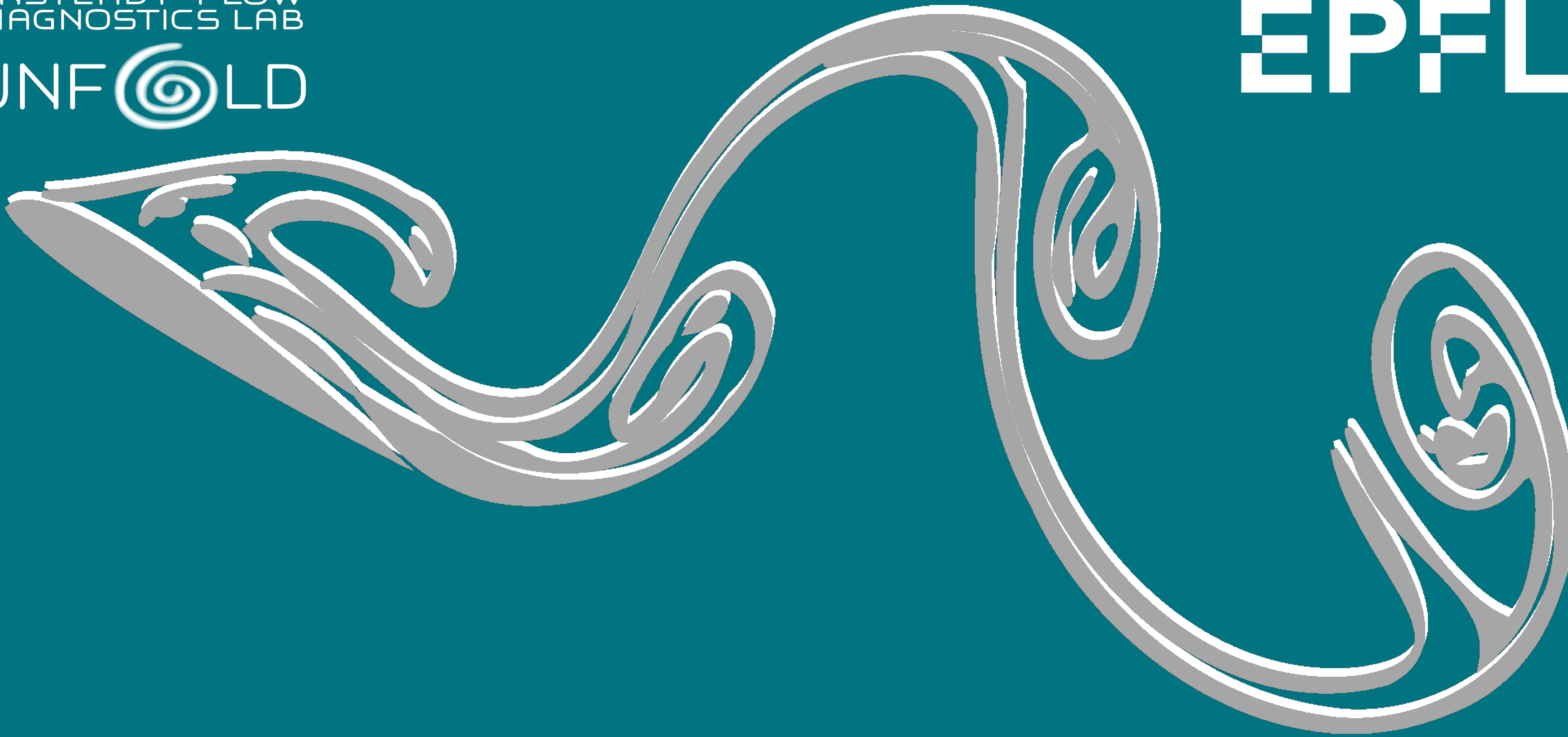
UNSTEADY FLOW DIAGNOSTICS LAB
UNFOLD

EPFL



Strain gauge measurements

An introduction

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FNSNF
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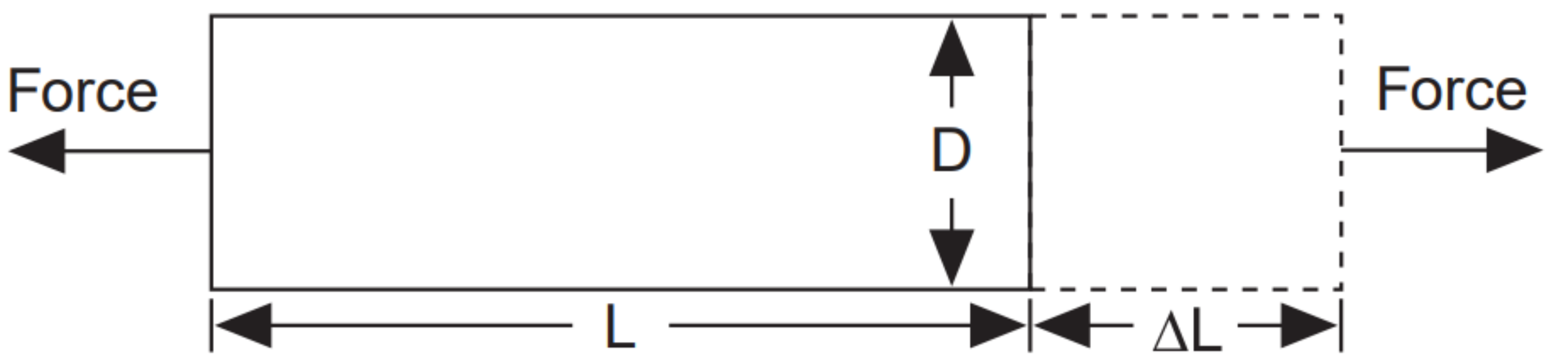
What is strain?

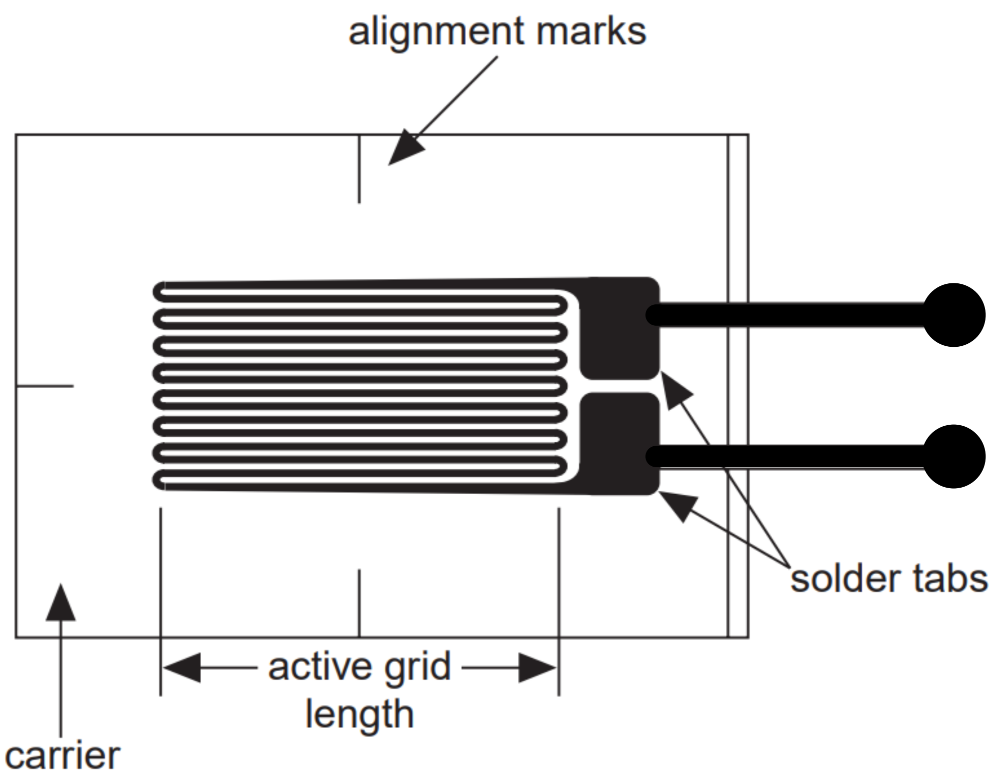
Strain is the amount of deformation of a body due to an applied force.



