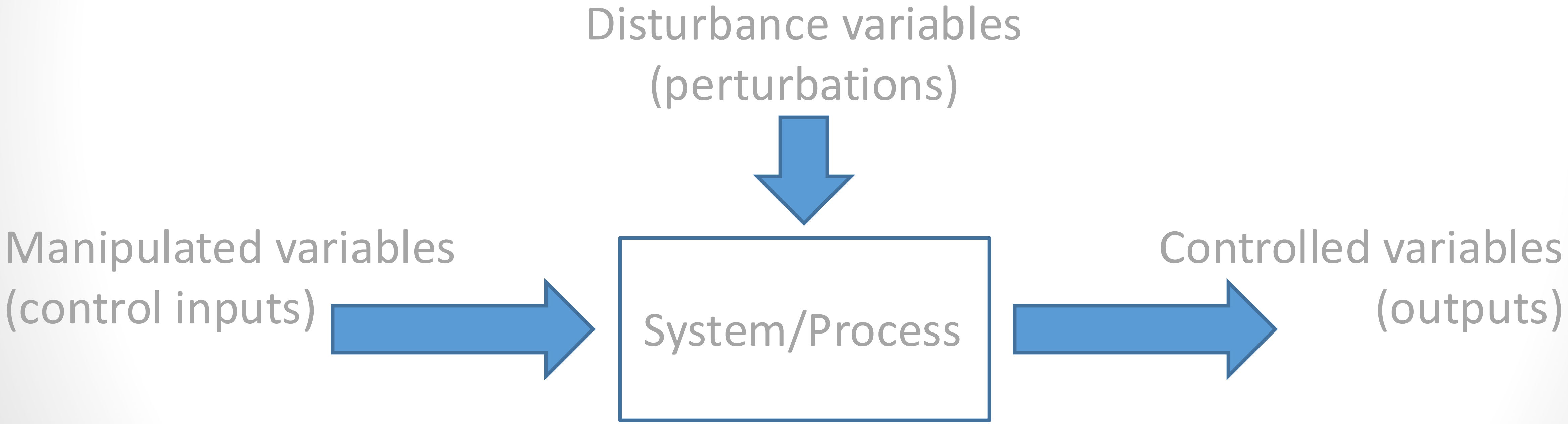
Positive feedback



- Other examples: blood clotting, menstrual cycle, …
- Less common in nature (and control systems) than negative feedback

System and system variables

- Physical -> our complete body, rocket, blood glucose regulation
- Abstract -> economy of a country, information systems



System and system variables

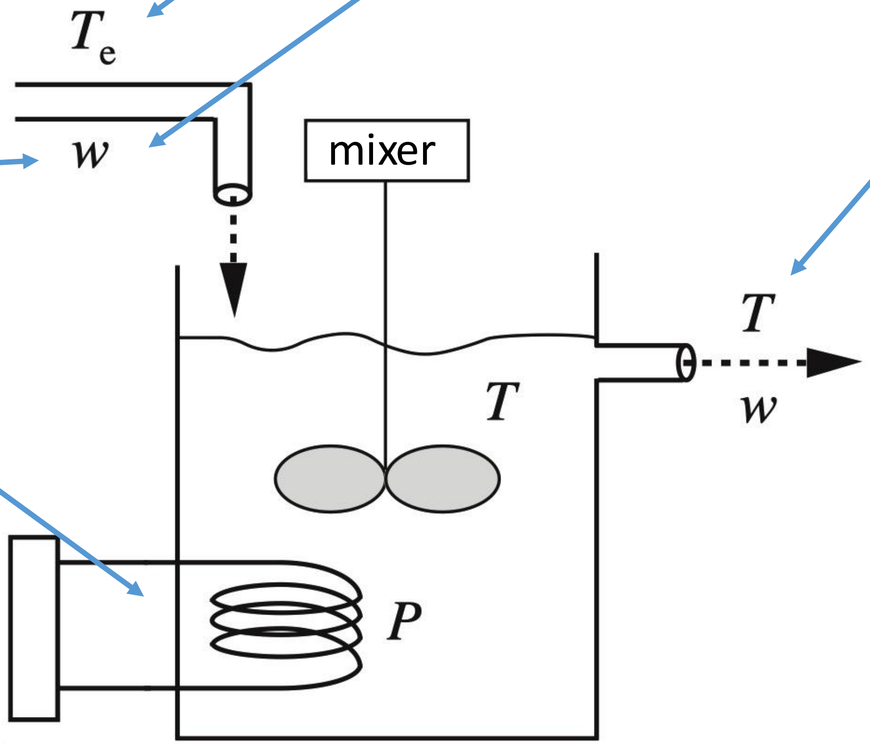
Disturbance variables (perturbations)

If *w* cannot be manipulated

Manipulated variables (control inputs)

