Problem 1

1. $E_g = K_g w \quad T_b = K_b i$ (1)

$E_g - R_1 i - L\frac{di}{dt} - R_2 i = 0$ (1) (3)

$J\frac{dw}{dt} = T(t) - f_R w - K_b i$ (2) (3)

7

3. $E_g(s) = K_g W(s) \quad T_b(s) = K_b I(s)$

$K_g W(s) - R_1 I(s) - LsI(s) - R_2 I(s) = 0$ (1) (3)

$Js\,W(s) = T(s) - f_R W(s) - K_b I(s)$ (2)

$W(s) = \frac{(R_1 + R_2 + Ls)}{K_g} I(s)$

6

$(Js + f_R)\frac{(R_1+R_2+Ls)}{K_g} I(s) = T(s) - K_b I(s)$

$$\frac{I(s)}{T(s)} = \frac{K_g}{(Js+f_R)(R_1+R_2+Ls) + K_b K_g}$$ (3)

2. $x_1 = w \quad x_2 = \dot{w} \quad x_3 = i$ (6)

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -f_R/J & -K_b/J \\ 0 & K_g/L & -\frac{R_1+R_2}{L} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1/J \\ 0 \end{bmatrix} \quad C = [0 \;\; 0 \;\; R_2] \quad D = 0$$

4. $G(s) = \dfrac{9}{(0.1s+0.4)(8+2s)+1.8} = \dfrac{45}{s^2+8s+25}$

$\omega = 5 \quad \zeta = 0.8$

4

$K = \dfrac{45}{25}$ (steady-state value) (1)

$\zeta > \dfrac{1}{\sqrt{2}} \rightarrow$ no resonance (3)

5. $|G(j\omega)| = \dfrac{45K}{\sqrt{\tau^2\omega^2+1}\sqrt{(25-\omega^2)^2+(8\omega)^2}}$

$|G(j\omega)|_{\omega=5} = \dfrac{45K}{\sqrt{25\tau^2+1}\,(40)} = 45$ (1) (2.5)

$\varphi = -\pi/2 - \arctan(5\tau) = -3\pi/4$ (2) (2.5)

$\arctan(5\tau) = \pi/4 \quad \tau = \dfrac{1}{5}$ (1)

7

$K = 40\sqrt{2}$ (1)

Problem 2

1) $m\ddot{x}_3 + f_2\dot{x}_3 + k_2 x_3 = f_2\dot{x}_2 + k_2 x_2$ 2pt

$(f_1+f_2)\dot{x}_2 + (k_1+k_2)x_2 = k_2 x_3 + f_2\dot{x}_3 + k_1 x_1 + f_1\dot{x}_1$ 2pt

Laplace domain

$(ms^2 + f_2 s + k_2)X_3 = (f_2 s + k_2)X_2$ 2pt

$[(f_1+f_2)s + k_1 + k_2]X_2 = (k_2 + f_2 s)X_3 + (k_1 + f_1 s)X_1$ 2pt

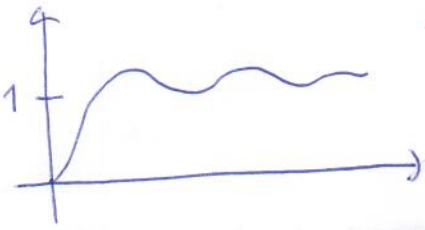
$$G(s) = \frac{(f_1 s + k_1)(f_2 s + k_2)}{ms^2[(f_1+f_2)s + k_1 + k_2] + (f_2 s + k_2)(f_1 s + k_1)}$$ 2pt

3) $G(s) = \frac{f_2 s + k_2}{ms^2 + f_2 s + k_2}$ 3pt

$$G'(s) = \frac{0.2s + 10}{0.1s^2 + 0.2s + 10} = \frac{2(s+50)}{s^2 + 2s + 100} = \frac{100.\left(\frac{s}{50}+1\right)}{s^2+2s+100}$$

$K = 1 \quad \omega_0 = 10 \quad \zeta = 0.1$ (underdamped) 3pt

$t_s = \frac{3}{\zeta\omega_0} = 3$ seconds, (5%) or 4 seconds (2%)



