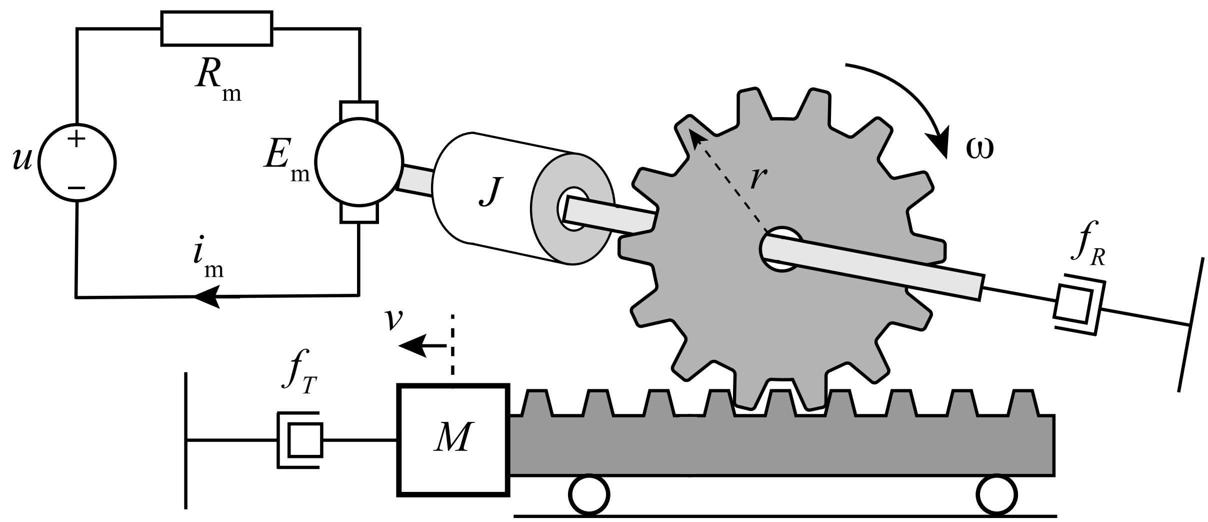
ME-221 Midterm Exam —— Spring 2019

Problem 1 (35 points)

Consider the mechanical system depicted below. A DC motor is connected to a pinion rack mechanism with a mass-damper load. The current flowing through the motor control circuit $i_m(t)$ with resistance R_m is supplied by the input voltage source $u(t)$ and the output of the system is the velocity $v(t)$ of the mass M. The motor-torque and back-emf (denoted by E_m) constants are given by K_t and K_m, respectively. The motor has an inertia denoted by J and the angular velocity of the shaft as well as the gear (with radius r) is given by ω. The parameters f_R and f_T represent the viscous damping coefficients for the motor shaft and the mass-damper unit, respectively.



1. (5 points) Derive the equations of motion.

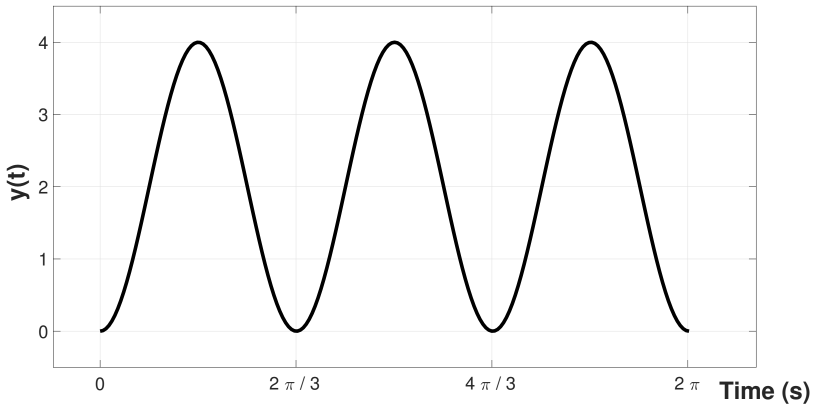
2. (10 points) Find the transfer function from the source voltage to the mass velocity. What is the order of this system?

