Problem 1

$F(i, x)$

a) $m\ddot{x} = \frac{L}{2}\frac{i^2}{(D-x)^2} - 2kx - b\dot{x}$ (2)

$$\frac{d}{dt}(\rho V) = \rho q_{in} - \rho q_{out}$$

$$\frac{dV(h)}{dt} = q_{in} - q_{out}$$

$$V(h) = \int_0^h A(h')\,dh' = \frac{\pi R^2}{3H^2}h^3$$

$$\boxed{\frac{dV(h)}{dh} = \frac{\pi R^2}{H^2}h^2}$$

(3) $\frac{dh}{dt} = \frac{H^2}{\pi R^2}\frac{q_{in} - q_{out}}{h^2} = \frac{H^2}{\pi R^2}\frac{q_{max}\frac{x}{d} - c\sqrt{h}}{h^2}$

b) At equilibrium $\dot{x} = 0, \dot{h} = 0, \ddot{x} = 0$

$$\frac{L}{2}\frac{\bar{i}^2}{(D-\bar{x})^2} = 2k\bar{x} \quad \text{and} \quad q_{max}\frac{\bar{x}}{d} = c\sqrt{\bar{h}}$$

$\bar{x} = \frac{d}{2} \rightarrow \bar{h} = \left(\frac{q_{max}}{2c}\right)^2 \quad \bar{i}^2 = \frac{2kd\,(D - \frac{d}{2})^2}{L}$ $\frac{16 \times 50 \times 2}{400}$ (3)

$= 1\,\text{cm} \quad \bar{h} = 7\,\text{cm} \quad \bar{i} = 4\,A$

$$x_1 = x \qquad u = i$$

$$x_2 = \dot{x} \qquad y = q_{out} = c\sqrt{h} = c\sqrt{x_3}$$

$$x_3 = h$$

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ m\dot{x}_2 &= \frac{L}{2}\frac{i^2}{(D-x_1)^2} - 2kx_1 - bx_2 \\ \dot{x}_3 &= \frac{H^2}{\pi R^2}\frac{q_{max}\frac{x_1}{d} - c\sqrt{x_3}}{x_3^2} \end{aligned}\right\} (5)$$

$$\delta\dot{x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} \to 0 & \frac{\partial f_1}{\partial x_2} \to 1 & \frac{\partial f_1}{\partial x_3} \to 0 \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \to -b/m & \frac{\partial f_2}{\partial x_3} \to 0 \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} \to 0 & \frac{\partial f_3}{\partial x_3} \end{bmatrix} \begin{matrix} \delta x_1 \\ \delta x_2 \\ \delta x_3 \end{matrix} + \begin{bmatrix} \frac{\partial f_1}{\partial u} \to 0 \\ \frac{\partial f_2}{\partial u} \\ \frac{\partial f_3}{\partial u} \to 0 \end{bmatrix} \partial u$$

$$L\frac{\bar{i}}{(D-x_1)^2} = 100$$

$$y = \begin{bmatrix} \underbrace{\frac{\partial g}{\partial x_1}}_{0} & \underbrace{\frac{\partial g}{\partial x_2}}_{0} & \frac{\partial g}{\partial x_3} \end{bmatrix} \begin{matrix} \delta x_1 \\ \delta x_2 \\ \delta x_3 \end{matrix}$$

format (5)

values (2)

$$\frac{H^2}{\pi R^2}\frac{q_{max}/d}{\bar{X}_3^2} = 0.03$$

$$\cancel{c x_3^{1/2}}$$

$$\frac{c}{2}x_3^{-1/2} = 0.005$$

Problem 2

1) $r(t_p) = K\left(1 + e^{-\zeta\pi/\sqrt{1-\zeta^2}}\right)$

$t_p = \dfrac{\pi}{w_0\sqrt{1-\zeta^2}}$ $\quad M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 0.5$

$\downarrow$

$\underline{\zeta = 0.215}$ (2)

$\underline{K = 0.02\ m}$ (1)

$t_s = \dfrac{4}{w_0 \cdot \zeta} = 2$ $\quad \underline{w_0 = 10}$ (2) $\quad \rightarrow t_p = \dfrac{\pi}{10\sqrt{1-0.215^2}} = 0.32\ s$