1pt

2) 3 pts

Problem 3

3pt

a) $\frac{d^2x}{dt^2} = 4e^{-t}\cos\left(\frac{t}{2}\right)\sin\left(\frac{t}{2}\right)$ $\quad x(0)=2,\ \dot{x}(0)=3$

$$s^2X(s) - sx(0) - x'(0) = \frac{2}{(s+1)^2+1}$$

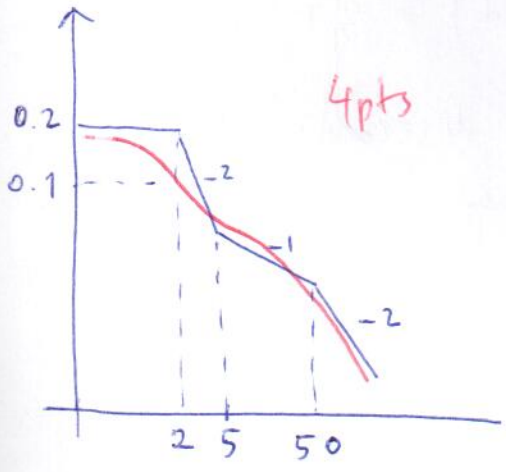
$$X(s) = \frac{2s^3+7s^2+10s+8}{s^2(s^2+2s+2)} \qquad 3pt$$

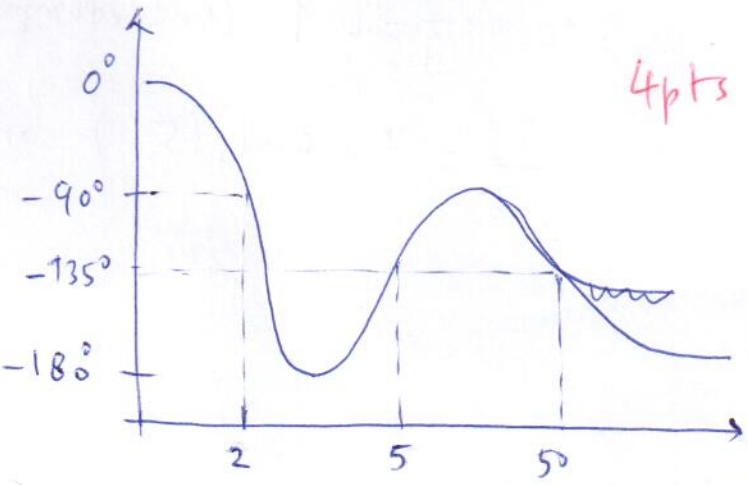
$$X(s) = \frac{A}{s^2} + \frac{B}{s} + \frac{Cs+D}{s^2+2s+2} \qquad A=4,\ B=1,\ C=1,\ D=1$$

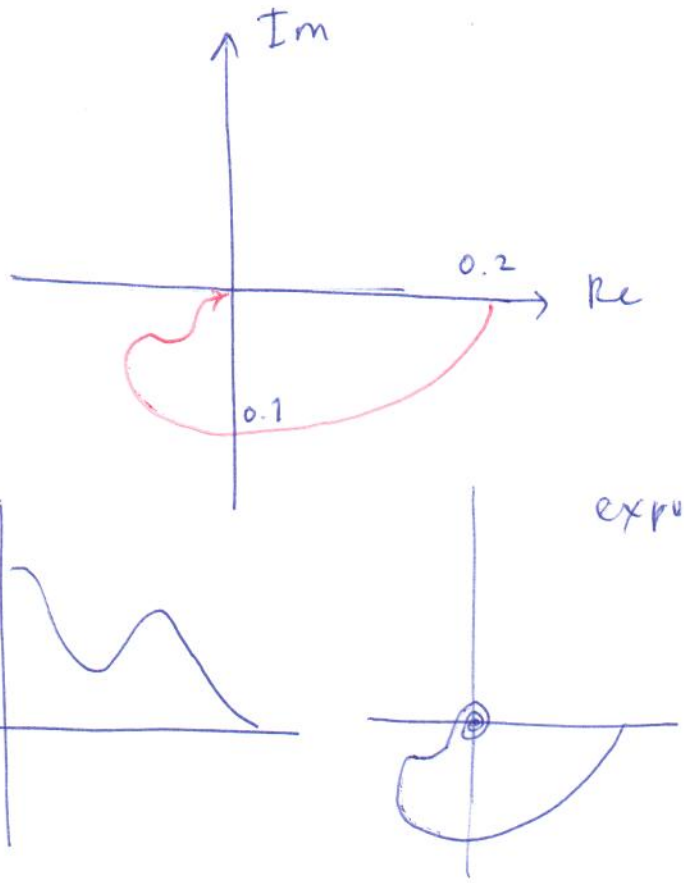
$$= \frac{4}{s^2} + \frac{1}{s} + \frac{s+1}{(s+1)^2+1}$$

