ME-280: Fluid Mechanics
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Problem Set 5: Control Volume Analysis

1 Vaccination

A hypodermic syringe (see Figure 1) is used to apply a vaccine. If the plunger is moved forward at a steady rate of $20\,mm/s$ and if vaccine leaks past the plunger at 0.1 of the volume flow rate out of the needle opening, calculate the average velocity of the needle exit flow. The inside diameters of the syringe and the needle are $20\,mm$ and $0.7\,mm$.
Hint: You can use a deforming or a non-deforming control volume to solve this exercise.

Figure 1: See problem 1.

2 How to anchor a pipe?

Water flows through a horizontal, 180° pipe bend as is illustrated in Figure 2. The flow cross-sectional area is constant at a value of $9000\,mm^2$. The flow velocity everywhere in the bend is $15\,m/s$. The absolute pressures at the entrance and the exit of the bend are $p_{in} = 210\,kPa$ and $p_{out} = 165\,kPa$, respectively. Calculate the horizontal (x and y) components of the anchoring force needed to hold the bended pipe in place. To obtain the force on the pipe you need to choose your control volume wisely.
Hint: Think about the role of atmospheric pressure $p_{atm} = 101\,kPa$.

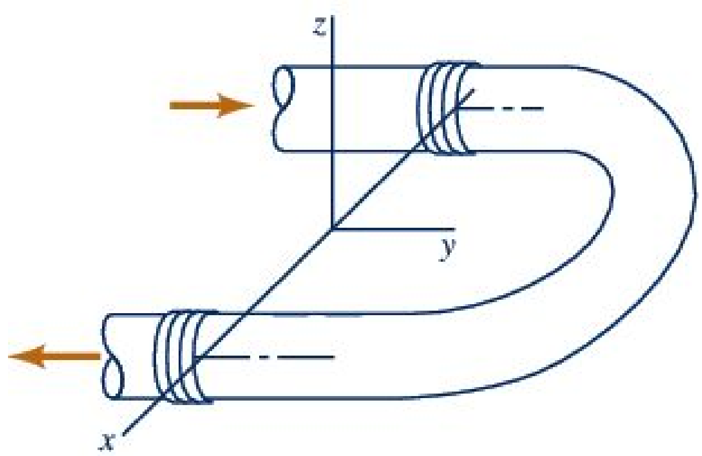

Figure 2: See problem 2.

3 How to anchor a plate?

A horizontal circular jet of air strikes a stationary flat plate as indicated in Figure 3. The jet velocity is $40\,m/s$ and the jet diameter is $30\,mm$. If the air velocity magnitude remains constant as the air flows over the plate surface in the directions shown, determine:

(a) the magnitude of F_A, which is the anchoring force required to hold the plate stationary.

(b) the fraction of mass flow along the plate surface in each of the two directions shown.

(c) the magnitude of F_A, the anchoring force required to allow the plate to move to the right at a constant speed of $c = 10\,m/s$.

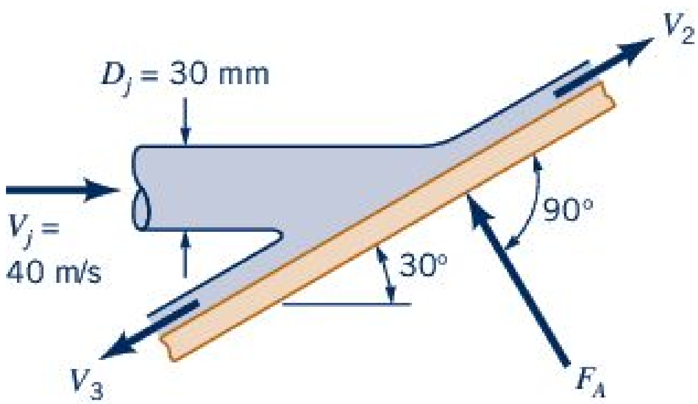

Figure 3: See problem 3.

4 Jet ski

The thrust developed to propel the jet ski shown in Figure 4 is a result of water pumped through the vehicle and exiting as a high-speed water jet. For the conditions shown in the figure, what flow rate is needed to produce a $1.3\,kN$ thrust? Assume the inlet and outlet jets of water are free jets.

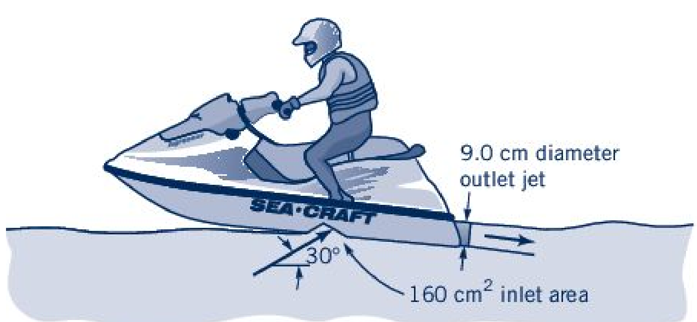

Figure 4: See problem 4.