$G(s) = K\dfrac{w_0^2}{s^2+2\zeta w_0 s + w_0^2} = \dfrac{2}{s^2+4.3s+100}$ (2)

$= \dfrac{100}{ms^2+bs+k} = \dfrac{1/m}{s^2+\frac{b}{m}s+\frac{k}{m}} \cdot 100$

$m = \cancel{0.5}\ kg$ 50 $\quad k = \cancel{50}\ N/m$ 5000 $\quad b = \cancel{2.15}\ N.s/m$ 215 (3)

2) damping coefficient "b"

decrease overshoot $\rightarrow$ ~~decrease~~ increase $\zeta \rightarrow$ increase b

settling time : $\dfrac{4}{w_0\zeta} \rightarrow$ decreases (5)

4) $\zeta = 0.215 \rightarrow$ resonance $\quad w_r = w_0\sqrt{1-2\zeta^2} = 10\sqrt{1-\ldots} = 9.52$ (3)

$\zeta = \dfrac{2}{\sqrt{5}} = 0.89 \rightarrow$ no resonance

3) $G(s) = \dfrac{1}{4s^2+16s+20} = \dfrac{0.25}{s^2+4s+5}$ (1)

$u(t) = \frac{4}{5} t \left(\varepsilon(t) - \varepsilon(t-5) \right)$

$= \frac{4}{5}\left[t\,\varepsilon(t) - (t-5)\varepsilon(t-5) - 5\,\varepsilon(t-5) \right]$

$U(s) = \dfrac{4}{5s^2} - \dfrac{4e^{-5s}}{5s^2} - \dfrac{4e^{-5s}}{s}$ (3)

I the rest is 3 points

$\dfrac{-(2s+4)}{(s^2+4s+5)^2}$

$A_1 s^2 + 4A_1 s + 5A_1 + A_2 s^2 + A_3 s$

$Y(s) = \dfrac{1}{5s^2}\dfrac{1}{s^2+4s+5} - \dfrac{1}{5s^2}\dfrac{e^{-5s}}{s^2+4s+5} - \dfrac{1}{s}\dfrac{e^{-5s}}{s^2+4s+5}$ 1 (1)

$\dfrac{A_1}{s^2} + \dfrac{A_2}{s} + \dfrac{A_3 s + A_4}{s^2+4s+5}$

$\dfrac{A_1}{s} + \dfrac{A_2 s + A_3}{s^2+4s+5}$

(2)

$A_1 = 0.2$

$A_2 = -0.16$

$A_3 = 0.16$

$A_4 = 0.44$

$A_1 = 1$

$A_2 = -1$

$A_3 = -4$

$0.2t - 0.16 + 0.16e^{-2t}\cos t + 0.44e^{-2t}\sin t$

$A_1 s^2 + \cancel{4A_1 s} + \cancel{5A_1}$

$A_2 s^3 + 4A_2 s^2 + \cancel{5A_2 s}$

$A_3 s^3 + A_4 s^2$

50

12,5 kg

10,75

Problem 3

a) $\left(s^2Y(s)-2s\right)+3\left(sY(s)-2\right)+2Y(s)=2sU(s)+4U(s)$

$$Y(s)=\underbrace{\frac{2s+4}{s^2+3s+2}U(s)}_{\text{forced}}+\underbrace{\frac{2s+6}{s^2+3s+2}}_{\text{free}} \qquad \underline{\underline{(2)}}$$

$$U_1(s)=\frac{1}{s} \qquad =\frac{2}{s+1}U(s)+\frac{2(s+3)}{(s+2)(s+1)}$$

$$Y_1(s)=\cancel{\frac{A_1}{s}+\frac{A_2}{s+2}+\frac{A_3}{s+1}}$$

$$=\frac{A}{s}+\frac{B}{s+1} \qquad A=2,\ B=-2 \qquad \underline{y_1(t)=2-2e^{-t}} \quad \underline{(1)}$$

$$Y_2(s)=\frac{A}{s+2}+\frac{B}{s+1} \qquad A=-2,\ B=4 \qquad \underline{y_2(t)=4e^{-t}-2e^{-2t}} \quad \underline{(1)}$$