$$x(t) = 4t+1+e^{-t}\cos(t) \qquad 1pt\ (+3pt\ bonus)$$

b) $G(s) = \frac{8(s+5)}{(s+50)(s+2)^2} = \frac{0.2\left(\frac{s}{5}+1\right)}{\left(\frac{s}{50}+1\right)\left(\frac{s}{2}+1\right)^2}$ $\quad 2pts$







3pts

exponential term

2pts

c) $x = [\dot{x}, \dot{\theta}, x, \theta]^T$

$y = \theta = x_4$

3pts

5pts

$$A = \begin{bmatrix} 0 & 0 & 0 & mg/M \\ 0 & 0 & 0 & \frac{g}{\ell}\left(\frac{m+M}{M}\right) \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$B = [1/M \quad 1/\ell M \quad 0 \quad 0]$$

$$C = [0 \quad 0 \quad 0 \quad 1] \quad D = 0$$

or $x = [x, \dot{x}, \theta, \dot{\theta}]$ $y = x_3$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & mg/M & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{Mg+mg}{M\ell} & 0 \end{bmatrix}$$

$$B = [0 \quad 1/M \quad 0 \quad 1/M\ell]$$

$$C = [0 \quad 0 \quad 1 \quad 0]$$

$$D = 0$$

Problem 4

1) $G(s) = K \cdot s \cdot \frac{1}{5s+1} \frac{9}{s^2+1.2s+9}$ 4pt

at $w = 0.2$, $\log k + \log w - \log 5w = \log 2$

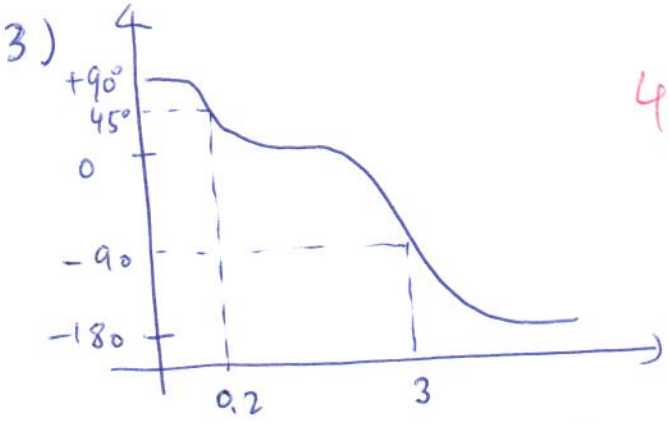
$\log 0.2k = \log 2$

$K = 10$ 1pt

$w_r = w_0\sqrt{1-2\zeta^2}$ $w_0 = 3$ rad/s $\rightarrow$ $\zeta = 0.2$ 2pt

2) $M_R = \frac{K}{2\zeta\sqrt{1-\zeta^2}} = \frac{10}{0.4\sqrt{1-0.04}} = 26$ 2pt

3)

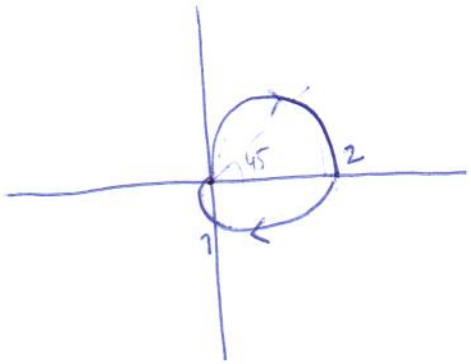


4pt

4) at $w = 3$ $|G(jw)| = 1$ 2pt $\phi = -90°$ 2pt

$y_{ss}(t) = 5 \sin(3t - \pi/2)$

5)



3pts