2 Free Jet

Water flows steadily from a large tank and exits through a constant diameter pipe as shown in Figure 2. The air in the tank is pressurized to $50kPa$. Assuming inviscid flow and gravitational acceleration $g = 10\,m/s^2$, determine

(a) the maximum possible height of water in the system, h.
(b) the water velocity in the pipe.
(c) the maximum pressure of water in the system and where it happens.

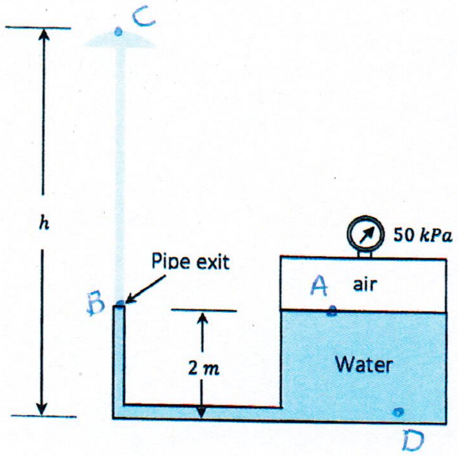

Figure 2: Problem 2

Assumptions:

1. As mentioned in the problem the flow is inviscid.
2. Water is an incompressible fluid.
3. The tank is large, so the flow is considered to be quasi-steady. Also, $V_A \simeq 0$.
4. There is a streamline from the water surface through the pipe to the highest point ($A \rightarrow B \rightarrow C$)

By these assumptions we can use Bernoulli eq.

(a) Bernoulli between points A and C:

$$\frac{P_A}{\gamma_w} + \frac{V_A^2}{2g} + H_A = \frac{P_c}{\gamma_w} + \frac{V_c^2}{2g} + H_c \quad \text{with} \begin{cases} P_A = 50\text{ kPa} & P_c = 0 \\ V_A = 0 & , V_c = 0 \\ H_A = 2m & H_c = ? \end{cases}$$

$$\Rightarrow \frac{50\text{ kPa}}{10^3 \frac{kg}{m^3} \cdot 10 \frac{m}{s^2}} + 0 + 2m = 0 + 0 + H_c \Rightarrow H_c = 5m + 2m = 7m$$

Why this solution is good:

1. All the assumptions are stated clearly.
2. Solution starts with the most general form of the equation.
3. Numbers are inserted in the equation and no term is canceled without any reasoning.
4. Quantities are inserted with units.

(b) Bernoulli between points B and C:

$$\frac{P_B}{\gamma_w} + \frac{V_B^2}{2g} + H_B = \frac{P_C}{\gamma_w} + \frac{V_C^2}{2g} + H_C \quad \text{with} \begin{cases} P_B = 0 & P_C = 0 \\ H_B = 2m & V_C = 0 \\ V_B = ? & H_C = 7m \end{cases}$$

$$\Rightarrow 0 + \frac{V_B^2}{2g} + 2m = 0 + 0 + 7m \Rightarrow V_B^2 = 2 \cdot 10 \frac{m}{s^2} (7m - 2m) = 100 \frac{m^2}{s^2}$$

$$\Rightarrow V_B = 10 \ m/s$$

(c) Bernoulli between any point in the system (D)

and point C

$$\frac{P_D}{\gamma_w} + \frac{V_D^2}{2g} + H_D = \frac{P_C}{\gamma_w} + \frac{V_C^2}{2g} + H_C \quad \text{with} \begin{cases} P_C = 0 \\ H_C = 7m \\ V_C = 0 \end{cases}$$

$$\Rightarrow \frac{P_D}{\gamma_w} + \frac{V_D^2}{2g} + H_D = 7m$$

So, maximum pressure means minimum $\frac{V_D^2}{2g} + H_D$. At the bottom of the tank $V_D = 0$, $H_D = 0$, so maximum pressure happens at the bottom of the tank

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$$\frac{P_D}{\gamma_w} + 0 + 0 = 7m \Rightarrow P_D = 10^3 \frac{kg}{m^3} \cdot 10 \frac{m}{s^2} \cdot 7m = 70 \ kPa$$

5. There is a clear reasoning to find the answer.