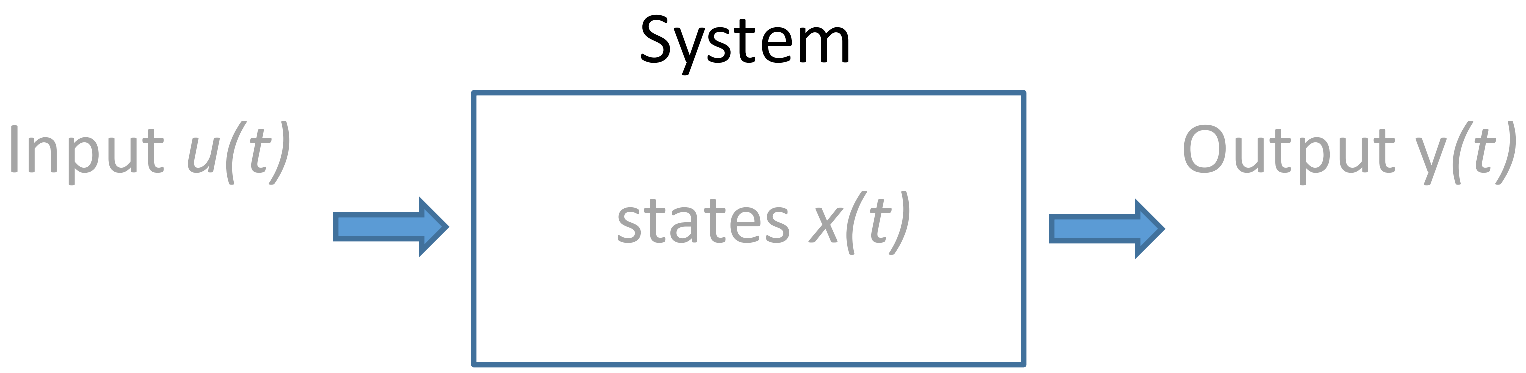
If *w* can be manipulated

Controlled variables (outputs)



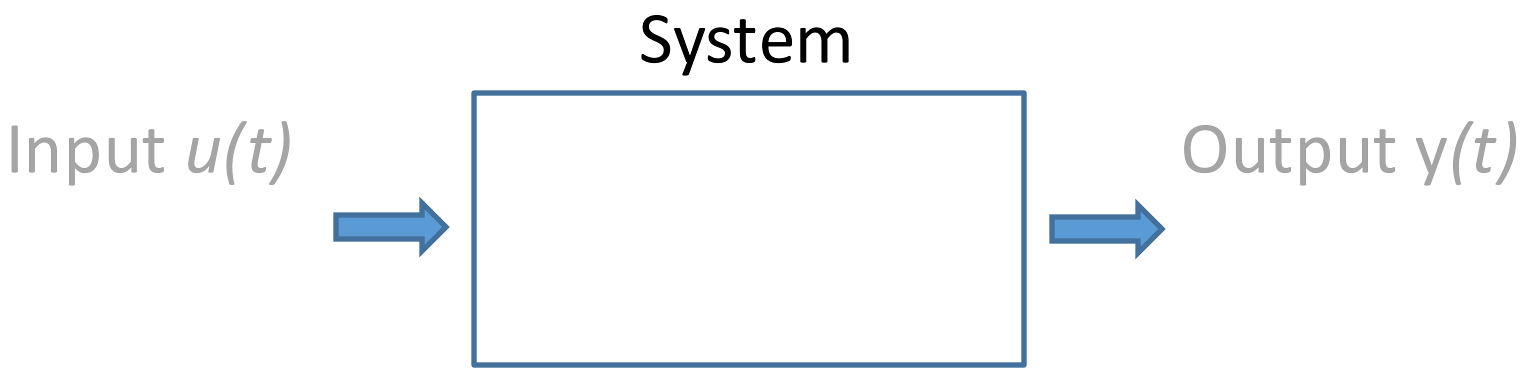
Static and dynamic systems

- Static systems
  - $y(t)$ at t depends only on the input $u(t)$ at the time t
  - Mathematically: $y(t) = f(u(t))$, e.g., $y(t) = 2u(t)$
  - Memoryless systems



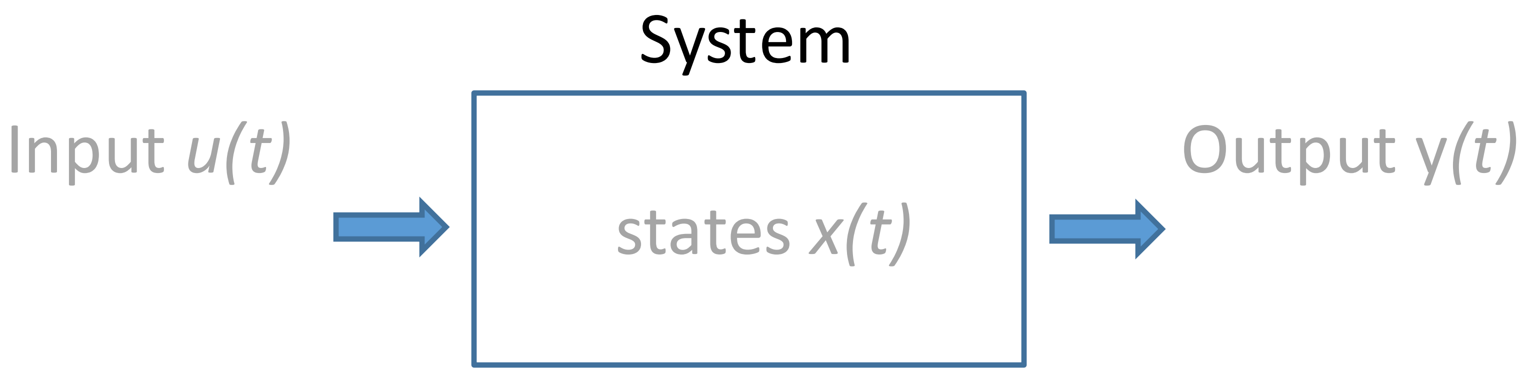
Static and dynamic systems

- Dynamic systems
  - *y(t)* depends on the input *u(t)* at the time *t* and at other times
  - Systems with memory