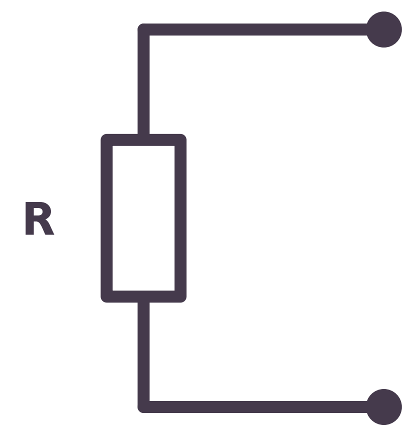
$$\epsilon = \frac{\Delta L}{L}$$

Measuring strain with a strain gauge

bonded metallic strain gauge



electronic equivalent at rest



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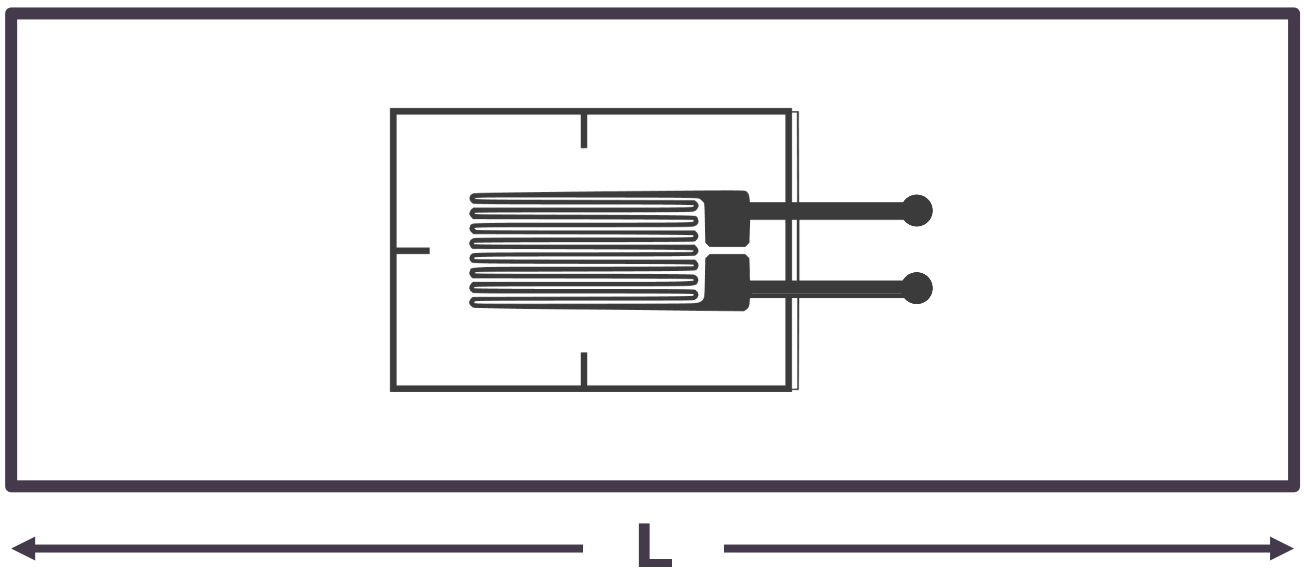
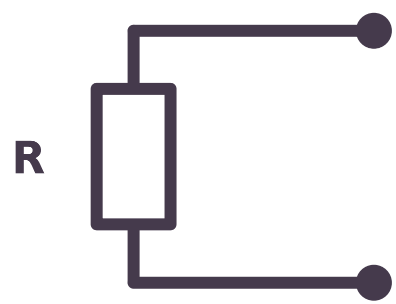
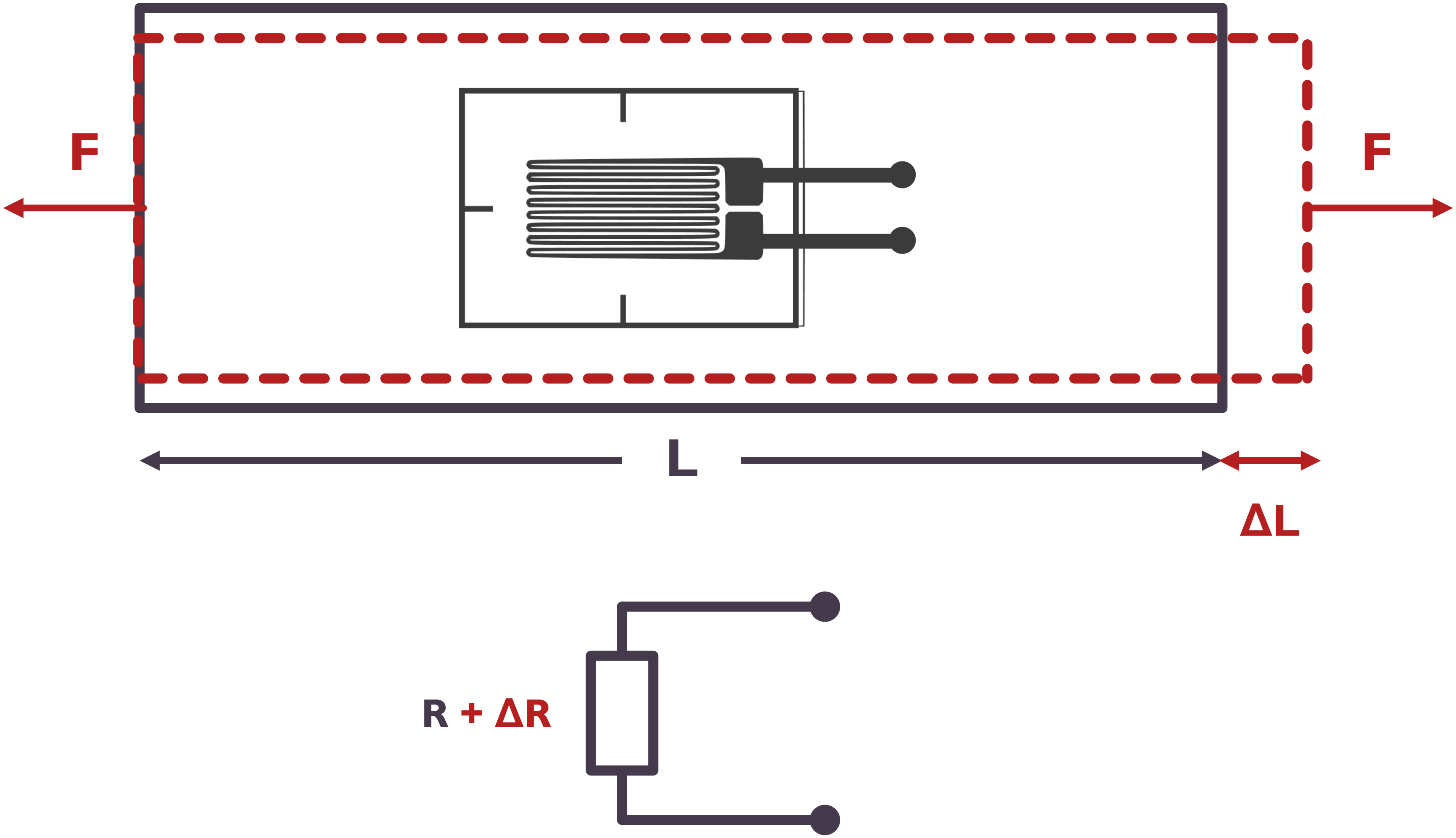
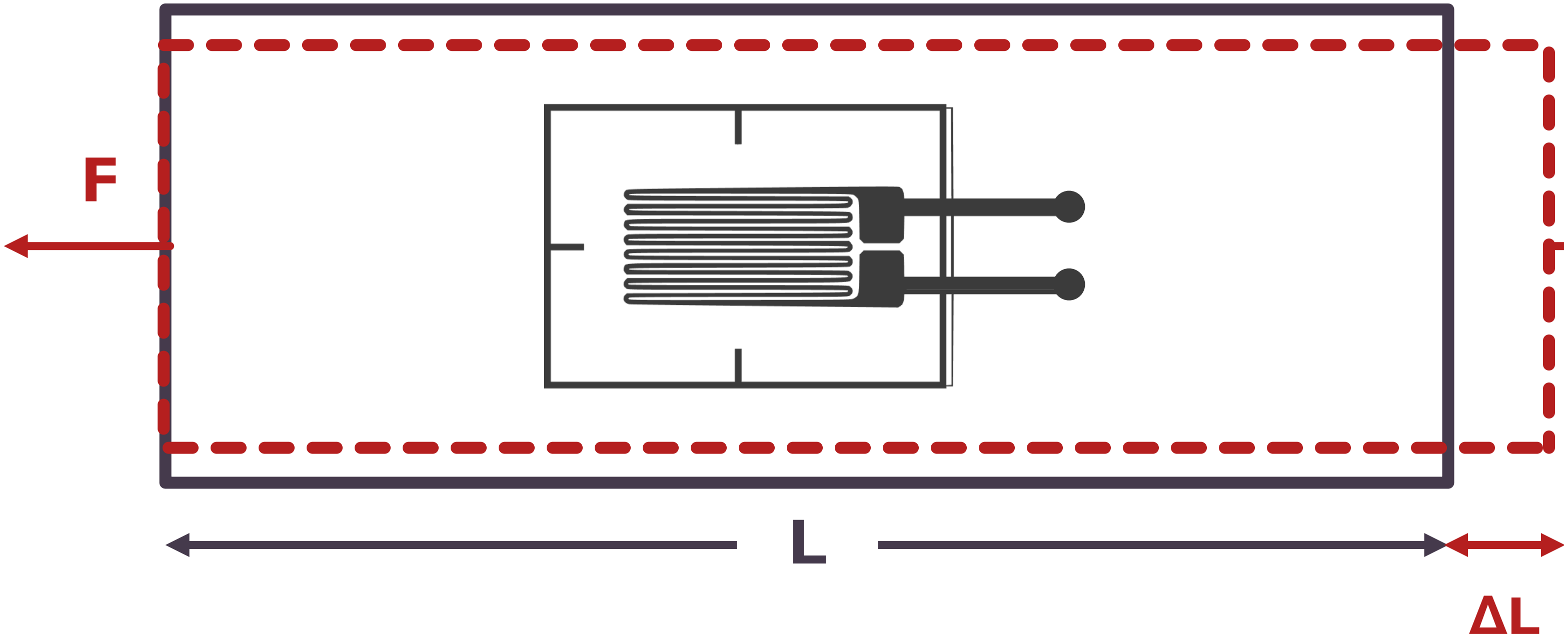


