Feedforward Control

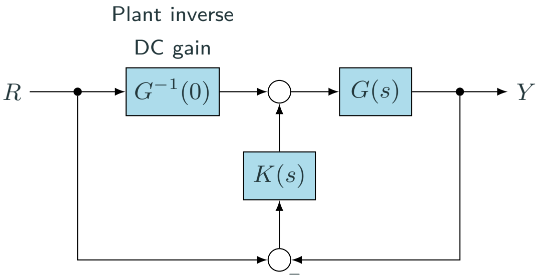
Feedforward Control

Goal Zero steady-state error

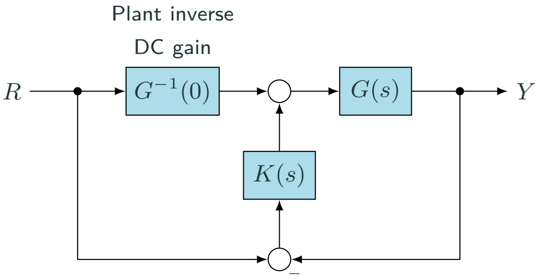
Solution Add an integrator to 'learn' the required steady-state input for zero steady-state error

Problem Integrators typically decrease damping / stability of the system

Idea Compute the required steady-state input and add it to the system



Feedforward Control - Analysis



System response

$$\begin{aligned} Y &= G(s)(G^{-1}(0)R + K(s)(R - Y)) \\ (1 + G(s)K(s))Y &= G(s)G^{-1}(0)R + G(s)K(s)R \\ Y &= \frac{G(s)G^{-1}(0) + G(s)K(s)}{1 + G(s)K(s)} R \end{aligned}$$

Feedforward Control - Analysis

System response

$$Y = \frac{G(s)G^{-1}(0) + G(s)K(s)}{1 + G(s)K(s)} R$$

Steady-state output in response to a step input $R = \frac{1}{s}$

$$\begin{aligned}
\lim_{t \to \infty} y(t) &= \lim_{s \to 0} sY(s) \\
&= \lim_{s \to 0} s \frac{G(s)G^{-1}(0) + G(s)K(s)}{1 + G(s)K(s)} \frac{1}{s} \\
&= \frac{G(0)G^{-1}(0) + G(0)K(0)}{1 + G(0)K(0)} \\
&= 1
\end{aligned}$$

If we can estimate the steady-state gain of the system, $G(0)$, then we can achieve zero steady-state error without the destabilizing influence of the integrator.

Example

Design a Feedforward Controller

Goal: Overshoot around 20% and zero steady-state error

$$G(s) = \frac{1}{s^2 + 1.4s + 1}$$

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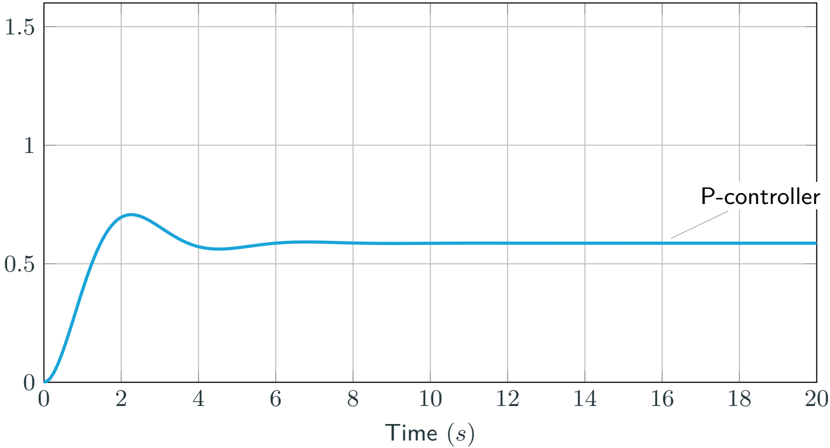
$$G(s) = \frac{1}{s^2 + 1.4s + 1}$$

Design proportional controller $K(s) = K_p$ to give approximately 20% error $\rightarrow \zeta \approx 0.45$