Static and dynamic systems

- Dynamic systems
  - *y(t)* depends on the input *u(t)* at the time *t* and at other times
  - Systems with memory
  - Memory quantified through state variables (or ”states”)



Static and dynamic systems

- Dynamic systems are often represented as ODEs (ordinary differential equations)
  - A general description:
    - $$\frac{dx}{dt} = f(x(t), u(t), t), \quad x(t_0) = x_0$$

      e.g., $$\frac{dx}{dt} = -x(t) + 2u(t)$$
    - $$y(t) = g(x(t), u(t), t)$$

      e.g., $$y(t) = 2x(t)$$

Static and dynamic systems

- Static or dynamic? if needed test for different values of t:
  - $y(t) = u(t) - u(t-1)$
  - $y(t) = u^2(t)$
  - $y(t) = u(t^2)$
  - $y(t) = tu(t)$

Types of systems studied in this course

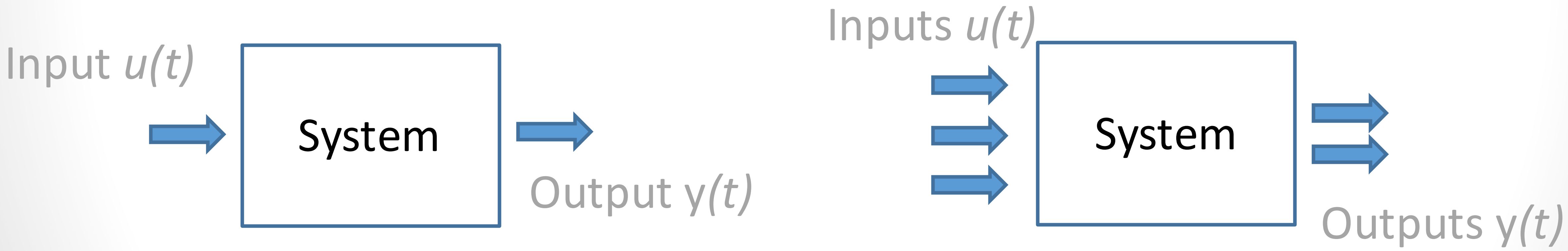
- Dynamic / Static
- Deterministic / Stochastic

Types of systems studied in this course

- Dynamic / Static
- Deterministic / Stochastic
  - Deterministic – knowing the current state of the system, $x(t)$, the future outputs $y(t)$ can be exactly predicted from $u(t)$.
  - Stochastic – random events/variables affect the output

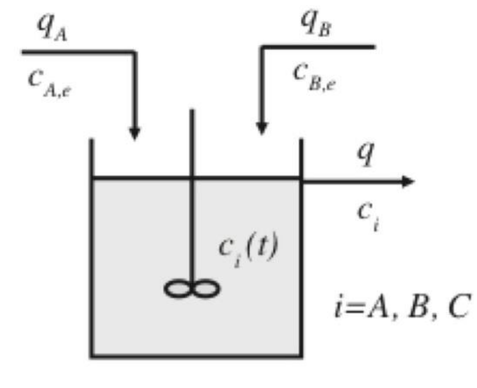
Types of systems studied in this course

- Dynamic / Static
- Deterministic / Stochastic
- Single input – single output / Multivariable



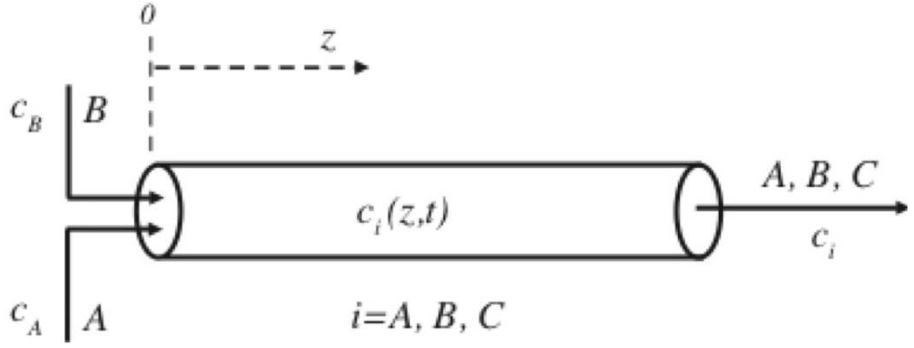
Types of systems studied in this course

- Dynamic / Static
- Deterministic / Stochastic
- Single input – single output / Multivariable
- Local parameters / Distributed parameters