5 Snowplow

A snowplow mounted on a truck clears a path of $3.6\,m$ width through heavy wet snow, as shown in Figure 5. The snow is $20\,cm$ deep and its density is $160\,kg/m^3$. The truck travels at $48\,km/h$. The snow is discharged from the plow at an angle of 45° above the horizontal, as shown in Figure 5. Estimate the force required to push the plow. Note that the problem is two-dimensional.

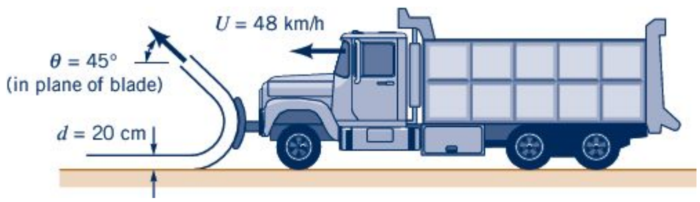

Figure 5: See problem 5.

6 Water power

Water is to be moved from one large reservoir to another at a higher elevation as indicated in Figure 6. The loss of available energy associated with $Q = 0.07\,m^3/s$ pumped from section (1) to (2) is loss $l = \left(61\,\bar{v}^2/2\right)\,m^2/s^2$, where $\bar{v}$ is the average velocity of water in the $20\,cm$ inside diameter piping involved.

(a) Select a control volume which allows you to simplify the general energy equation (RTT with first law of thermodynamics) to a sum over two surfaces which is valid for the *steady-in-the-mean flow* of this exercise. State all assumptions which you apply to arrive at a 1-D equation.

(b) Determine the amount of shaft power in the pump required to pump the water from (1) to (2).

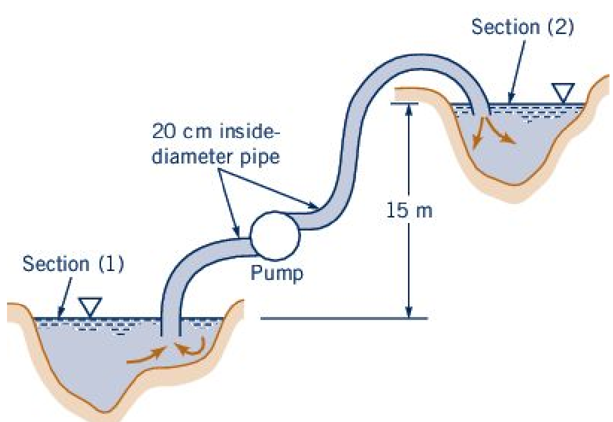

Figure 6: See problem 6.

7 Boundary layer flow

The flow of a viscous fluid over a plate results in the development of a region of reduced velocity adjacent to the wetted flat surface as depicted in Figure 7. The region of reduced flow velocity is called *boundary layer*. Inside the boundary layer the viscous force is significant but outside of it the flow remains inviscid. At the leading edge of the plate, the velocity profile may be considered uniformly distributed with a value U_∞. All along the outer edge of the boundary layer, the fluid velocity component parallel to the plate is also U_∞.

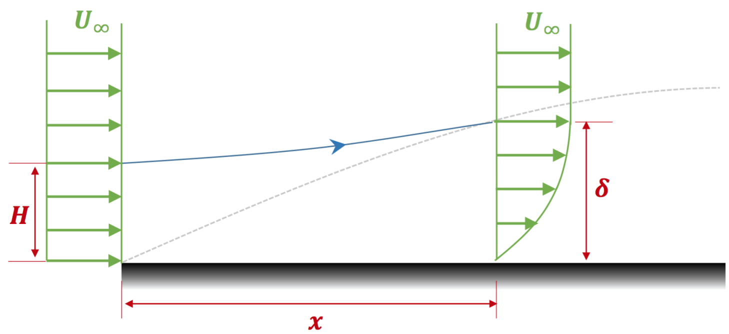

Figure 7: See problem 7.

(a) Approximate the x-direction velocity profile $u(y, \delta(x))$ inside the boundary layer by a second order polynomial. Here, y is the plate normal direction. Determine the polynomial by considering the following constraints:

- The velocity at the wall is zero.
- The velocity at $y = \delta$ is U_∞.
- The shear at $y = \delta$ is zero.

(b) A streamline that is initially located at height H, after a length x, intersects the boundary layer tip at height δ. Using mass conservation for an appropriate control volume of unit length along the cross stream direction z, find a relation between H and $\delta(x)$.

(c) Assuming that the only force acting on the flow is wall friction (we assume $p = \text{const.}$), write the momentum equation for the same control volume to find an integral equation for $\delta(x)$.
Hint: Use the relation of (b) to eliminate H.

(d) Differentiate the equation to find an ODE of the form $\delta' = f(x, \delta)$, and solve it by using an appropriate boundary condition.