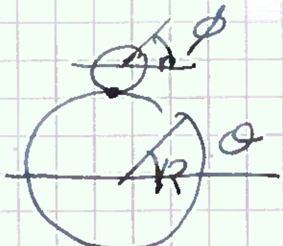
Combinations of schemes.

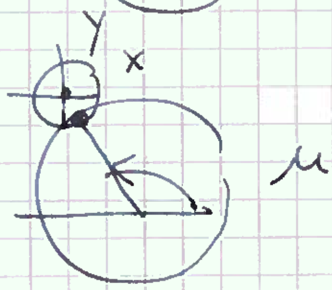
Example IV

Ball & Wheel.
Bille sur une roue



Liaison cinématique des vitesses:

$$\mu = \frac{r}{R}\phi + \theta$$



μ : angle du point de contact [rad]
θ : angle de la roue [rad]
ϕ : angle de la bille [rad].

x, y positions de la bille

Le lagrangien:

$$L = E_{cin} - E_{pot}$$

$$E_{cin} = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{y}^2 + \frac{1}{2} I_R \dot{\theta}^2 + \frac{1}{2} I_r \dot{\phi}^2$$

I_R : inertie de la roue [kg·m²]
I_r : inertie de la bille [kg·m²]
m : masse de la bille [kg]

$E_{pot} = m g y$

$$x = (r+R)\cos(\mu) = (r+R)\cos\left(\frac{r}{R}\phi + \theta\right)$$

$$y = (r+R)\sin(\mu) = (r+R)\sin\left(\frac{r}{R}\phi + \theta\right)$$

2 coordonnées généralisées : θ et ϕ

Mécanique analytique : formule de Lagrange

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = F_q$$

$q \to \theta$
$q \to \phi$

$F_\phi = 0 \qquad F_\theta = \tau$ couple [N·m] (moment de force).

$$\text{(A)}\quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta}\right) - \frac{\partial L}{\partial \theta} = \tau \qquad \text{(B)}\quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot\phi}\right) - \frac{\partial L}{\partial \phi} = 0$$

$$\cancel{E_{cin}} = \quad \dot x = -(r+R)\sin\left(\frac{r}{R}\phi + \theta\right)\left(\frac{r}{R}\dot\phi + \dot\theta\right)$$

$$\dot y = (r+R)\cos\left(\frac{r}{R}\phi + \theta\right)\left(\frac{r}{R}\dot\phi + \dot\theta\right)$$

$$\frac{1}{2} m \dot x^2 + \frac{1}{2} m \dot y^2 =$$

$$\frac{1}{2} m (r+R)^2 \left(\frac{r}{R}\dot\phi + \dot\theta\right)^2 \qquad (\text{car } \sin^2 + \cos^2 = 1)$$

$$E_{cin} = \frac{1}{2} m (r+R)^2 \left(\frac{r}{R}\dot\phi + \dot\theta\right)^2 + \frac{1}{2} I_R \dot\theta^2 + \frac{1}{2} I_r \dot\phi^2$$

$$E_{pot} = mg(r+R)\sin\left(\frac{r}{R}\phi + \theta\right)$$

$$\mathcal{L} = E_{cin} - E_{pot}$$

$$\text{(A)} \to \frac{\partial \mathcal{L}}{\partial \dot\theta} = I_R \dot\theta + m(r+R)^2\left(\dot\theta + \frac{r}{R}\dot\phi\right) \quad ✓$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot\theta} = I_R \ddot\theta + m(r+R)^2\left(\ddot\theta + \frac{r}{R}\ddot\phi\right)$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = -mg(r+R)\cos\left(\frac{r}{R}\phi + \theta\right)$$

$$\boxed{\text{(A)}\quad \left(I_R + m(r+R)^2\right)\ddot\theta + m(r+R)^2\frac{r}{R}\ddot\phi + mg(r+R)\cos\left(\frac{r}{R}\phi + \theta\right) = \tau}$$

$$\text{(B)} \to \frac{\partial \mathcal{L}}{\partial \dot\phi} = \frac{r}{R} m (r+R)^2 \left(\frac{r}{R}\dot\phi + \dot\theta\right) + I_r \phi$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot\phi} = m(r+R)^2\left(\frac{r}{R}\ddot\phi + \ddot\theta\right) + I_r \ddot\phi$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = -mg(r+R)\frac{r}{R}\cos\left(\frac{r}{R}\phi + \theta\right)$$

$$\boxed{\text{(B)}\quad \frac{r}{R} m (r+R)^2 \ddot\theta + \left(I_r + \frac{r^2}{R^2} m (r+R)^2\right)\ddot\phi + mg(r+R)\frac{r}{R}\cos\left(\frac{r}{R}\phi + \theta\right) = 0}$$

$$\begin{bmatrix} I_R + m(r+R)^2 & \frac{r}{R}\, m(r+R)^2 \\ \frac{r}{R}\, m(r+R)^2 & I_r + \frac{r^2}{R^2}\, m(r+R)^2 \end{bmatrix}\begin{bmatrix}\theta'' \\ \phi''\end{bmatrix}$$

$$= \begin{bmatrix} -mg\,(r+R)\cos\left(\frac{r}{R}\phi+\theta\right) + \tau \\ -mg\,(r+R)\,\frac{r}{R}\cos\left(\frac{r}{R}\phi+\theta\right)\end{bmatrix}$$

<u>Etats</u>

$$\begin{aligned} x_1 &= \theta \\ x_2 &= \dot{\theta} \\ x_3 &= \phi \\ x_4 &= \phi' \end{aligned}$$

<u>entrée</u> : couple $\tau = u$

<u>sortie</u> position de contact $y = \mu = \frac{r}{R}\phi + \theta$

modèle d'état :

$$\begin{cases} \dot{\underline{x}} = \underline{f}(\underline{x}, u) \\ y = h(\underline{x}) \end{cases}$$

non linéaire.