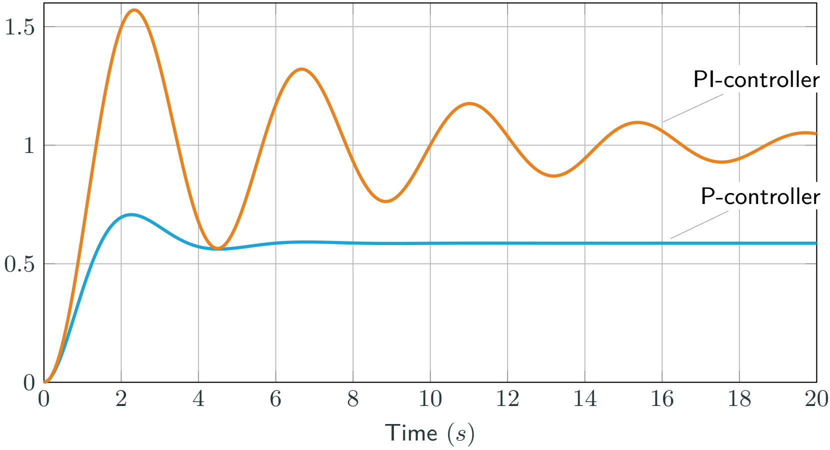
$$\begin{aligned} \frac{Y}{R} &= \frac{GK_p}{1 + GK_p} \\ &= \frac{K_p}{s^2 + 1.4s + 1 + K_p} \end{aligned}$$

$$\begin{aligned} \omega_n^2 &= 1 + K_p \\ 2\zeta\omega_n &= 1.4 \qquad \rightarrow \qquad \zeta = 0.45 = \frac{0.7}{\sqrt{1 + K_p}} \qquad \rightarrow \qquad K_p = 1.4 \end{aligned}$$

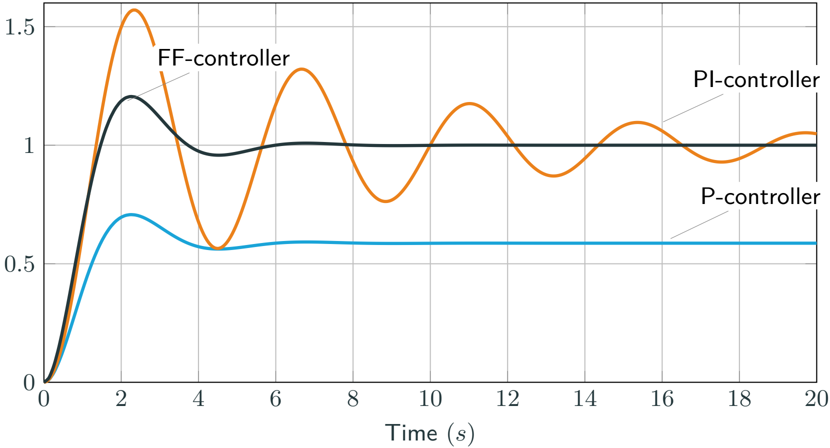
Example - Closed-Loop Step Response



Example - Closed-Loop Step Response

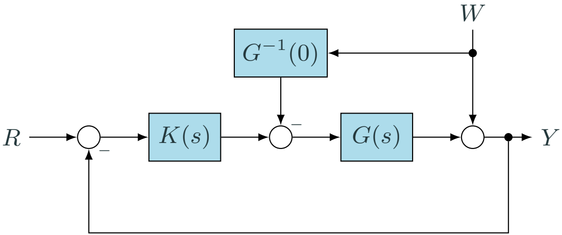


Example - Closed-Loop Step Response



Feedforward Disturbance Rejection

If a constant disturbance can be estimated or measured, then it can also be mitigated via feedforward control



Summary - Feedforward Control

Idea Compute the required steady-state input to the system, and add this to the input

- When possible, this removes the need for an integrator, which can reduce the damping of the system
- If possible to estimate the steady-state impact of a disturbance, this can also be removed

Much more advanced feedforward controllers are possible

- Compute the required input for a desired ***dynamic*** trajectory
- Drive the system with this pre-computed trajectory, and then use feedback only to deal with errors
- Possible to handle some very complex non-linear and constrained system in this way