Homogeneous system, *c(t)*

Local parameters

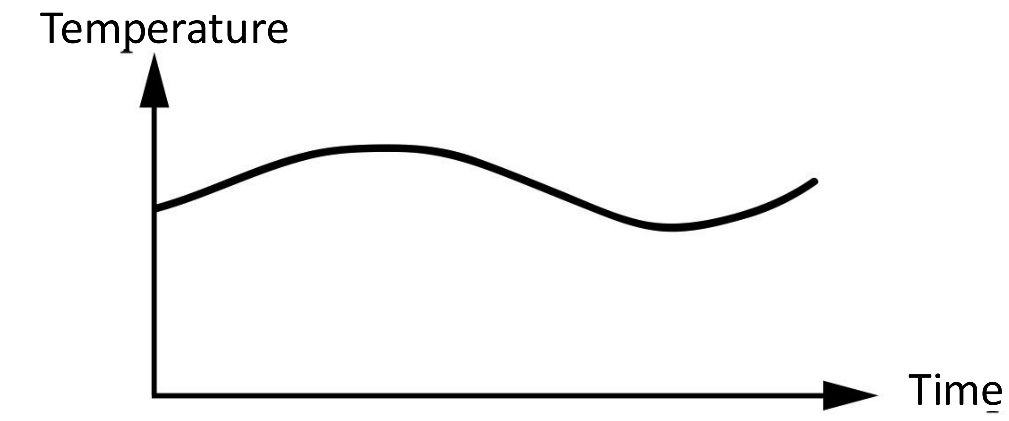


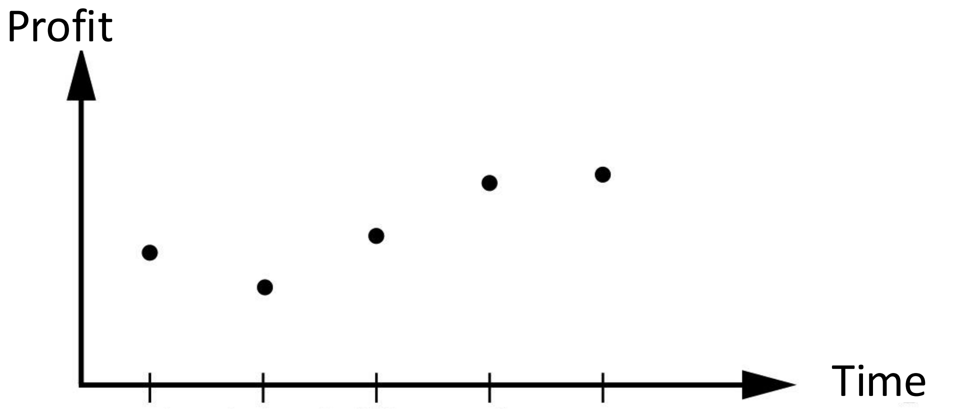
Spatial and temporal changes, *c(z,t)*

Distributed parameters

Types of systems studied in this course

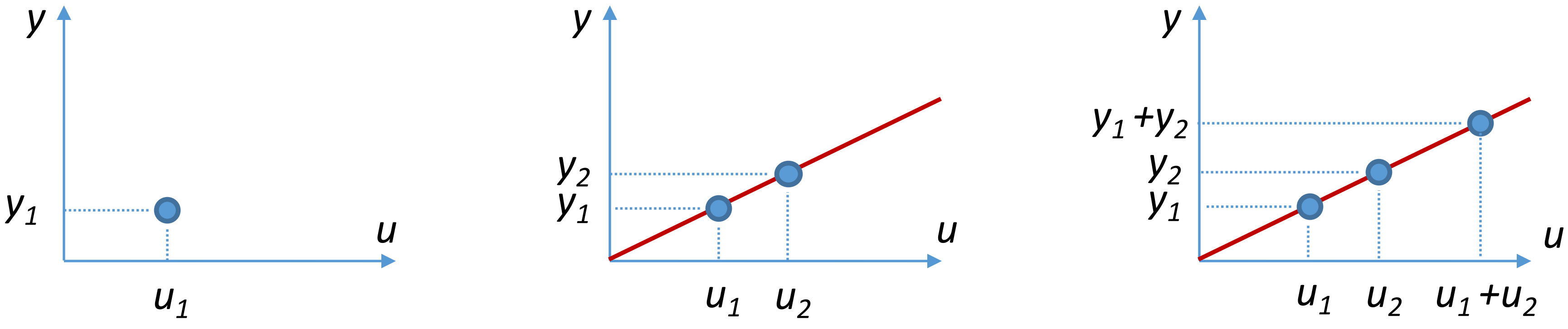
- Dynamic / Static
- Deterministic / Stochastic
- Single input – single output / Multivariable
- Local parameters / Distributed parameters
- Continuous / Discrete





Properties of dynamic systems

- Linearity (homogeneity and additivity)



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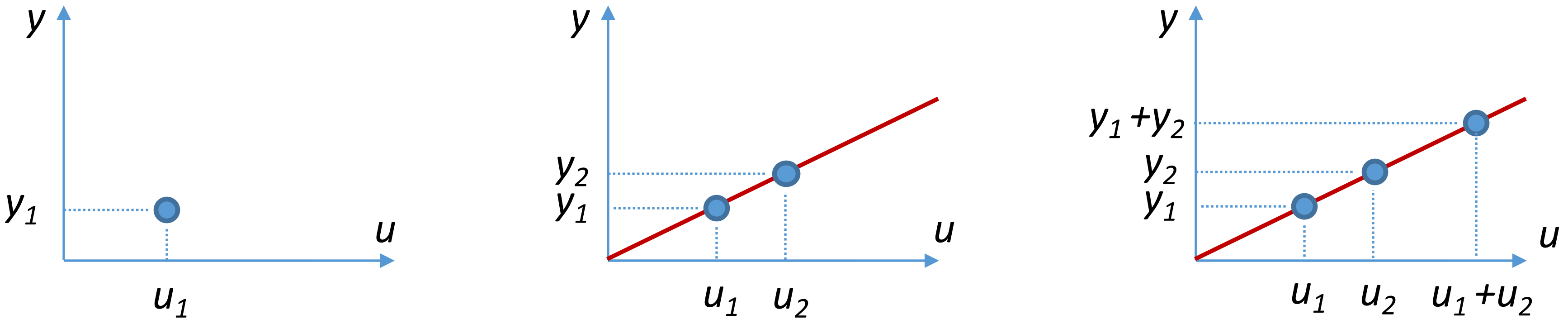
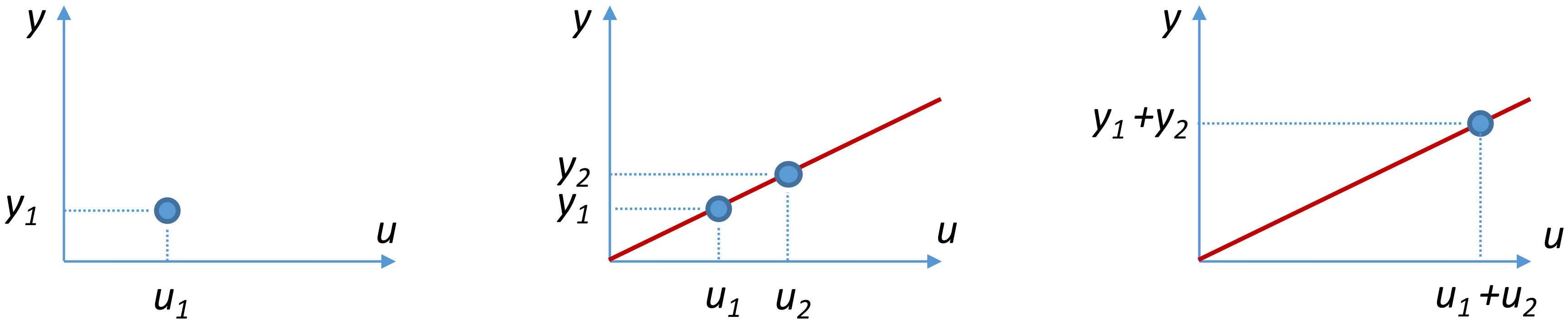