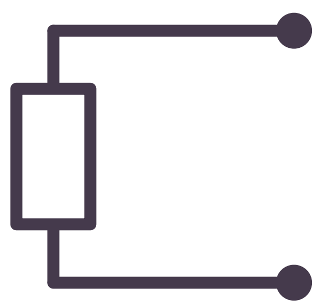


Measuring strain with a strain gauge



Measuring strain with a strain gauge

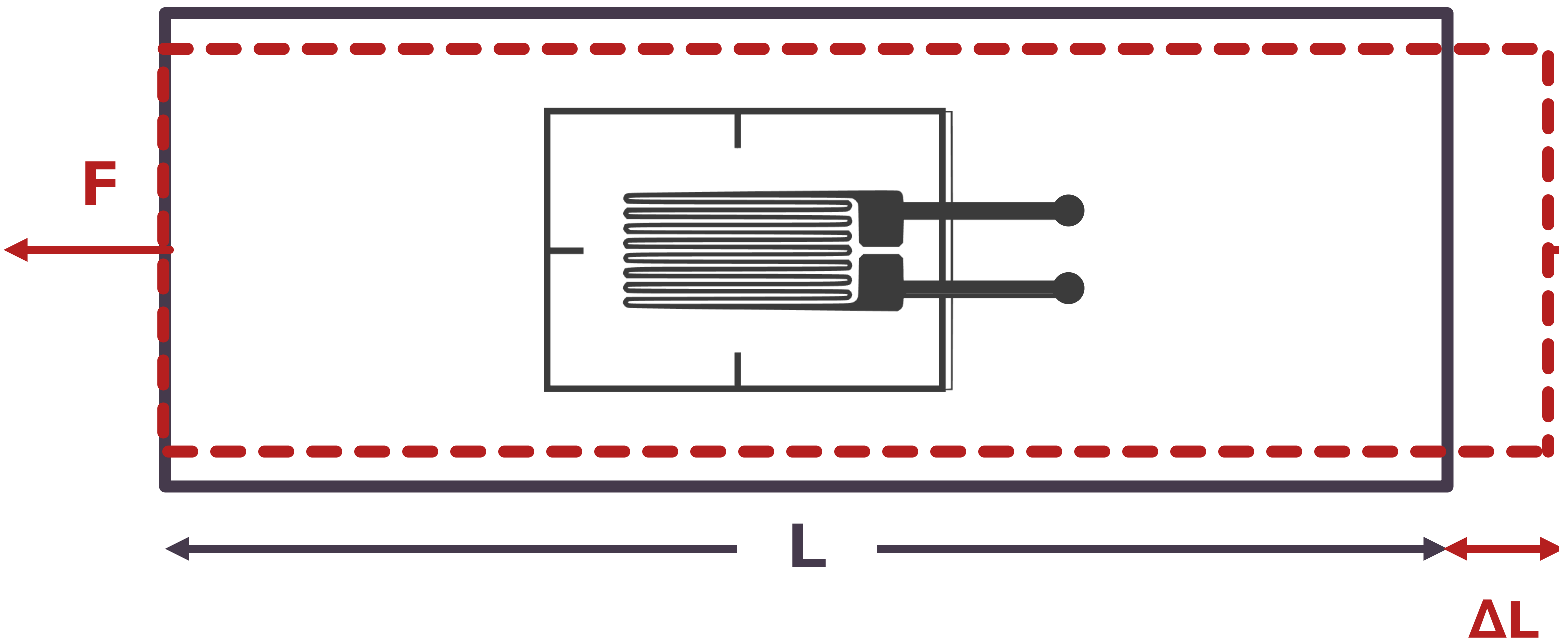




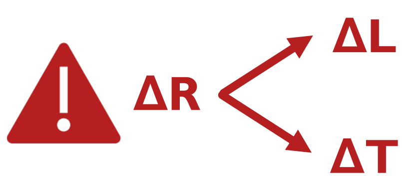
Gauge factor (sensitivity)

$$\mathrm{GF} = \frac{\Delta R/R}{\Delta L/L} = \frac{\Delta R/R}{\epsilon}$$

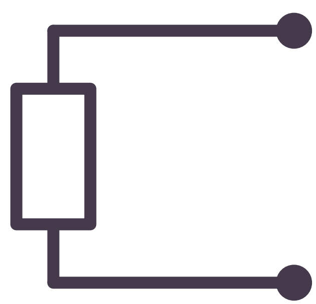
Measuring strain with a strain gauge



sensitive to



R + ΔR



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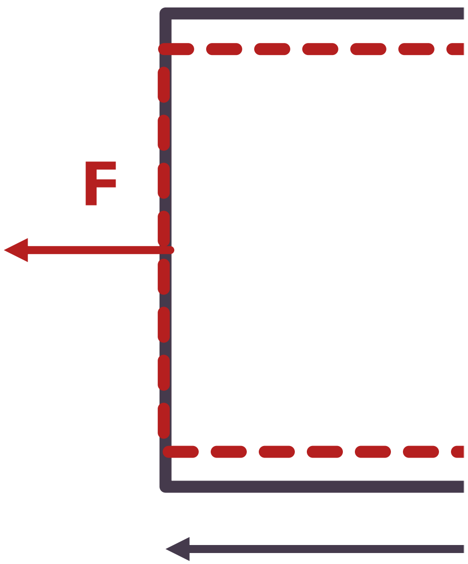
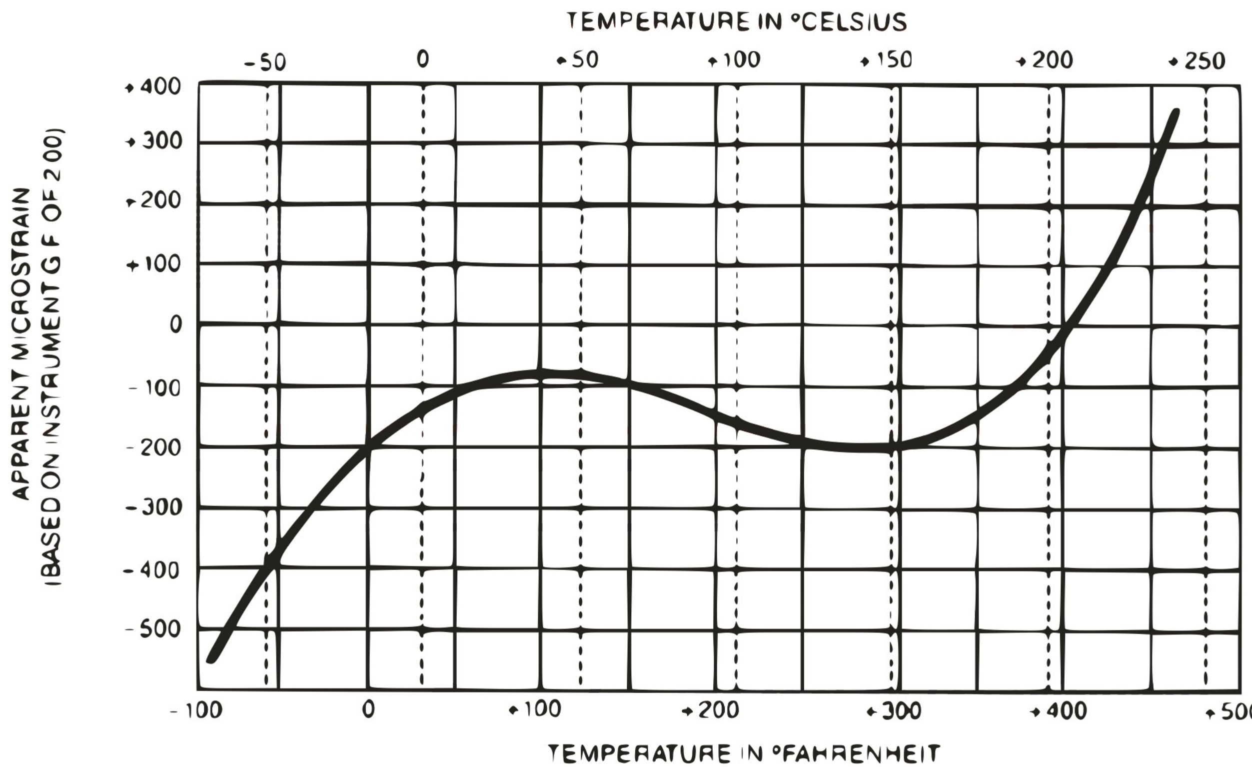
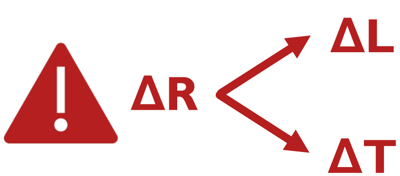
Measuring strain with a strain gauge





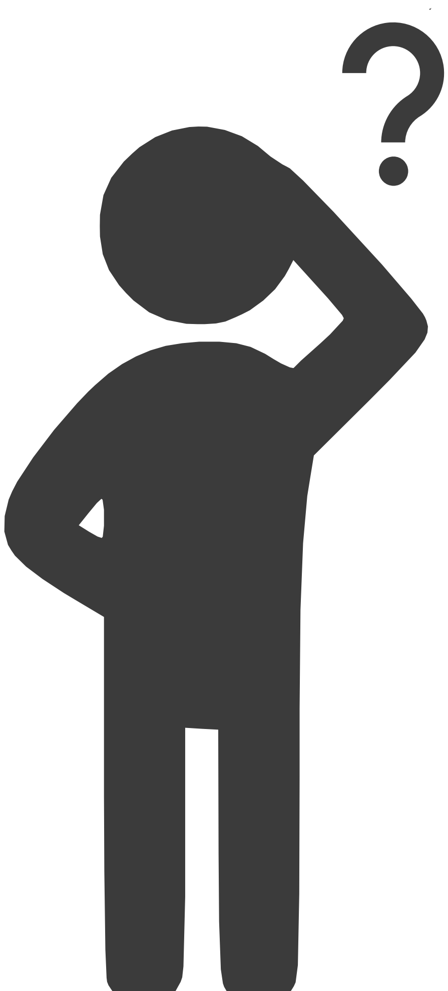
sensitive to



R + ΔR

$$GF = \frac{\Delta R/R}{\Delta L/L} = \frac{\Delta R/R}{\epsilon}$$

Practical measurement



How do we measure a change in resistance ΔR that is:

