Control Systems : Exam Formula Sheet

Zieger-Nichols

Type	K_P	T_i	T_d
P	$\frac{1}{aL}$		
PI	$\frac{0.9}{aL}$	$3.3L$	
PID	$\frac{1.2}{aL}$	$2L$	$0.5L$

Type	K_P	T_i	T_d
P	$0.5K_{pc}$		
PI	$0.45K_{pc}$	$0.83T_c$	
PID	$0.6K_{pc}$	$0.5T_c$	$0.125T_c$

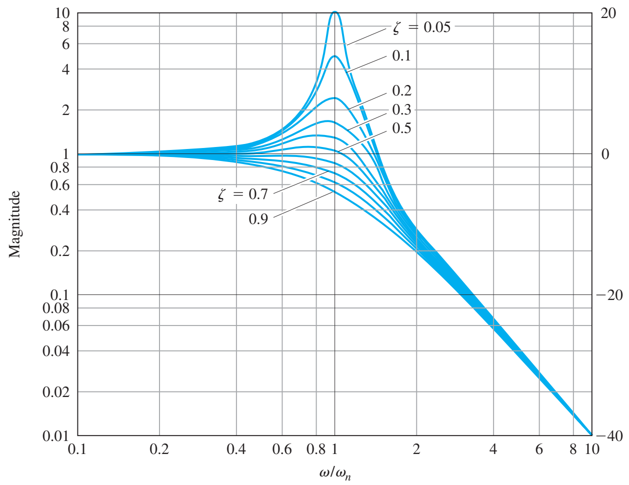
Second order systems

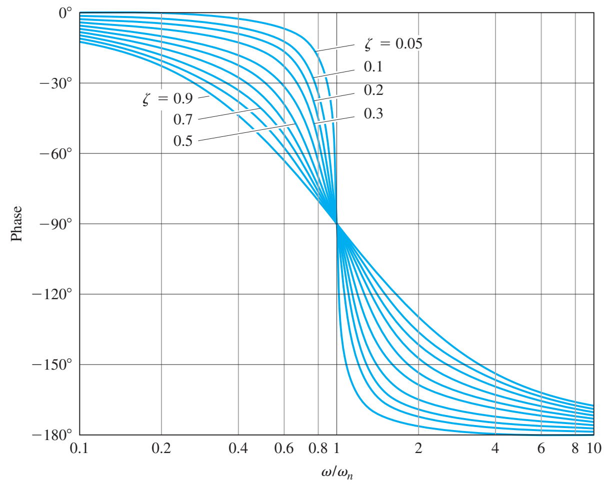
$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{\omega_d}$$

$$M_p \times 100\% = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

$$T_s = \frac{-\ln\delta}{\zeta\omega_n} = \frac{4}{\zeta\omega_n} = \frac{4}{\sigma}$$

$$G(j\omega) = \frac{1}{(j\omega/\omega_n)^2 + 2\zeta(j\omega/\omega_n) + 1}$$

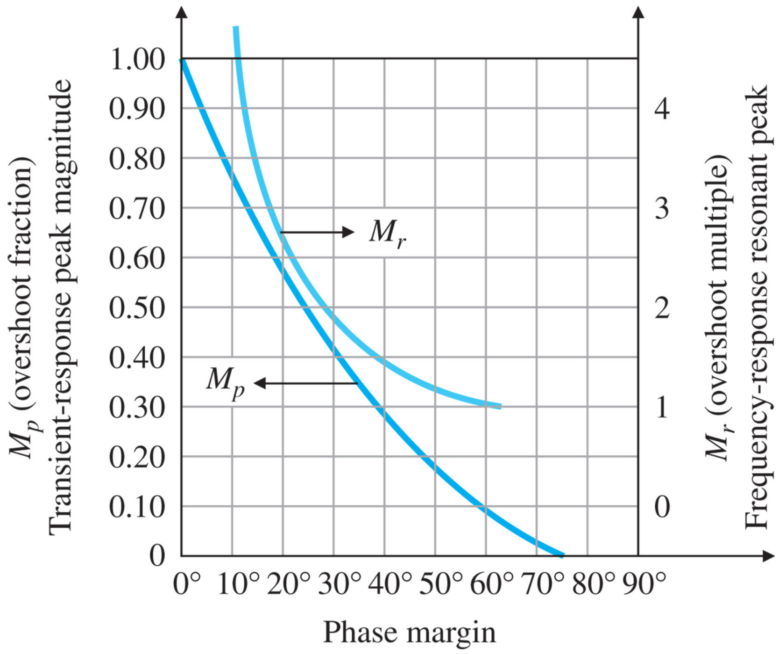




System Types

Type	$r(t) = 1$	$r(t) = t$	$r(t) = t^2$
0	$\frac{1}{1+\gamma}$	∞	∞
1	0	$\frac{1}{\gamma}$	∞
2	0	0	$\frac{1}{\gamma}$

