ME – 221
MATLAB® ASSIGNMENT 1

You are expected to upload 2 files:

1. A **report** (.pdf file) that addresses the questions by showing the equations derived, the plots generated by the MATLAB script, and your interpretation of the results.

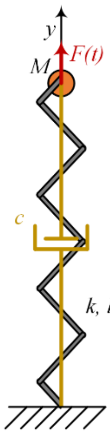
2. The **MATLAB file** (.m file), written to simulate the systems. A MATLAB file is uploaded to serve as a template.

File naming: Surname_SCIPER_Assignment1.pdf and Surname_SCIPER_Assignment1.m

Important!

- All plots, developments, and results must appear in the report.
- When using `ode45` you may need to adjust the error tolerance
```
opts = odeset('RelTol',1e-4,'AbsTol',1e-6);
```
- Use enough points for the time vector.
- Go over the example from the tutorial to see how to use `ODE45` properly.

Consider the following system:



The system comprises mass M in a gravity field g. The mass M is attached to a spring (k) and a linear damper (c). The mass m is forced by $F(t)$. The governing dynamical equation of motion of the system is:

$$M\ddot{y}(t) + c\dot{y}(t) + f_k(t) = F(t) - Mg \tag{1}$$

The force applied by the spring is:

$$f_k = k\Delta \tag{2}$$

Where $\Delta = y(t) - l_0$ represents the relative length of the spring (i.e., the instantaneous length of the spring minus its initial length).

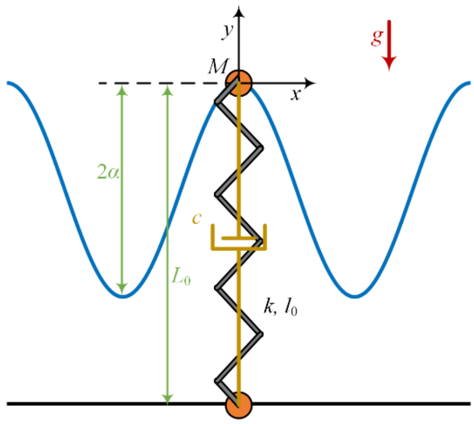
The parameters of the system are:

$$M = 1.2\text{kg}, \quad k = 300\,\text{N/m},$$
$$c = 35\text{Ns/m}, \quad g = 10\,\text{m/s}^2, \quad l_0 = 0.15\,m$$

1) Perform order reduction to the governing equation of motion to implement it in MATLAB `ode45` function. Present the modified equations.

2) Simulate the dynamic response to the harmonic force $F = 20\sin(\omega t)$ from the rest position of the spring, for the following frequencies: $\omega = \{1, 5, 17\}$ rad/s. Simulate the response for at least 10s, display the results graphically, and comment on what you observe.
   Remarks:
   - You were given the function `A1_P1_2025ode.m`, which already realizes the order reduction correctly.
   - The rest position of the spring $y(0)$ is found when the spring counteracts gravity in the absence of the harmonic force and when the particle has stopped moving.
   - Use enough points in the time vector; you can try with 1000.

Now consider the modified system:



Instead of being driven by a harmonic force, consider that the system is constrained to move without friction on the blue sinusoidal rail, which is located at a distance L_0 from the origin, as shown in the figure below. The attachment point between the spring, damper, and the ground is free to move along the black rail without friction. The blue rail is described by the function:

$$y(x) = \alpha\left[\cos(\beta x) - 1\right] \tag{3}$$

With the following parameters:

$$\alpha = 0.15 \text{ m}, \quad \beta = 0.32\text{m}^{-1}, \quad l_0 = \{0.15, \quad 0.38 \quad 0.84\} \text{ m}, \quad L_0 = 0.4m$$

3) Without explicitly writing the Euler-Lagrange equations, proceed with the steps required to obtain them:
   a. Write the position vector $\mathbf{r}(x)$ of the bead mass M, according to the coordinate system shown in the figure.
   b. Find the expressions for kinetic energy, potential energy, and Rayleigh dissipation function.

The equation of motion of the system, which can be derived from the Lagrangian, is given by:

$$M\left[1 + \alpha^2\beta^2 \sin(\beta x)^2\right]\ddot{x} + c\alpha^2\beta^2 \sin(\beta x)^2 \dot{x} + \frac{1}{2}M\alpha^2\beta^3 \sin(2\beta x)\dot{x}^2 - Mg\alpha\beta\sin(\beta x)$$
$$+\left(k\left\{l_0 - L_0 + \alpha\left[1 - \cos(\beta x)\right]\right\}\right)\alpha\beta\sin(\beta x) = 0 \tag{4}$$

After order reduction, the equation of motion can be written as:

$$\mathbf{z} = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} x \\ \dot{x} \end{pmatrix}$$

$$\dot{\mathbf{z}} = \begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = \begin{pmatrix} \dot{x} \\ \ddot{x} \end{pmatrix} = \begin{pmatrix} z_2 \\ \dfrac{1}{M\left[1+\alpha^2\beta^2\sin\left(z_1\right)^2\right]} \begin{bmatrix} -c\alpha^2\beta^2\sin\left(\beta z_1\right)^2 z_2 - \\ \dfrac{1}{2}M\alpha^2\beta^3\sin\left(2\beta z_1\right)z_2^2 + Mg\alpha\beta\sin\left(\beta z_1\right) - \\ \left(k\left\{l_0 - L_0 + \alpha\left[1-\cos\left(\beta z_1\right)\right]\right\}\right)\alpha\beta\sin\left(\beta z_1\right) \end{bmatrix} \end{pmatrix}$$

4) Plot the position y(x) and the potential energy V(x) for $-4\pi \le x \le 4\pi$, with at least 1000 points, on the same graph. One figure for each of the three different values of l_0. Where do you expect the stable equilibrium points to be, and why are they different for each case? What do you expect to find outside of the plotted region? How would the potential energy impact the motion of the particle for different initial conditions?

5) Simulate the dynamic response for the following initial conditions $\{x(0),\dot{x}(0)\} = \{-0.1,0\},\{8,1.6\}$ for $l_0 = 0.38$m. Simulate for at least 400s and with at least 2000 timesteps. Show the results for each case using two graphs: 1) depicting the displacement $x(t)$ and velocity $\dot{x}(t)$ on the same graph and 2) depicting the potential energy $V(x)$, rail $y(x)$, and the bead trajectory $\mathbf{r}(x)$ on the rail (see the example below for different values than you have). Explain what happens in each simulation (i.e., the role of the initial conditions, the role of the potential energy, the trajectory of the particle, …)

<u>**Note:**</u> you were given the function `A1_P2_2025ode.m`, which already realizes the order reduction correctly.