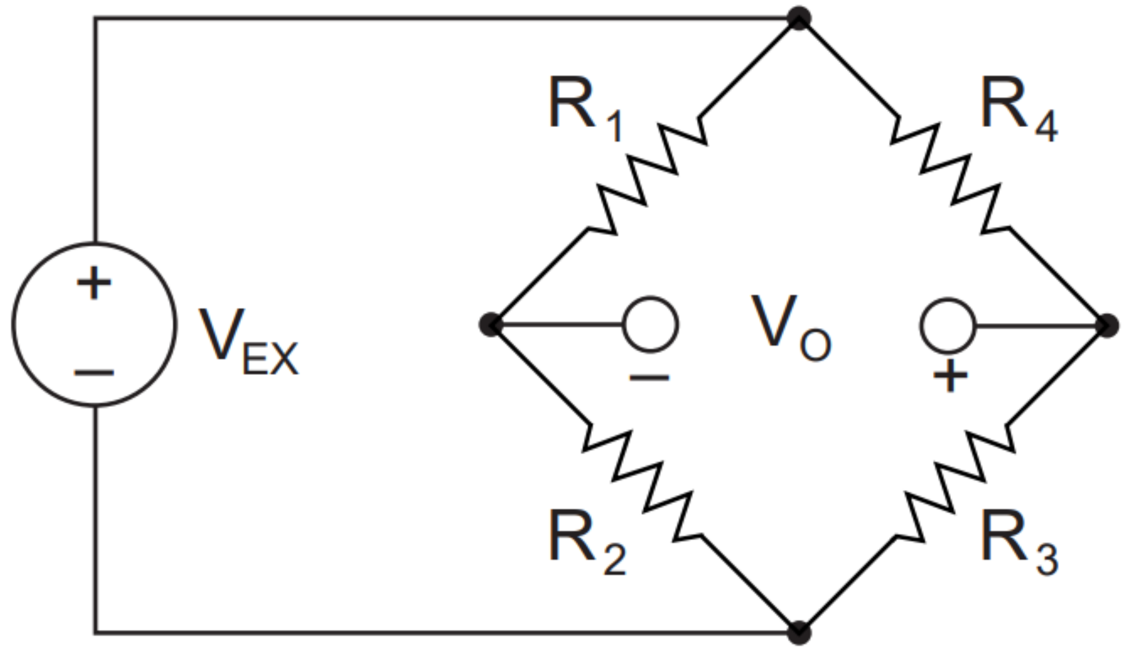
- tiny,
- sensitive to temperature?

Example: a test specimen undergoes a substantial strain of 500 με.

A strain gauge with a gauge factor GF = 2 will exhibit a change in electrical resistance of only 2 × (500 × 10–6) = 0.1%.

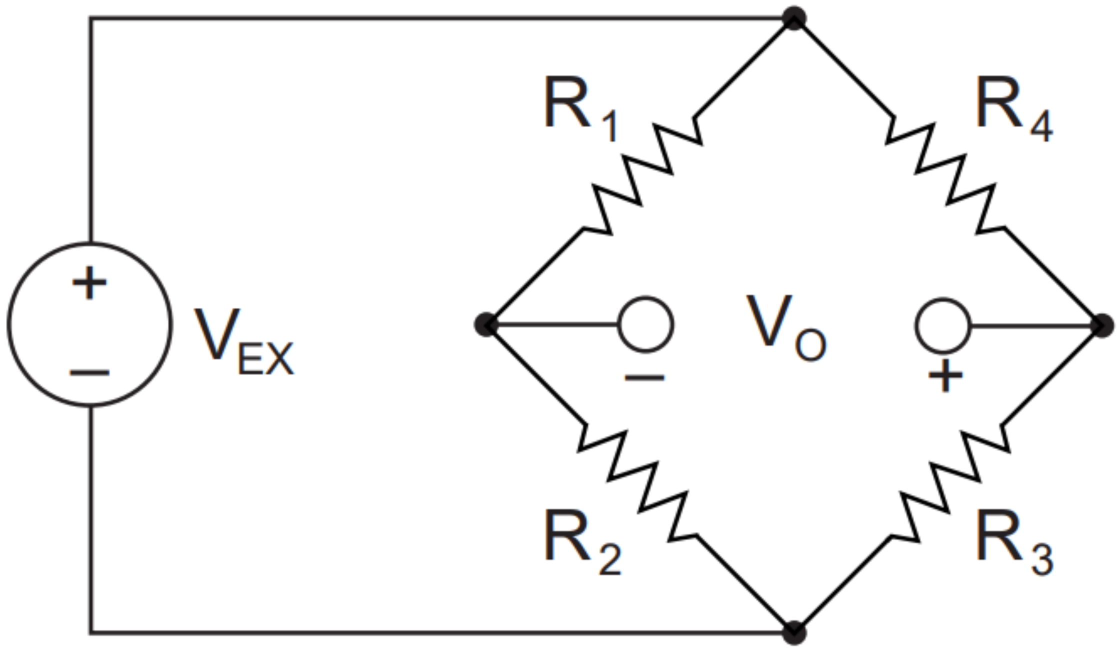
For a 120 Ω gauge, this is a change of only **0.12 Ω**

The Wheatstone bridge



$$\mathbf{V_O} = \left[\frac{R_3}{R_3 + R_4} - \frac{R_2}{R_1 + R_2}\right] \times \mathbf{V_{EX}}$$

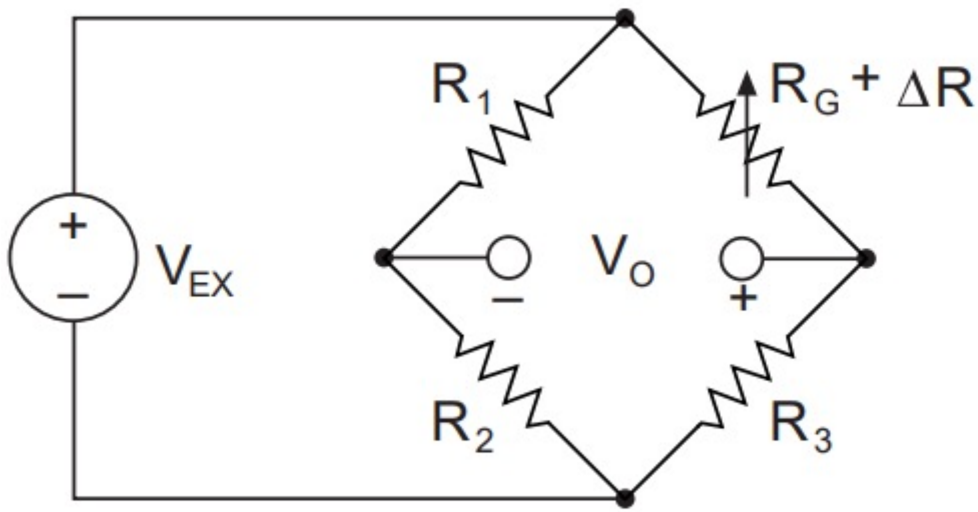
The Wheatstone bridge



balanced bridge

$$V_O = \left[\frac{R_3}{R_3 + R_4} - \frac{R_2}{R_1 + R_2}\right] \times V_{EX} = 0 \text{ iff } \frac{R_1}{R_2} = \frac{R_4}{R_3}$$

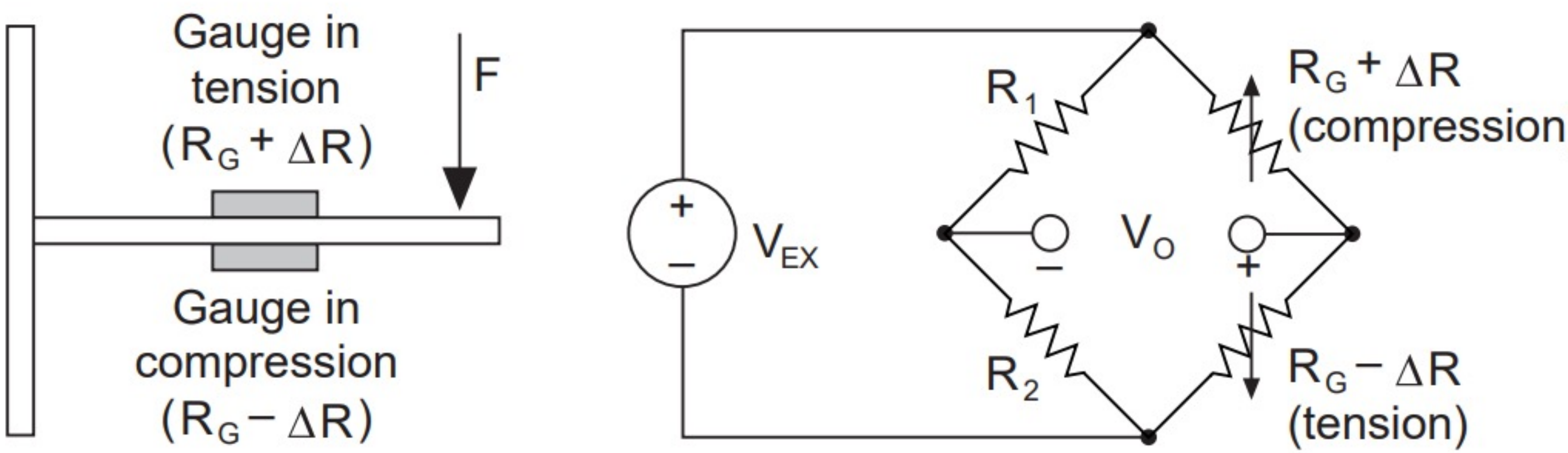
The Wheatstone bridge – quarter bridge



$$\mathrm{GF} = \frac{\Delta R / R_G}{\epsilon}$$

$$\frac{V_O}{V_{EX}} = -\frac{\mathrm{GF} \times \epsilon}{4}\left(\frac{1}{1 + \mathrm{GF} \times \frac{\epsilon}{2}}\right)$$