The output of a linear system excited by any linear combination of inputs u_1 and u_2 is equal to the same linear combination of the corresponding outputs y_1 and y_2: $au_1 + bu_2$ ----> $ay_1 + by_2$

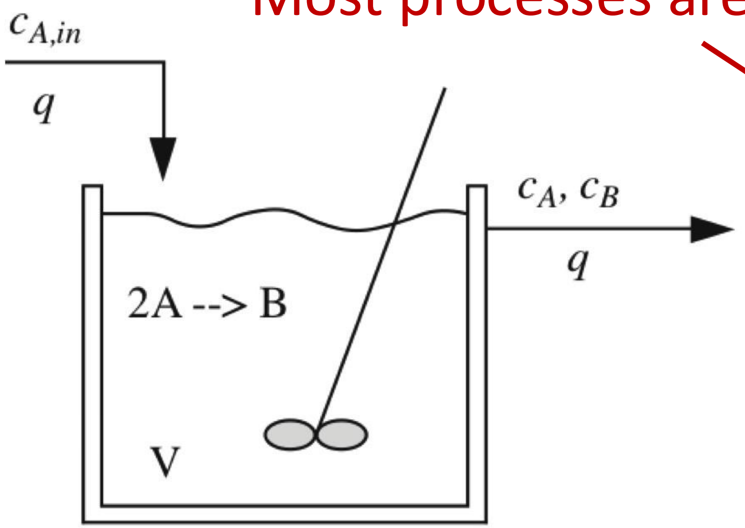
inputs system outputs

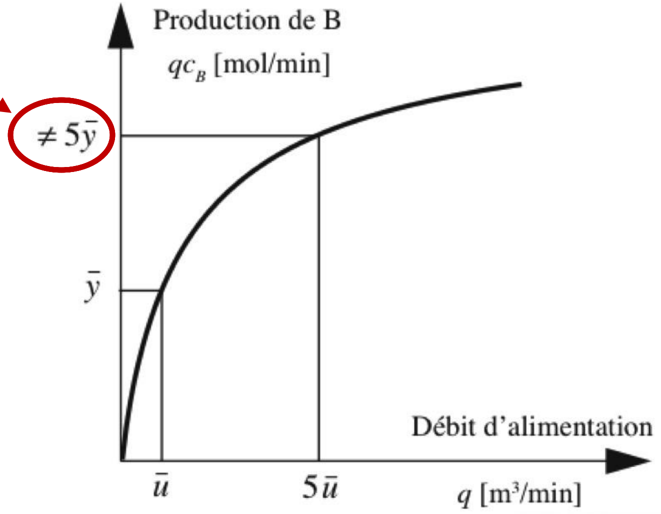
Properties of dynamic systems

- Linearity (homogeneity and additivity)



Most processes are nonlinear





Properties of dynamic systems

- Time-invariance (stationarity) – system not changing in time, i.e., whose parameters are time-invariant

