

2 Free Jet

Water flows steadily from a large tank and exits through a constant diameter pipe as shown in Figure 2. The air in the tank is pressurized to 50 kPa . Assuming inviscid flow and gravitational acceleration $g = 10 \text{ m/s}^2$, determine

- the maximum possible height of water in the system, h .
- the water velocity in the pipe.
- the maximum pressure of water in the system and where it happens.

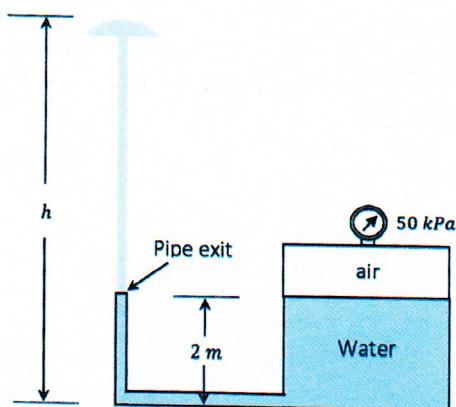


Figure 2: Problem 2

a) I can use the Bernoulli equation in this system: f ①

$$\frac{1}{2} \cancel{V_1^2} + p_1 + \cancel{5gz_1} = \frac{1}{2} \cancel{V_2^2} + \cancel{p_2} + \cancel{5gz_2} \quad ? \text{ ②}$$

Inserting the numbers in SI-units f ③

$$\begin{aligned} 50 \cdot 10^3 + 10^3 \cdot 10 \cdot 2 &= 10^3 \cdot 10 \cdot z_2 \\ 50 + 20 &= 10 \cdot z_2 \\ \underline{\underline{z_2 = 7 \text{ m}}} \end{aligned}$$

The height is 7 m .

(✓) ④

Comments:

① Missing:

- State all assumptions required to apply the Bernoulli equation
- Define your streamline

② State why terms cancel, e.g. by " $p_2 = 0$ "

③ Wrong "="-signs because physical quantities have units which must appear in all equations.

Defining everything "SI" does not make the equations below correct.

④ It is good practise to use a final sentence but be precise:

"The max. possible height of the jet is 7 m ."

b) The Bernoulli equation applies for the same reasoning as in a). Selecting a streamline between [✓] the top of the jet and exit of the pipe ^{OK} ⑥

$$\frac{1}{2} \rho v_e^2 + p_e + \rho g z_e = 7 [\text{m}] \rho g \quad (v) \quad ⑦$$

$$\frac{v_e^2}{2g} + \frac{p_e}{\rho g} + z_e = 7 [\text{m}] \quad ? \quad ⑧$$

with $\leftarrow$ gauge pressure *very good!*

$$p_e = 0 [\text{Pa}], z_e = 2 [\text{m}], g = 10 \left[\frac{\text{m}}{\text{s}^2} \right],$$

$$\rho = 10^3 \left[\frac{\text{kg}}{\text{m}^3} \right]$$

$$\Rightarrow \frac{v_e^2}{2g} = 7 [\text{m}] - 2 [\text{m}]$$

$$\Rightarrow \underline{v_e = 10 \left[\frac{\text{m}}{\text{s}} \right]}$$

c) It is obvious that the maximum of the pressure must be at the bottom of the tank:

$$p_B @ z_B = 0 \quad \text{f} \quad ⑩$$

Comparing with top of the jet

$$\frac{v_B^2}{2g} + \frac{p_B}{\rho g} + z_B = 7 \text{ m} \quad \text{very good}$$

$$\frac{v_B^2}{2g} + \frac{p_B}{\rho g} + z_B = 7 \text{ m}$$

$$\Rightarrow p_B = 10^3 \frac{\text{kg}}{\text{m}^3} \cdot 10 \frac{\text{m}}{\text{s}^2} \cdot 7 \text{ m} = 70 \text{ kPa}$$

The pressure is 70 kPa.

$\rightarrow$ ④

⑤ It is correct to do this, if assumptions are written in a)

⑥ Better indicate points in the figure of the problem.

⑦ Better start with the most general equation and remove terms based on justifications. Here, it is OK because you follow results from a)

⑧ Allow the reader to understand your calculations and note down the operations, e.g. by $(: \rho g = x)$

⑨ The brackets are not necessary here!

⑩ We expect you to derive results "step-by-step", even if they seem trivial to you!