The Wheatstone bridge – half bridge

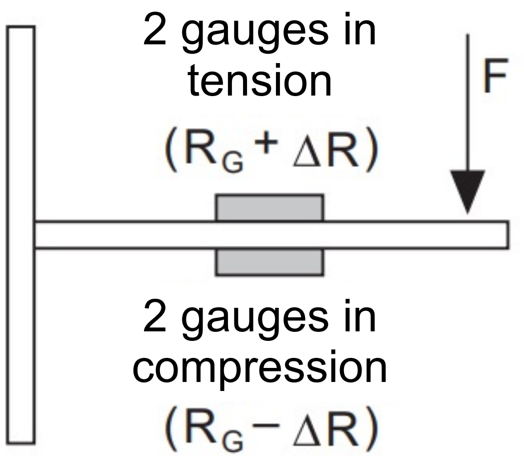


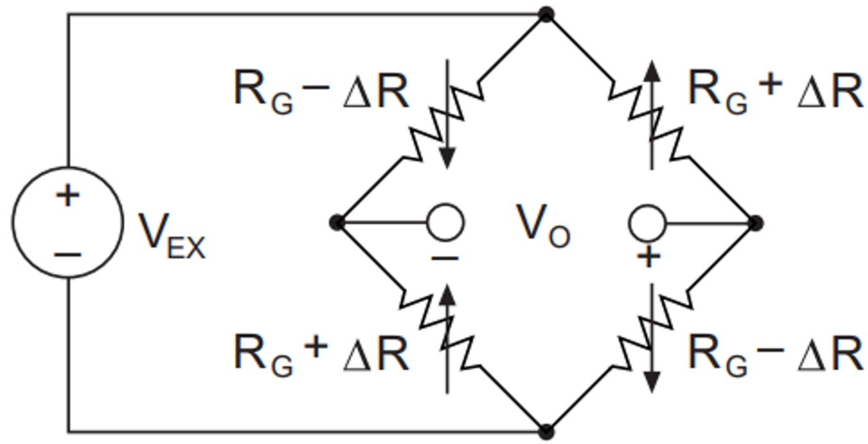
$$\mathrm{GF} = \frac{\Delta R / R_G}{\epsilon}$$

$$\frac{V_O}{V_{EX}} = -\frac{\mathrm{GF} \times \epsilon}{2}$$

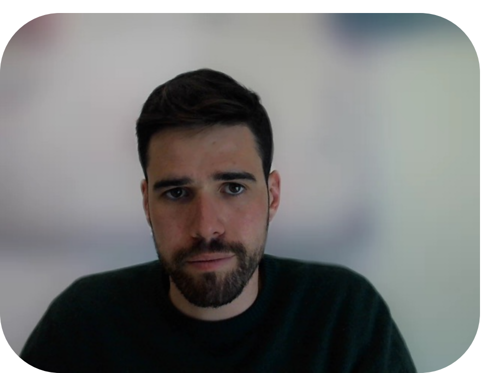
temperature compensated

The Wheatstone bridge – full bridge





$$\mathrm{GF} = \frac{\Delta R / R_G}{\epsilon}$$



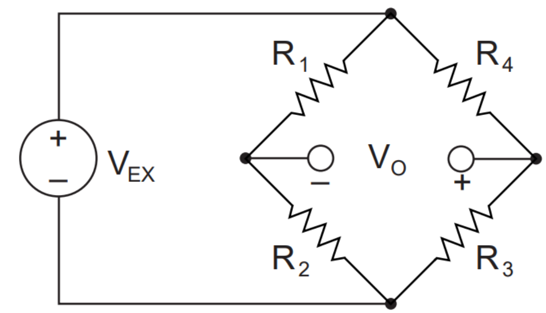
$$\frac{V_O}{V_{EX}} = -\mathrm{GF} \times \epsilon$$

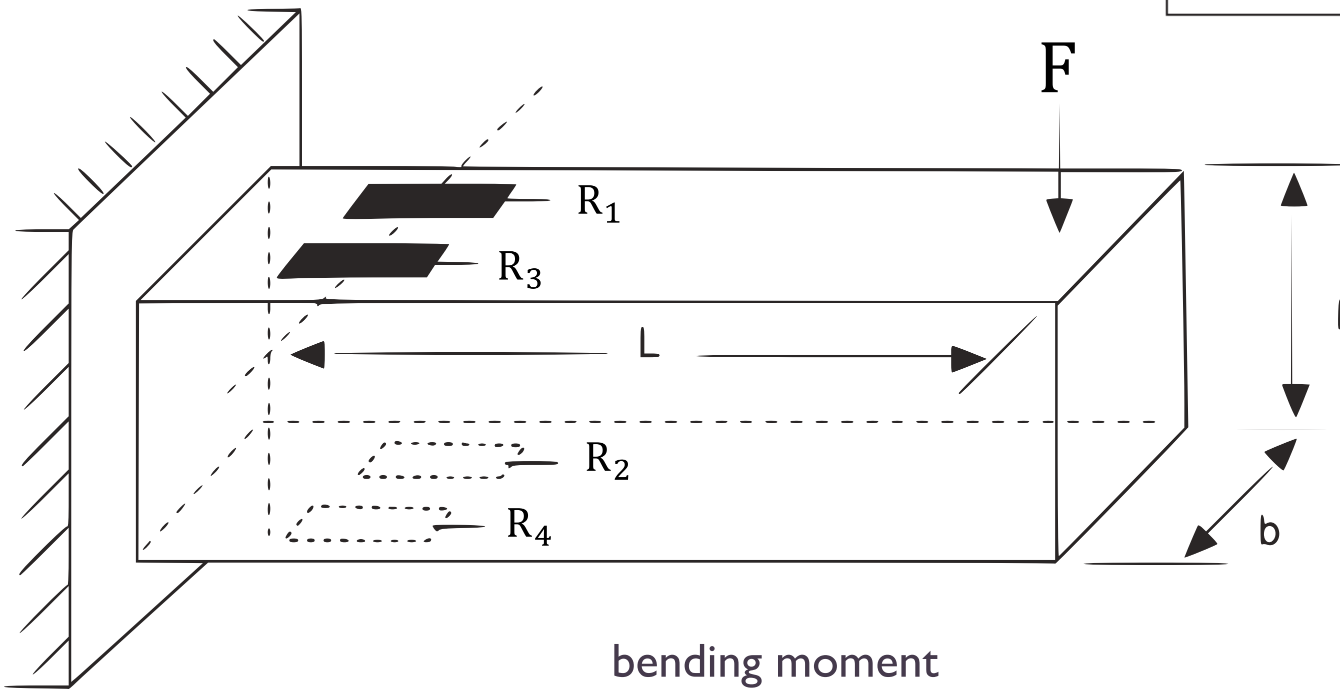
temperature compensated

Strain gauge position

How to position strain gauges on a specimen to monitor bending, axial, shear, and torsional loads?