$$y(t)=2+2e^{-t}-2e^{-2t}$$

$$u_2(t)=e^{-t}\varepsilon(t) \qquad U_2(s)=\frac{1}{s+1}$$

$$Y_1(s)=\frac{2}{(s+1)^2}\cancel{+\frac{B}{s+1}} \qquad \cancel{A=} \quad \underline{y_1(t)=2te^{-t}} \quad \underline{(1)}$$

$$\underline{y_2(t)=4e^{-t}-2e^{-2t}}$$

$$\underline{y(t)=\left(2+2e^{-t}-2e^{-2t}\right)+\left(2te^{-t}+4e^{-t}-2e^{-2t}\right)}$$

$\underline{(1)}$

b) $G(s) = \frac{4}{(s+2)^2}$ $\qquad u(t) = e^{-3t}\varepsilon(t)$

$K = \lim_{s\to 0} \frac{4}{(s+2)^2} = 1$ (2)

$u(s) = \frac{1}{s+3}$ $\qquad Y(s) = \frac{4}{(s+3)(s+2)^2} = \frac{\overset{4}{A}}{(s+2)^2} + \frac{\overset{-4}{B}}{s+2} + \frac{\overset{4}{C}}{s+3}$

$g(t) = 4te^{-2t}$ (2)

$y(t) = \int_0^t u(t-\tau)\, g(\tau)\, d\tau = \int_0^t e^{-3(t-\tau)} 4\tau e^{-2\tau} d\tau$

$= 4e^{-3t} \underbrace{\int_0^t e^{\tau} \tau\, d\tau}_{(1-e^t+te^t)}$ $\qquad = \underline{4e^{-3t} - 4e^{-2t} + 4te^{-2t}}$

(~~5~~) 4

5points

c) poles

$s_1 = -a$

$s_2 = -19.8$

$s_3 = -0.2$

$a > 1 \quad a < 0.04$

$a < -1$

$0 > a > -0.04$

$\dot{x}_2 + 4x_2 + 4x_1 = 4u$

$\ddot{y} + 4\dot{y} + 4y = 4u$

$(s^2+4s+4)Y(s) = 4U(s)$

$x_1 = y$

$x_2 = \dot{y}$

$\begin{bmatrix} 0 & 1 \\ -4 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{pmatrix} 0 \\ 4 \end{pmatrix}$

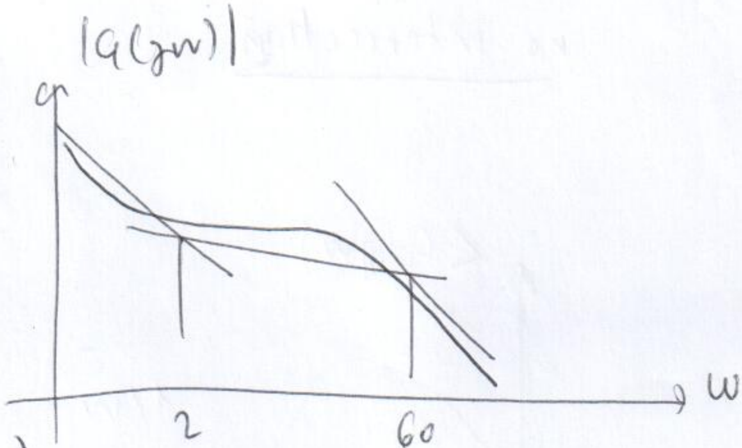
Problem 4

a) $G(s) = \dfrac{10}{s^2}\,\dfrac{(s+2)}{(s+60)}$ (3)

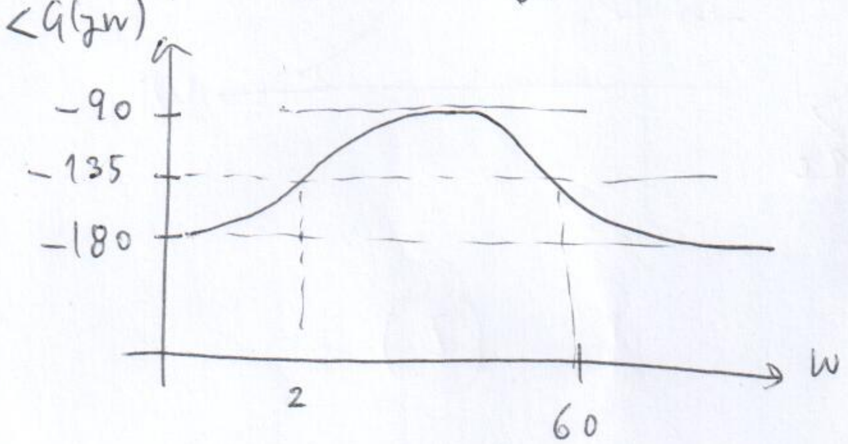
order = 3 (1)