  - Time-invariant system

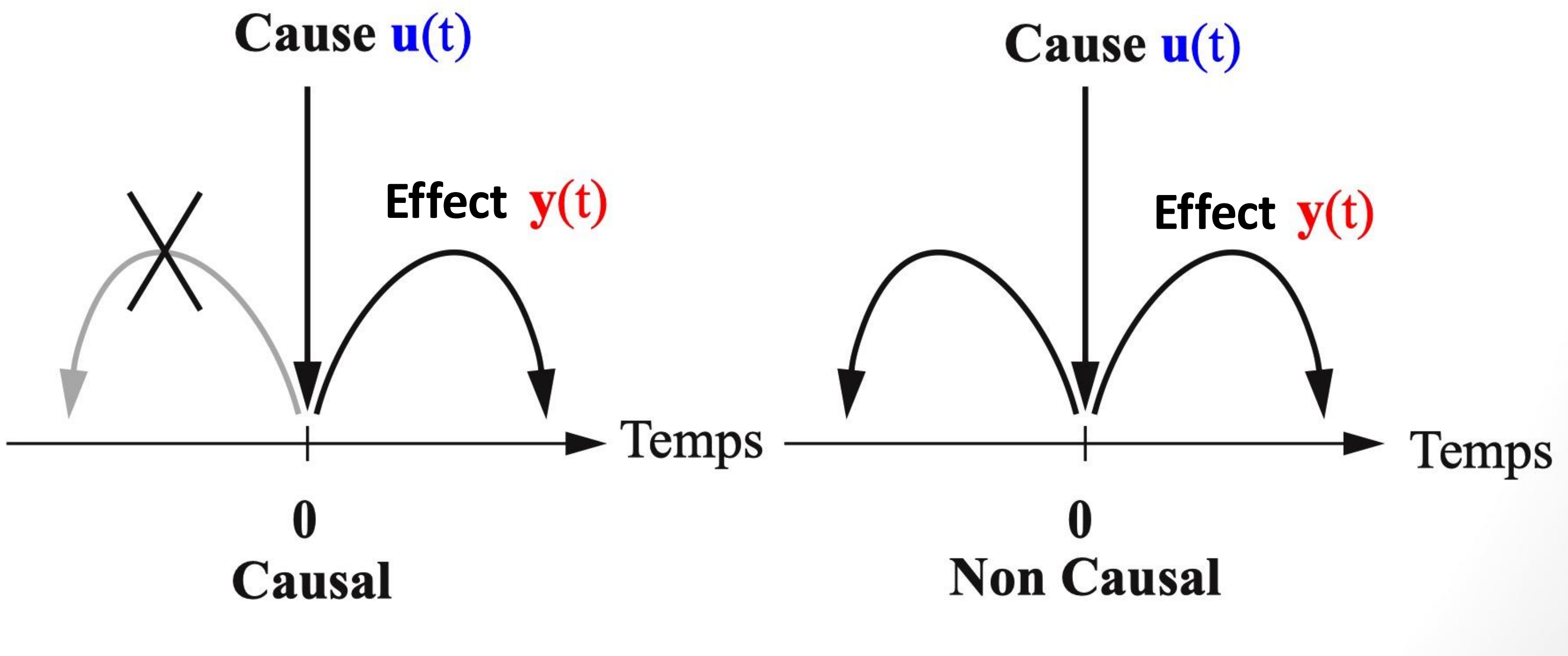
    $\ddot{y} + 3\dot{y} + 2y = 3\mathrm{u}$

  - Time-variant system

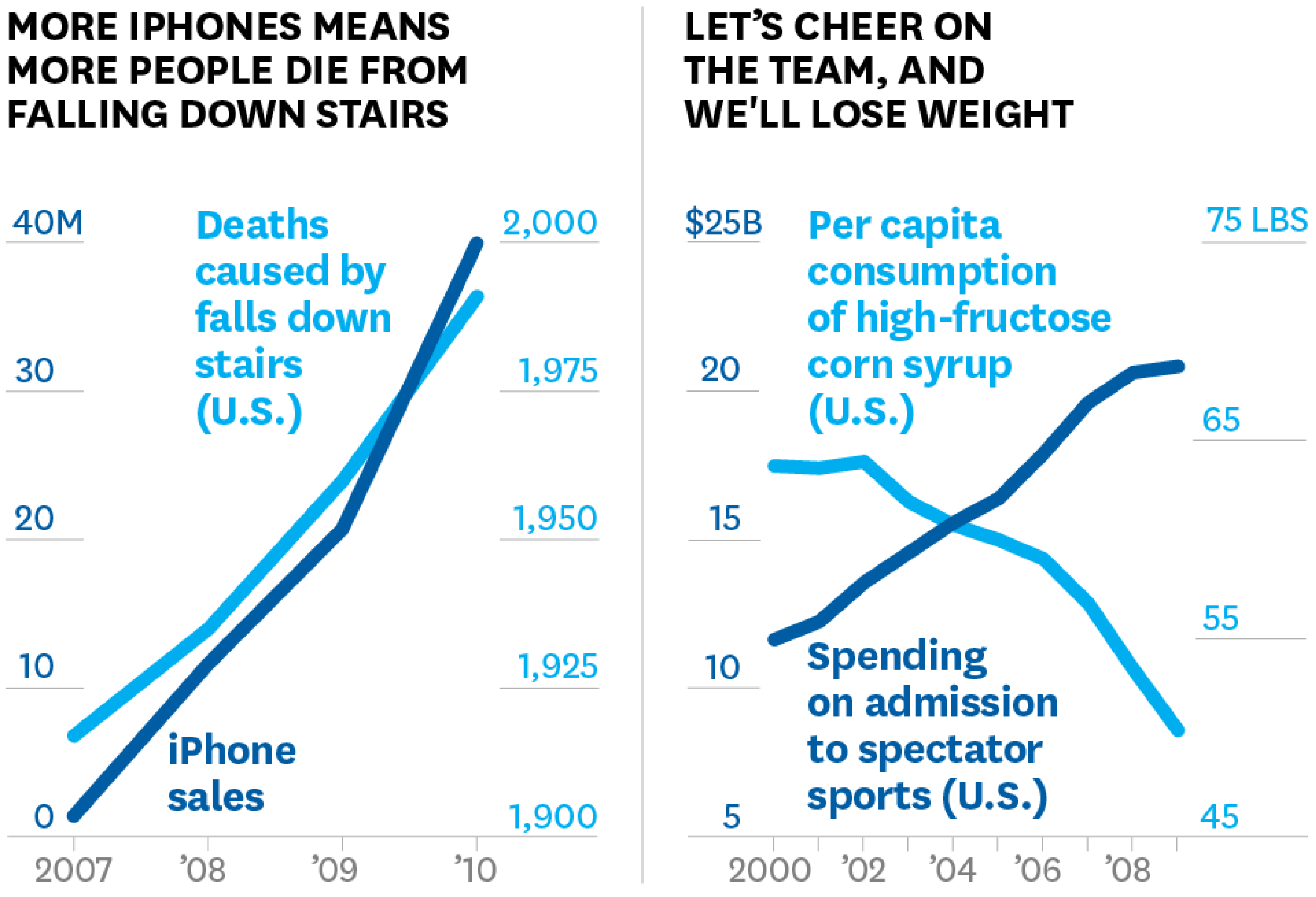
    $\ddot{y} + 3e^{-t}\dot{y} + 2y = 3\mathrm{u}$