Bending strain



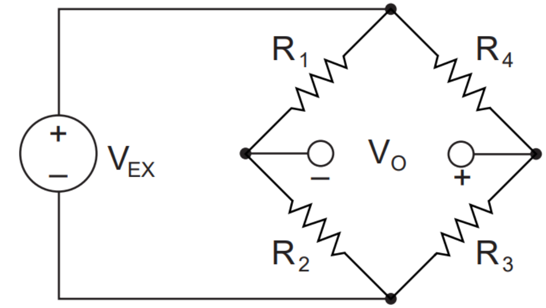


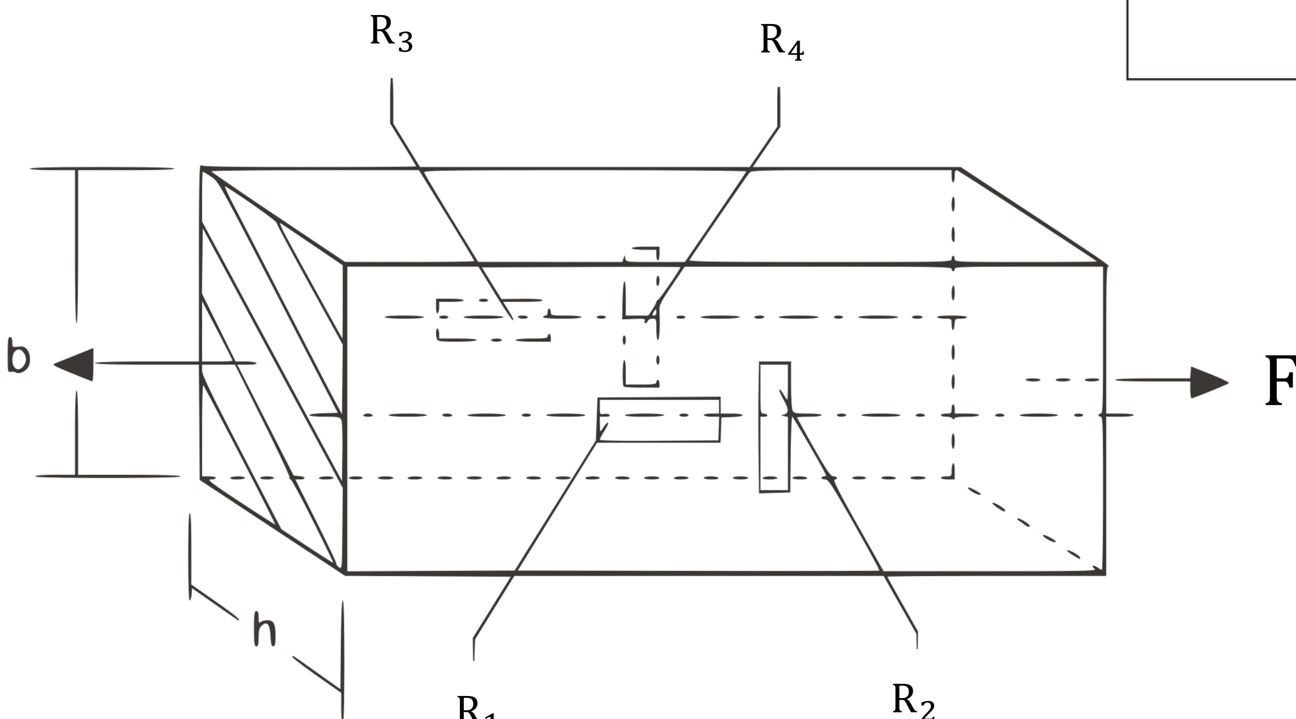
bending moment

strain

$$\epsilon \propto M = F \times L$$

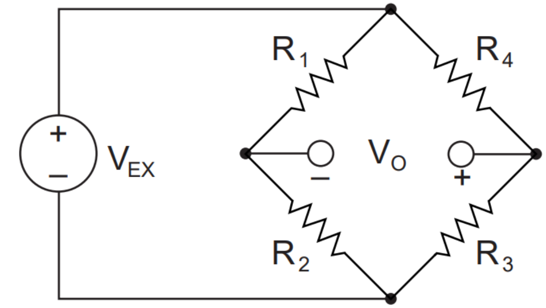
Axial strain

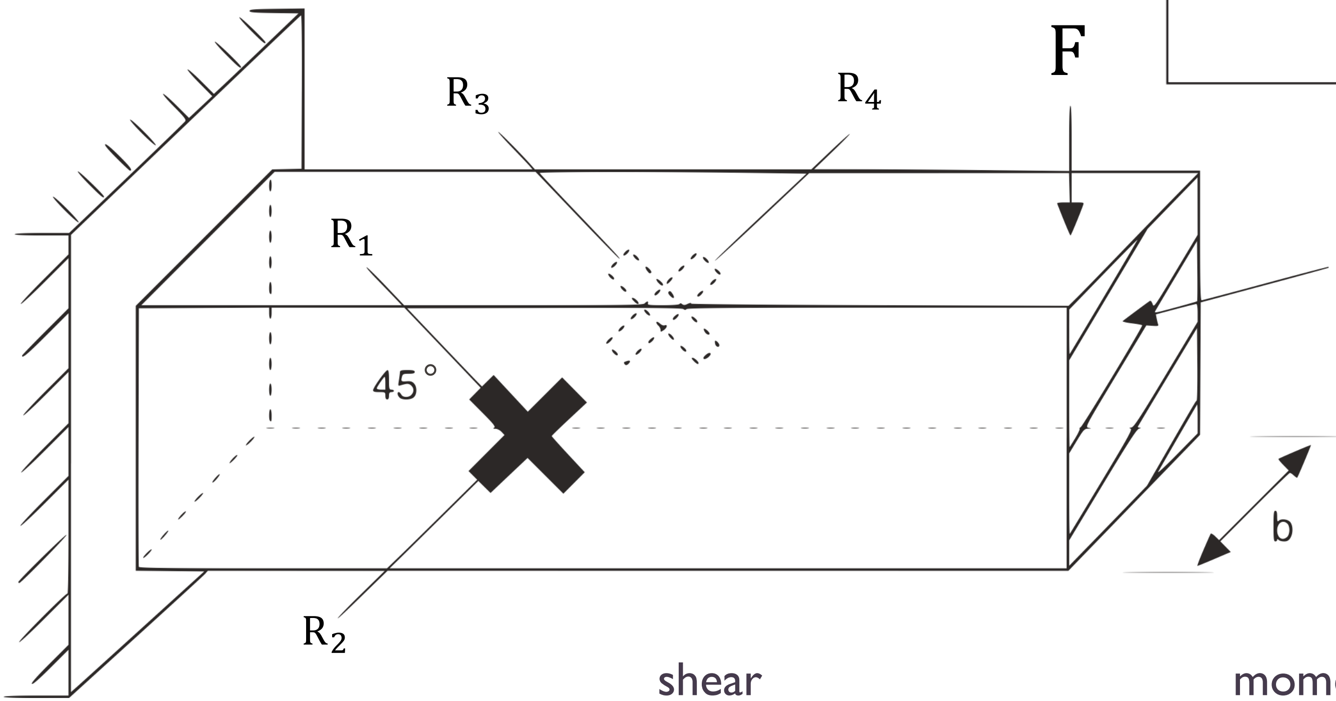




strain → $\epsilon \propto \sigma = \frac{F}{bh}$ ← stress

Shear strain





shear strain

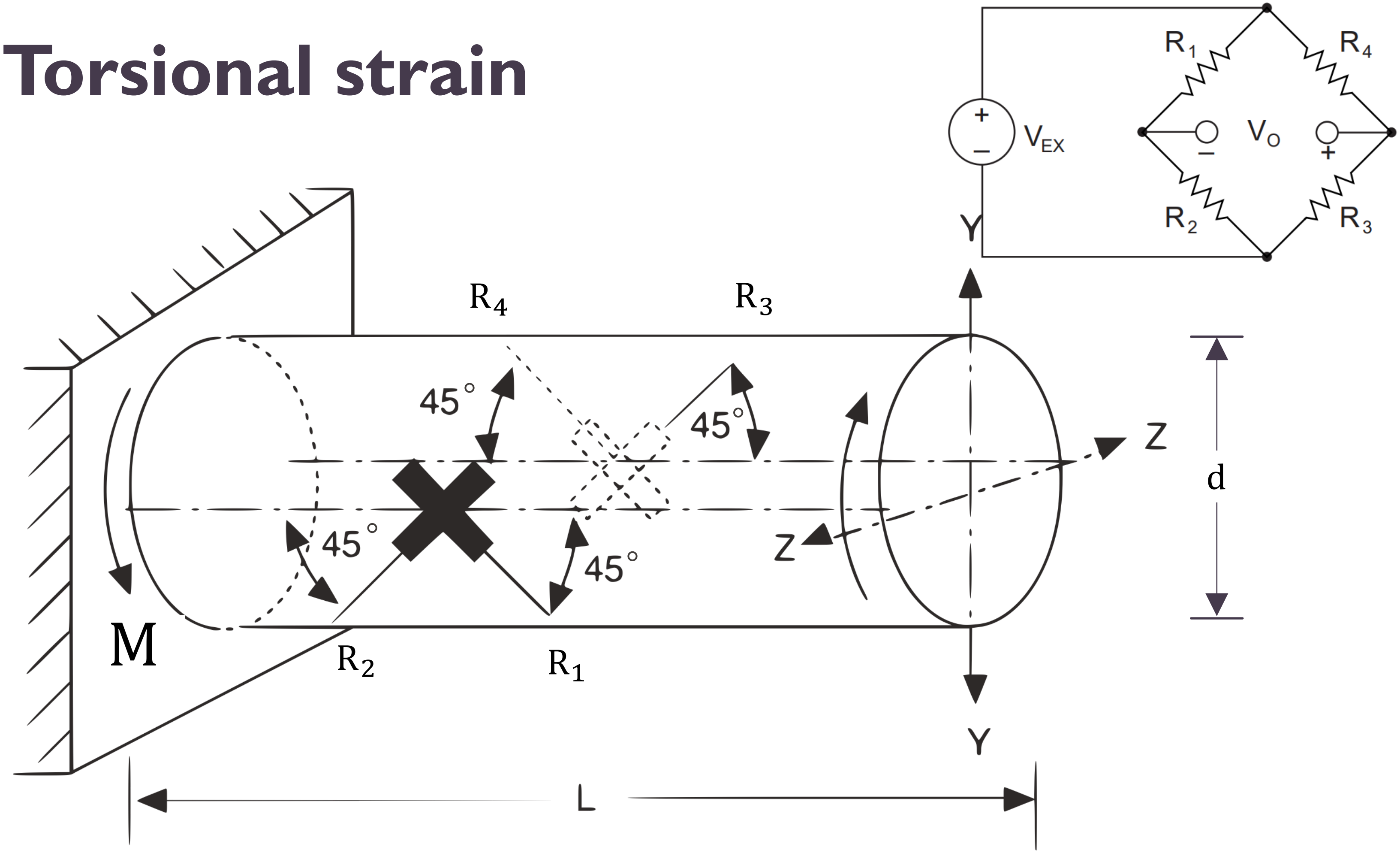
shear stress

moment of area

$$\gamma \propto \tau = \frac{F\times Q}{bI}$$

moment of inertia

Torsional strain



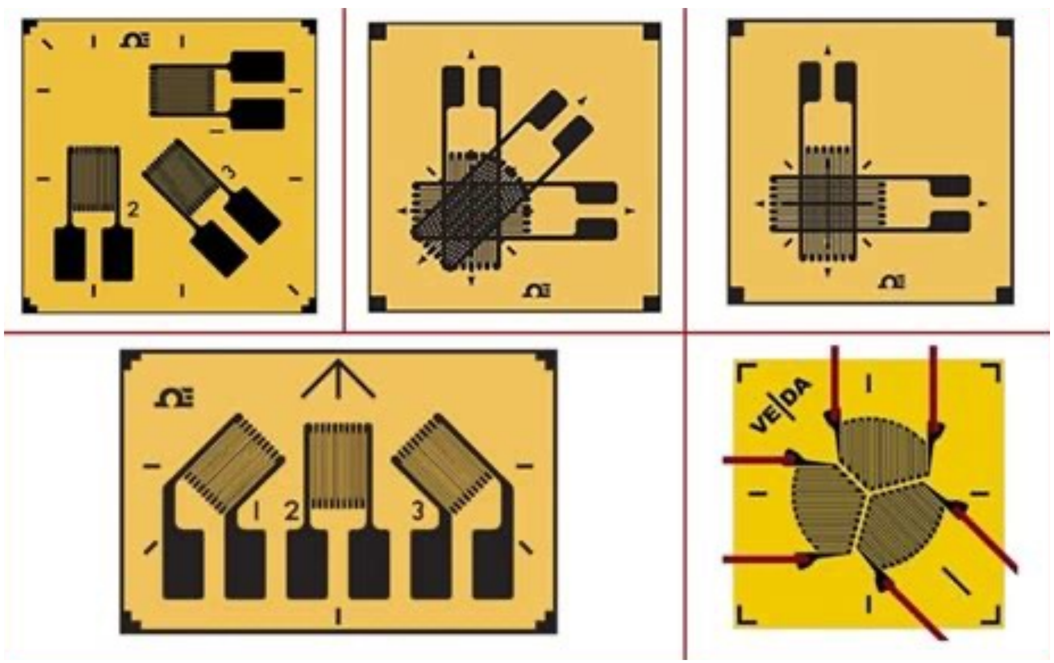
$$\gamma \propto \tau = \frac{M \times d/2}{J}$$

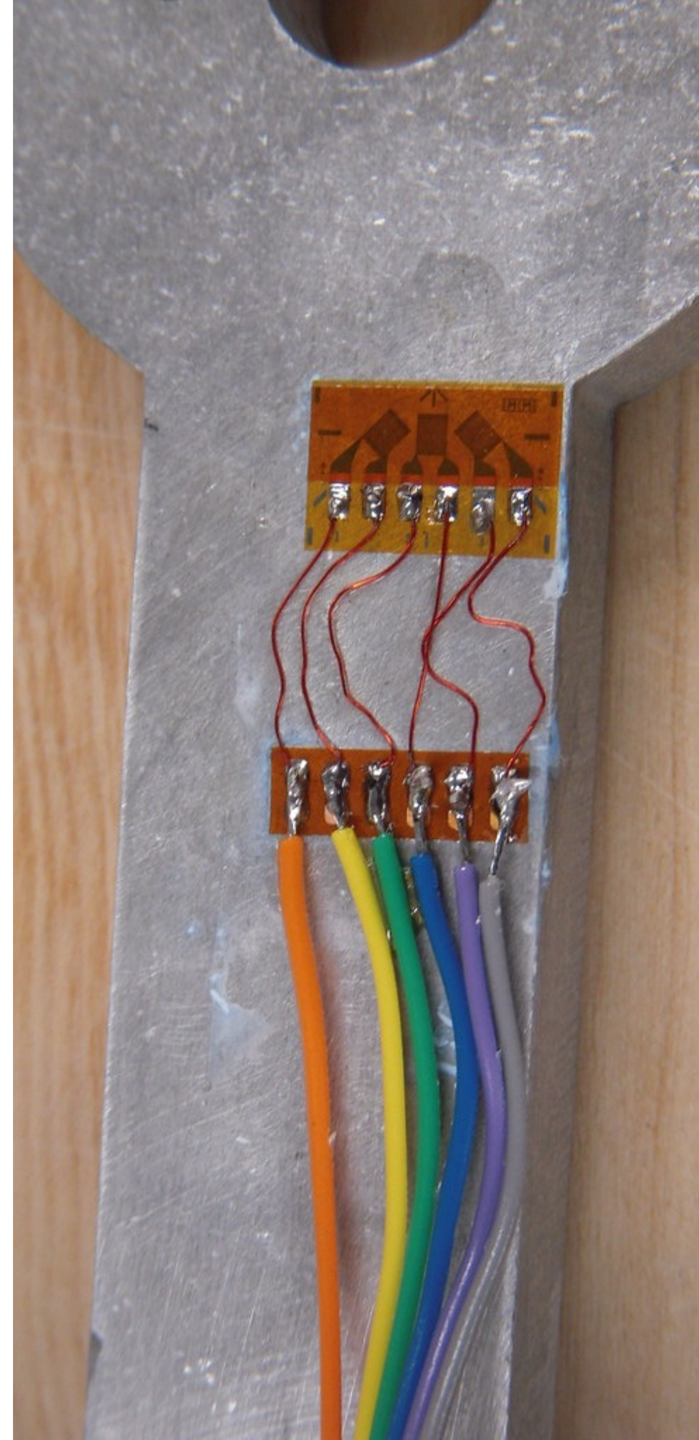
polar moment of inertia

Rosettes

For cases of:

- non uniaxial states for strain and stress
- limited space on specimen





Strain gauge selection and position

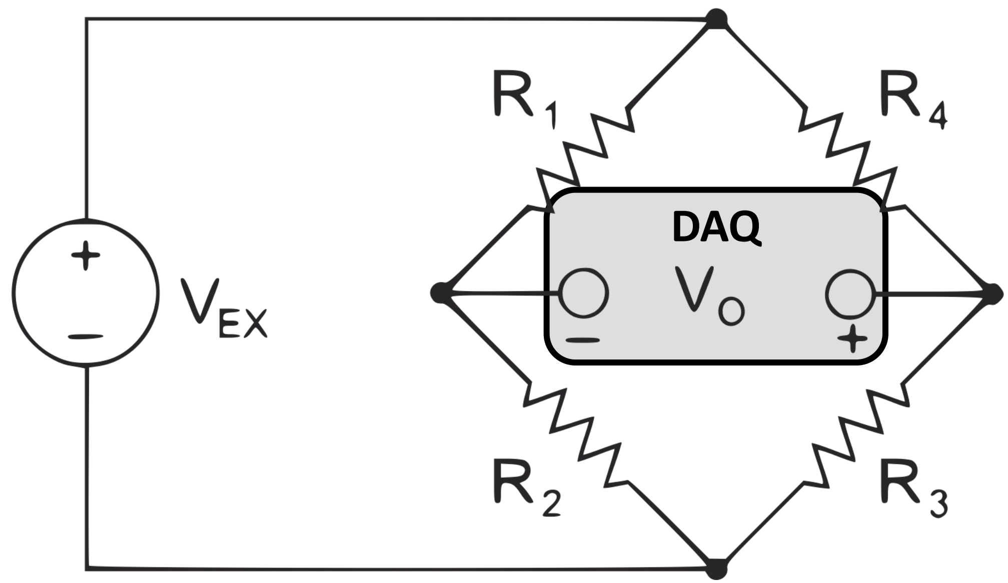
To measure forces or moments on a shaft:

1. Determine which components you wish to measure, forces or moments and direction.
2. Install strain gauges (type and position) such that:

$$\underbrace{\gamma \text{ or } \epsilon}_{\textbf{measured strain}} \propto \underbrace{M \text{ or } F}_{\textbf{force or moment of interest}}$$

3. Calibrate system to establish relationship between strain and force

Strain gauge calibration



How do we go from a voltage measurement to the physical force or moment we are interested in?

Calibration matrix

calibration matrix

forces and moments → $$[\mathrm{F}] = [\mathrm{R}][\mathrm{V_0}]$$ ← measured volts

for all 6 components

$$\begin{bmatrix} F_x \\ F_y \\ \vdots \\ M_z \end{bmatrix} = \begin{bmatrix} R_{11} & \cdots & R_{16} \\ \vdots & \ddots & \vdots \\ R_{61} & \cdots & R_{66} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_6 \end{bmatrix}$$

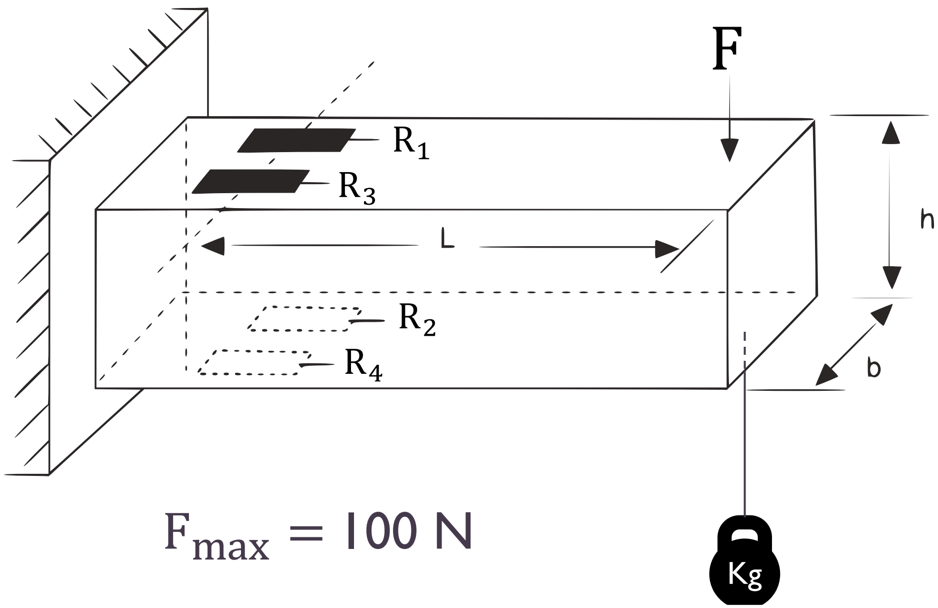
6x1	6x6	6x1
forces and moments	calibration matrix	voltage measurements