3. (10 points) The system parameters are given as $m = 1.6$ kg, $J = 0.9$ kg·m^2, $r = 0.25$ m, $R = 1\ \Omega$, $f_T = 16$ kg/sec, $f_R = 1$ kg·m^2/rad·sec, $K_m = 1$ V·sec/rad , $K_t = 1$ N·m/A. Plot the unit-step response of the system. Calculate the steady-state value of the output. When does the output reach 98% of its steady-state value?

4. (10 points) Calculate the output of the system to an input given by $u(t) = 4e^{-2(t-3)}\varepsilon(t-3)$ using the parameters given in part 3.

Problem 2 (40 points)

The unit step response of a second order undamped system that has no zeroes is plotted below.

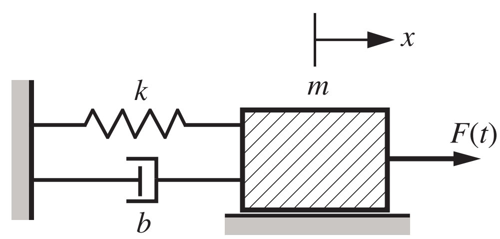


1. (10 points) Find the transfer function and the poles of the system. Calculate $y(t)$.

2. (5 points) Locate the poles on the complex plane. How would the stability of the system change if we move the poles to the right or to the left of the imaginary axis?

3. (10 points) Suppose that we created a new system by moving the poles to the left by 4 and the new poles are given by $p_1^* = -4 + p_1$ and $p_2^* = -4 + p_2$. Find the new transfer function. Sketch the unit step response of this new system. Denote the rise time, peak time, maximum overshoot, and settling time (2% criterion).

4. (15 points) Suppose that the new system represents a mass-spring-damper system as depicted on the right and the output is the displacement $x(t)$. Calculate the values of the mass m, spring coefficient k, and damping coefficient b. How would you change these parameters if we want to increase maximum overshoot without changing the natural frequency of the system. How would the settling time change as a result of this modification?



Problem 3 (25 points)

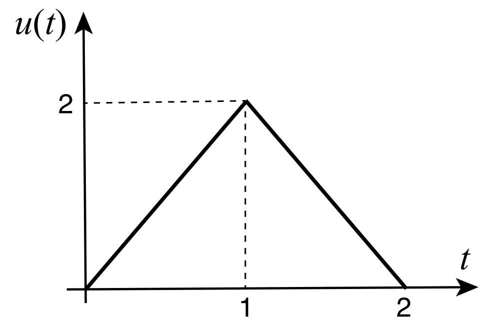
Solve the following three problems. They are not connected to each other.

a) (5 points) Consider an LTI system described by the following differential equation:

$$\ddot{y}(t) + 3\dot{y}(t) + 2y(t) = 3\dot{u}(t) + 2u(t)$$

Find the unit step response of the system. The initial conditions are $y(0) = 2$, $\dot{y}(0) = 0$, $u(0) = 0$.

b) (10 points) The impulse response of a system is given by the unit-step function, i.e. $g(t) = \varepsilon(t)$. Find the output of the system in response to the input signal shown on the right.



c) (10 points) A mechanical stabilization system is defined by the following differential equations:

$$M\ddot{r}(t) = M\omega^2(t)r(t) - 10[r(t) - 0.1]$$
$$2I(r)\dot{\omega}(t) = u(t) - 2\omega(t)$$

The input of the system is $u(t)$ and the output is the angular velocity $\omega(t)$. The inertia is given by $I(r) = Mr^2(t)$ where $M = 1$ kg. Derive a state-space representation of this non-linear system. Linearize the model around an equilibrium point corresponding to $\bar{\omega} = 3$ rad/sec. Obtain a state-space representation for the linearized system.

Note: The convolution operation is defined as:

$$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau) f_2(t - \tau) d\tau$$