Resonance and Phase Margin

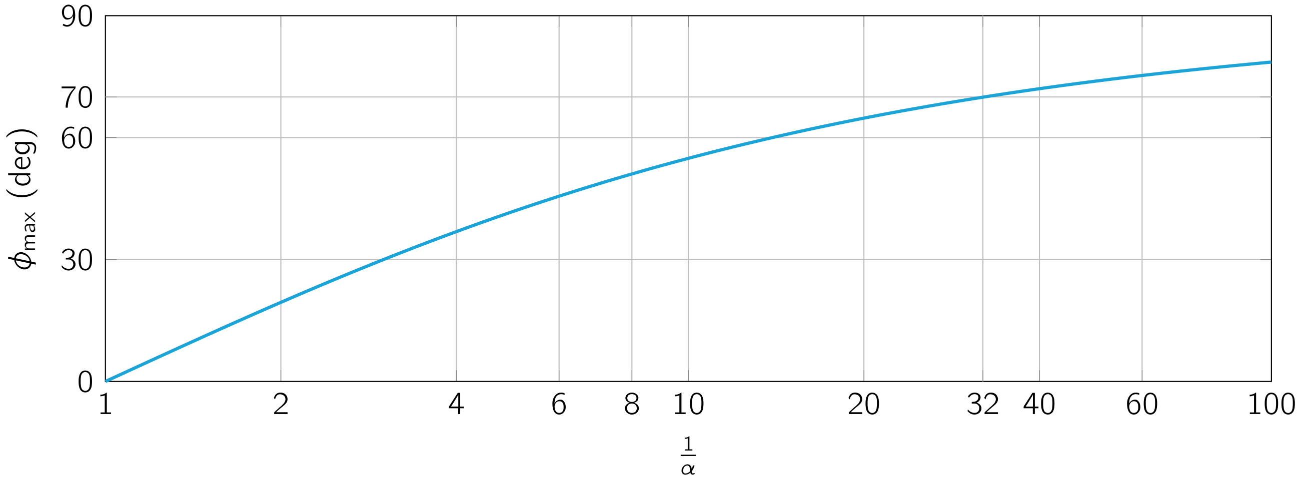


Tustin Approximation

$$s \approx \frac{2}{T}\frac{z-1}{z+1}$$

Phase Lead Compensators

$$\omega_{max} = \frac{1}{T_D\sqrt{\alpha}} \qquad \sin\phi_{max} = \frac{1-\alpha}{1+\alpha} \qquad \alpha = \frac{1-\sin\phi_{max}}{1+\sin\phi_{max}}$$



State Space

$$G(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D$$

Canonical Forms

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_1 s^{n-1} + \cdots + b_{n-1}s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n}$$

$$\dot{x} = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u \qquad \dot{x} = \begin{bmatrix} -a_1 & 1 & 0 \\ -a_2 & 0 & 1 \\ -a_3 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} u$$

$$y = \begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix} x \qquad y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$$

Ackermann's Formula

$$K = \begin{bmatrix} 0 & 0 & \dots & 1 \end{bmatrix} \mathcal{C}^{-1} \alpha(A) \qquad\qquad L = \alpha_e(A) \mathcal{O}^{-1} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

$$\mathcal{O} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} \qquad\qquad \mathcal{C} = \begin{bmatrix} B & AB & A^2B & \dots & A^{n-1}B \end{bmatrix}$$

Bessel polynomials

$n = 1$	$\theta_1(s) = s + 1$
$n = 2$	$\theta_2(s) = s^2 + 3s + 3$
$n = 3$	$\theta_3(s) = s^3 + 6s^2 + 15s + 15$
$n = 4$	$\theta_4(s) = s^4 + 10s^3 + 45s^2 + 105s + 105$
$n = 5$	$\theta_5(s) = s^5 + 15s^4 + 105s^3 + 420s^2 + 945s + 945$

LQR

$$u = -Kx$$

$$K = R^{-1}B^TP \qquad\qquad A^TP + PA - PBR^{-1}B^TP + Q = 0$$

Laplace Transforms

1	$\frac{1}{s}$
$t^n, for\ n = 0, 1, 2, \dots$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$
$e^{at} \cos \omega t$	$\frac{(s-a)}{(s-a)^2+\omega^2}$