Properties of dynamic systems

- Causality – an effect cannot occur before its cause
- Here it means that the system response in time (output) cannot precede the excitation (input)

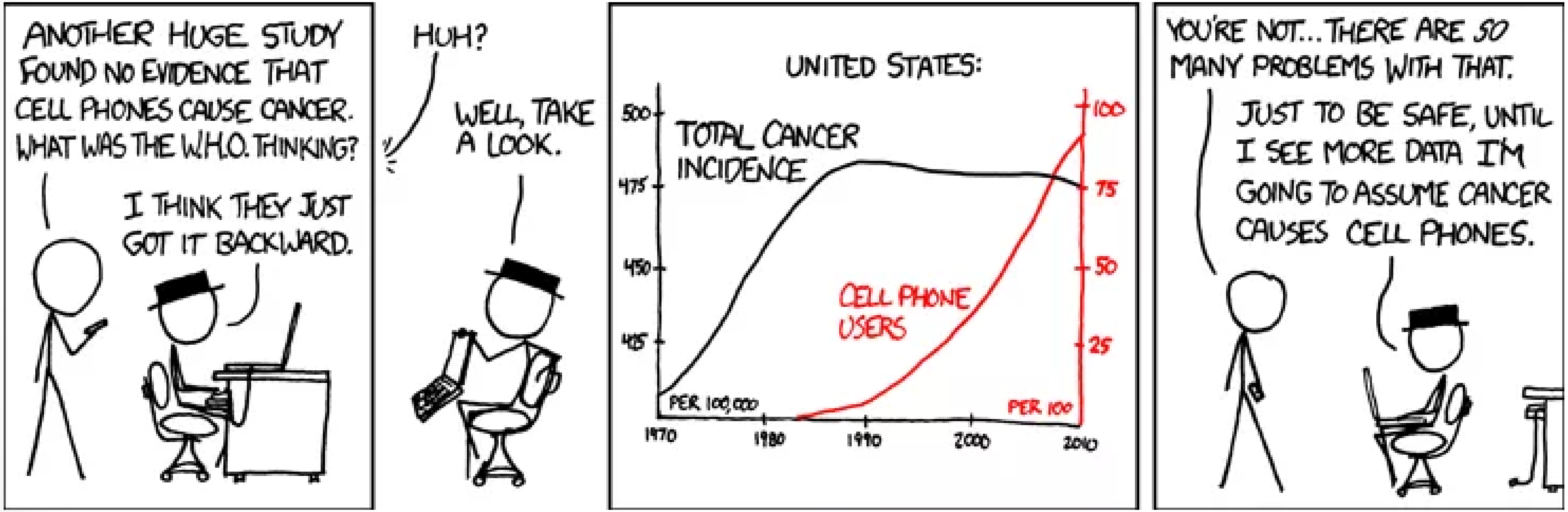


Do not mix correlation and causality!



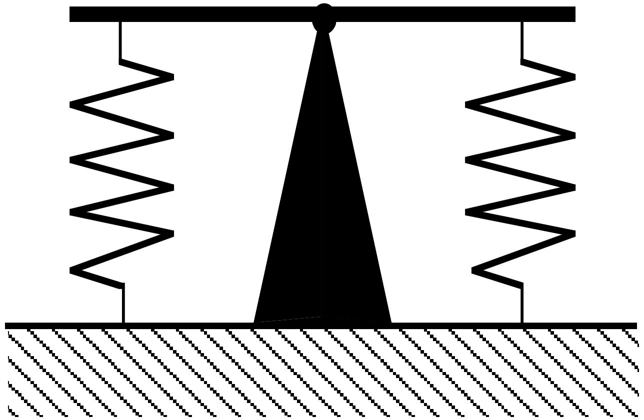
SOURCE TYLERVIGEN.COM
FROM "BEWARE SPURIOUS CORRELATIONS," JUNE 2015

Do not mix correlation and causality!

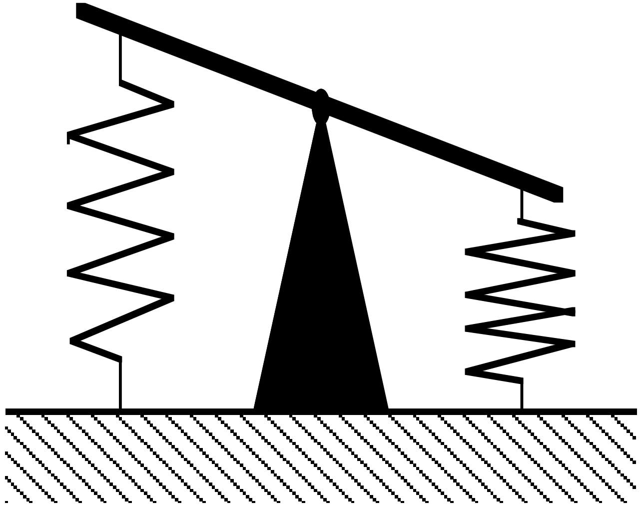


Initial conditions

Relaxed system A system is said to be relaxed if the system is causal and at the initial time t_0 the output of the system is zero, i.e., there is **no stored energy in the system**.