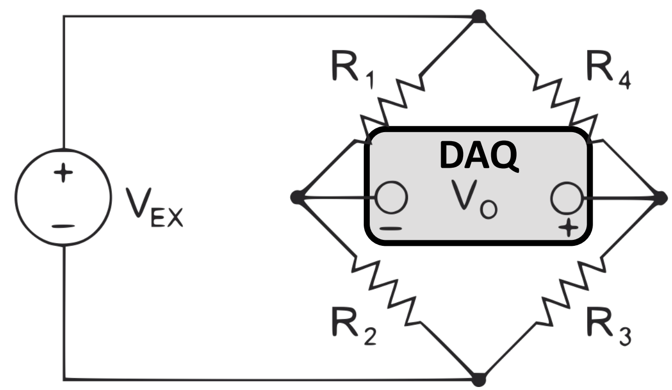
Calibration matrix

To obtain the calibration matrix

1. Obtain measurements with known weights, applying know forces and moments
2. Gradually increase weight from 0 to the maximum load you expect to measure
3. Repeat measurements
4. Solve $[\mathrm{V}_{\mathrm{cali}}] = [R]^{-1} [\mathrm{F}_{\mathrm{cali}}]$ for R using a linear regression

Calibration example



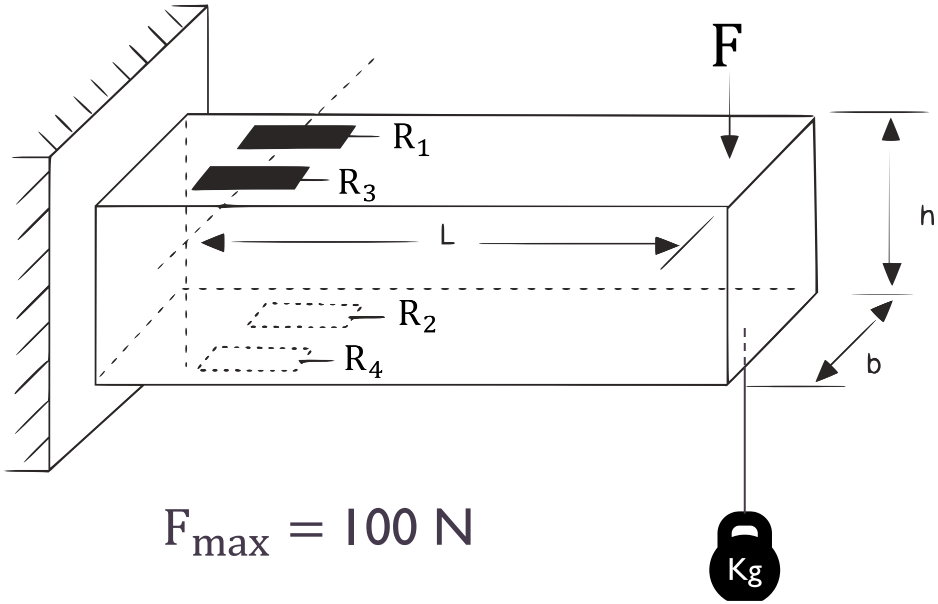


$F_{max} = 100\ N$

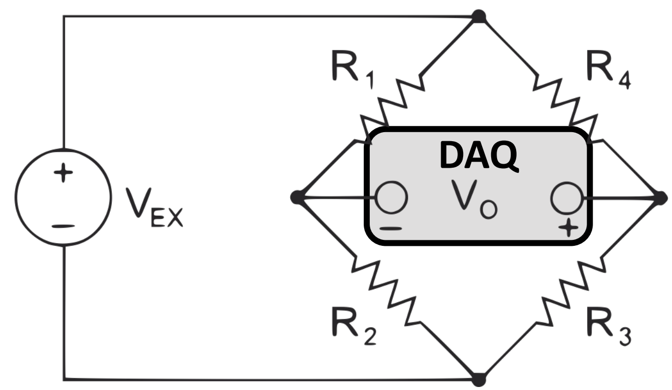
Apply weights of 2.5kg, 5kg, 7.5kg, 10kg to discretise your expected measurement range between 0-100N.

Generally, a pulley system is also used to apply an equal and opposite force.

Calibration example

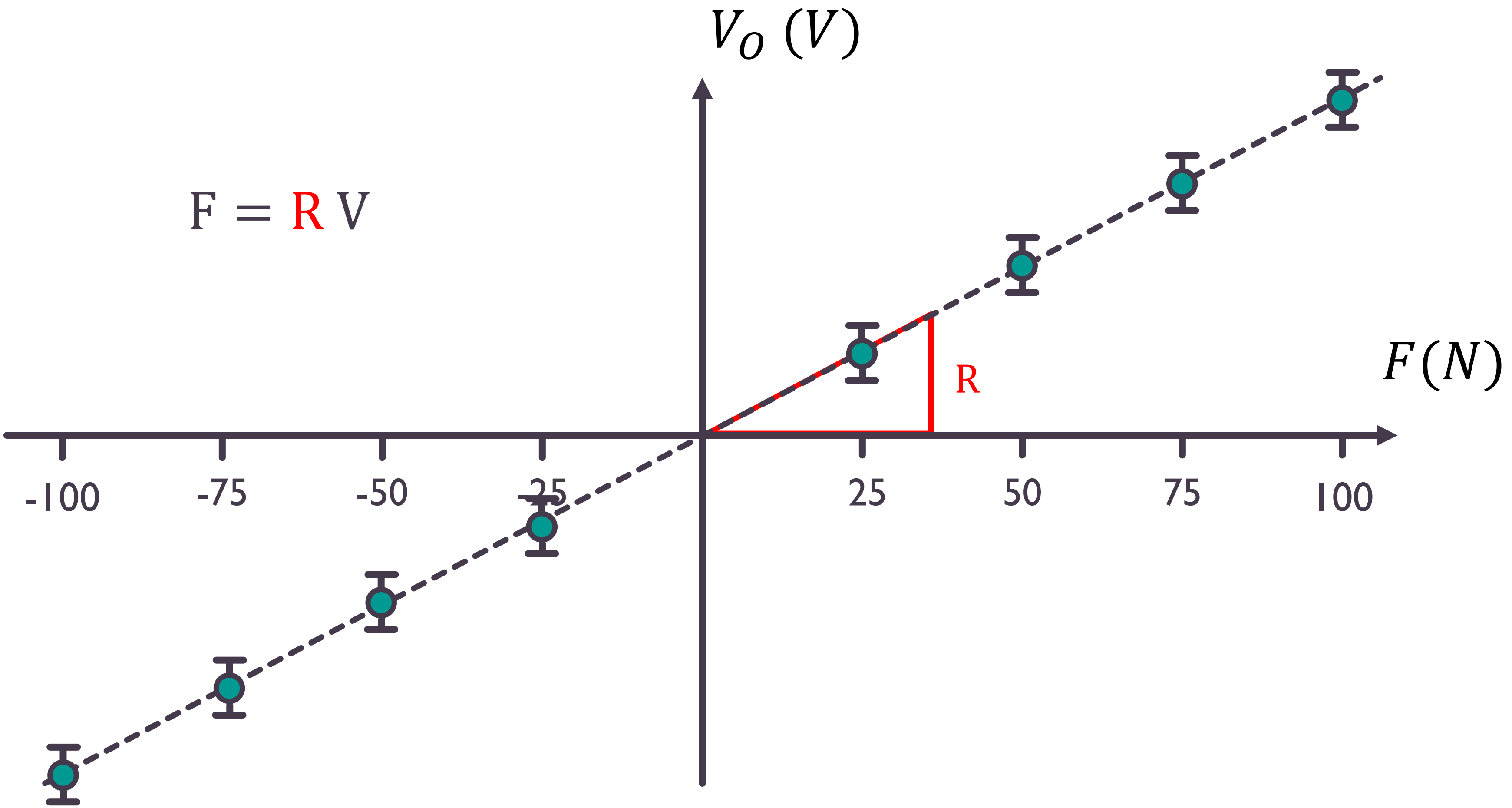


$F_{max} = 100$ N



- Apply weights of 2.5kg, 5kg, 7.5kg, 10kg to discretise your expected measurement range between 0-100N.
- Generally, a pulley system is used to apply an equal and opposite force as well.
- Repeating measurements helps quantify the measurement uncertainty.

Calibration example



Summary

- A strain gauge is a sensor whose measured electrical resistance varies with changes in strain.
- Several strain gauge types and configuration exist to measure forces and moment in different directions.
- Strain gauges are arranged into a Wheatstone bridge configuration to measure their variation in resistance.
- Once the system is connected, it can be calibrated to obtain the calibration matrix $[R]$ such that:

$$[F] = [R][V_0]$$