ME-221
Lecture 4 Example 5 (page 19)

The equation of motion is given by:

$$m\ddot{x}_1 = -k_1 x_1 - f_1 \dot{x}_1 - f_2 \dot{x}_2 \tag{1}$$

We have an input derivative and, thus, we need to a change of variables to obtain another set of equations that are in the standard state-space representation. We described this procedure on Page 18 of the same lecture. Let's first put (1) in the same form as shown on Page 18 by replacing x_1 and x_2 with input u and output y.

$$\ddot{y} + \frac{f_1}{m}\dot{y} + \frac{k_1}{m}y = -\frac{f_2}{m}\dot{u} \tag{2}$$

Here we define a new set of parameters $\beta_0 = 0$, $\beta_1 = -\frac{f_2}{m}$, and $\beta_2 = \frac{f_1 f_2}{m}$. They are calculated from the coefficients of (2). The new set of equations are given by:

$$\begin{aligned} \dot{x}_1 &= x_2 - \frac{f_2}{m}u \\ \dot{x}_2 &= -\frac{k_1}{m}x_1 - \frac{f_1}{m}x_2 + \frac{f_1 f_2}{m^2}u \\ y &= x_1 \end{aligned} \tag{3}$$

If we put the equations in matrix form, we will end up with the standard state-space representation of this system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k_1}{m} & -\frac{f_1}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -\frac{f_2}{m} \\ \frac{f_1 f_2}{m^2} \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$