Relaxed system



Perturbed system

Initially relaxed systems

A dynamic system is **initially relaxed** if its initial conditions correspond to an equilibrium point

$$\dot{y}(t) = -2y(t) + u(t) \quad \mathrm{y}(0) = y_0$$

Equilibrium point ($\dot{y}$=0)

$$0 = -2\bar{y} + \bar{u} \quad \rightarrow \quad \bar{y} = 0.5\,\bar{u}$$

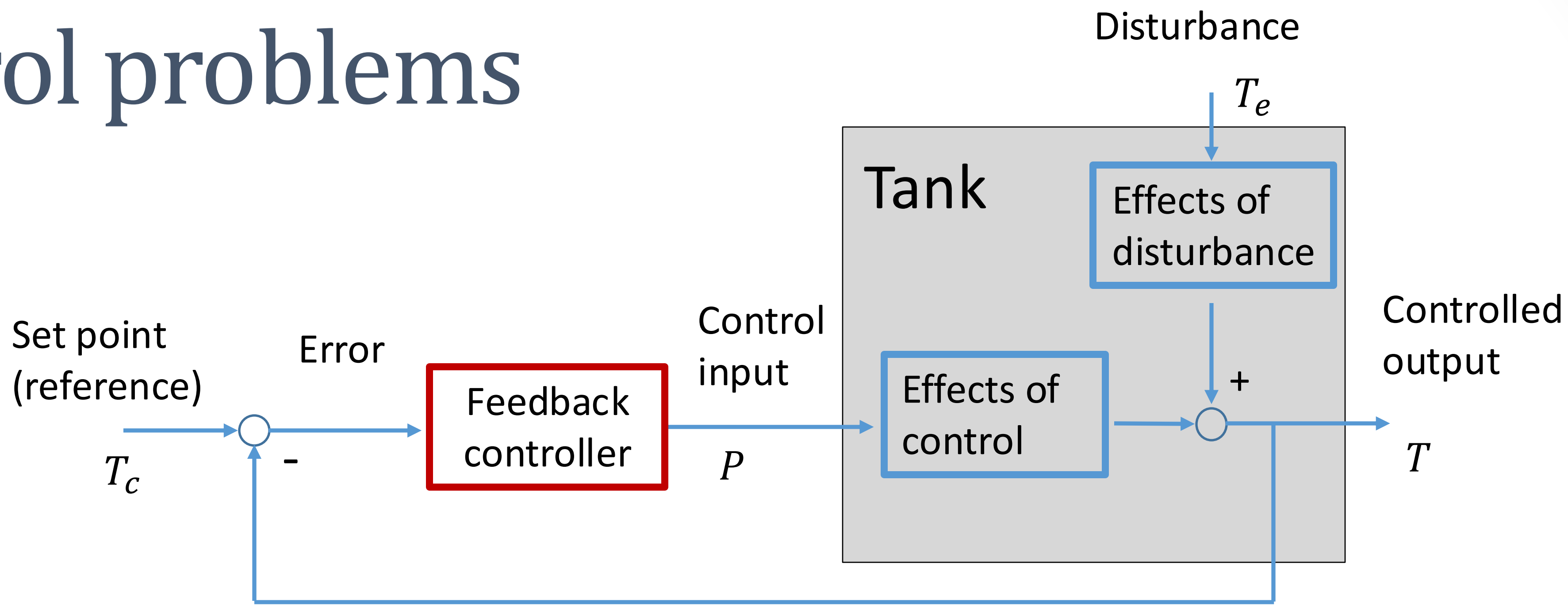
The equilibrium point depends on $\bar{u}$:

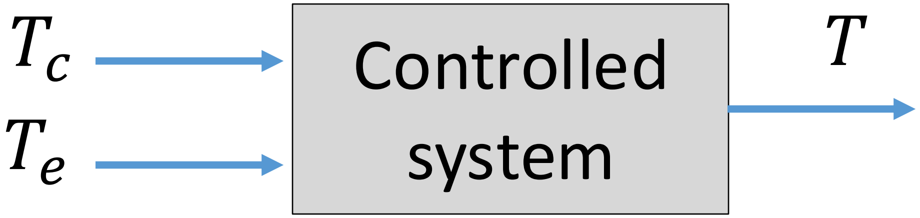
- For $\bar{u} = 0, \bar{y} = 0$ $\rightarrow$ system is relaxed if $y_0 = 0$
- For $\bar{u} = 2, \bar{y} = 1$ $\rightarrow$ system is relaxed if $y_0 = 1$

Systems studied in this course

System	**Represented by an equation**
Continuous dynamic	Differential
Local parameters	Ordinary
Linear	Linear
Time-invariant	Constant coefficients
Relaxed	Zero initial conditions

Control problems





Tracking: variation of $T_c, T(t)$ follows $T_c(t)$ – tracking of a reference trajectory

Regulation: : variation of $T_e, T(t)$ follows $T_c(t)$ – disturbance rejection