stable = no! (1)
(s^2)

b)



(3)



(7)

c) $u(t) = 48\sin(60t)$

$$y(t) = 0.03 \times 48 \sin(60t - 135^\circ) \quad (3)$$
$$= 1.44 \sin(60t - 135^\circ)$$

$$|G(j\omega)| = \frac{10}{\omega^2}\,\frac{\sqrt{\omega^2+4}}{\sqrt{\omega^2+60^2}}$$

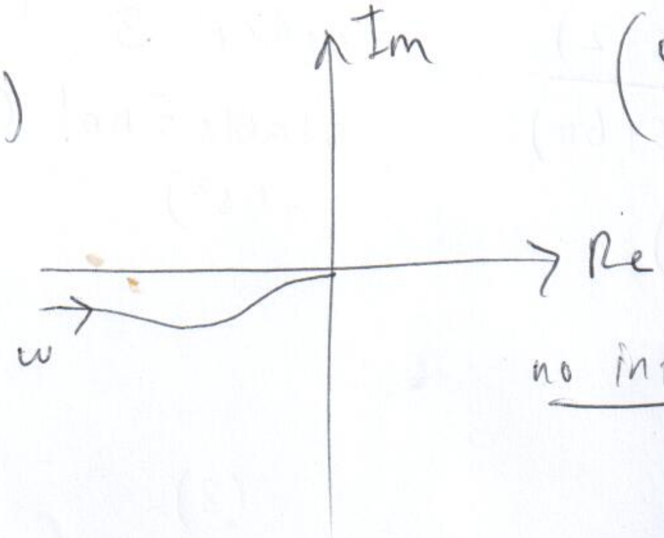
$\omega = 60$

$$|\;| = \frac{10}{60^2}\,\frac{60}{60\sqrt{2}}$$
$$\simeq 0{,}002$$
(2)

$$\angle G(j\omega) = -180 + \underset{\sim}{90} - 45 = -133^\circ$$

app
arctan(30) = 88°

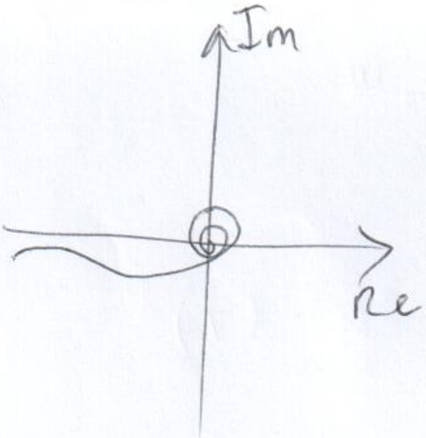
d)

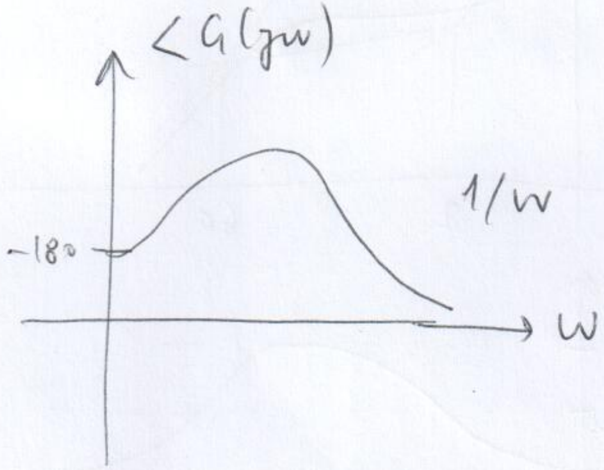


(5)

no intersection

e)





(3)

(2)