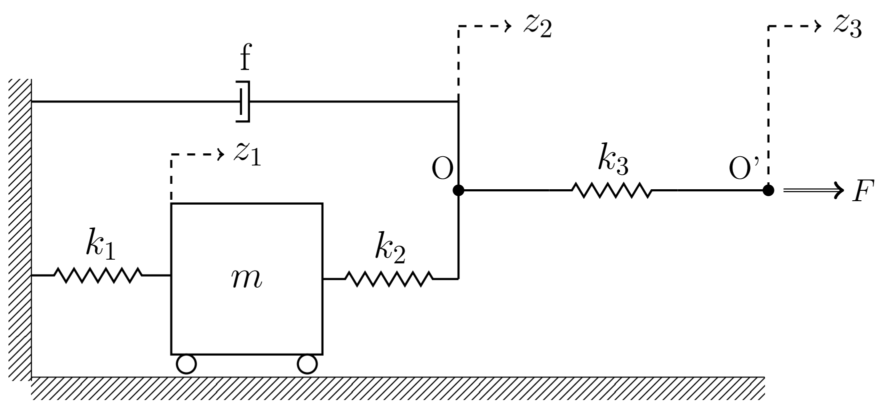
ME-221 —– Sample Exam Questions (from 2017)

Problem 1

Consider the mechanical systems shown below. The system is initially at rest. The displacements z_1, z_2 and z_3 are measured with respect to their equilibrium positions before the application of an external force F.



1. Derive the equations of motion. Note that O and O' points have zero mass. Does the displacement of the mass m depend on the stiffness coefficient k_3 ?
2. Formulate a state-space representation of the system by taking the force F as the input and the displacement z_1 as the output of the system.
3. Calculate the transfer function $\frac{Z_1(s)}{F(s)}$.
4. Propose a circuit diagram that is analogous to this mechanical system using the force-voltage analogy.

Problem 2

Consider the dynamical system described by the following state equations:

$$\begin{aligned} \dot{x}_1 &= 2x_1 + 2x_1x_2 + u & \qquad x_1(0) &= 0 \\ \dot{x}_2 &= x_2 + 3x_1x_2 & \qquad x_2(0) &= 0 \end{aligned}$$

1. Calculate the equilibrium point if $\bar{u} = 1$ and $\bar{x}_1$, $\bar{x}_2 \neq 0$.
2. Linearize the system around this equilibrium point. Write down the state equations.

Problem 3

a) Calculate the displacement $x(t)$ of a mechanical system described by the following differential equation:

$$m\ddot{x}(t) = F(t) - kx(t) - f\dot{x}(t) \qquad\qquad x(0) = \dot{x}(0) = 0$$

The system parameters are measured as $m = 2$ kg, $k = 10$ N/m, $f = 8$ Ns/m, and the input force $F(t)$ is given by:

$$F(t) = \begin{cases} 1\,\text{N} & \text{for } t < 5, \\ 0 & \text{for } t \geq 5. \end{cases}$$

b) Calculate the unit-step response of a system with the impulse response $g(t) = e^{-t} + e^{-2t}$ using convolution operation. Verify the result using Laplace transform.

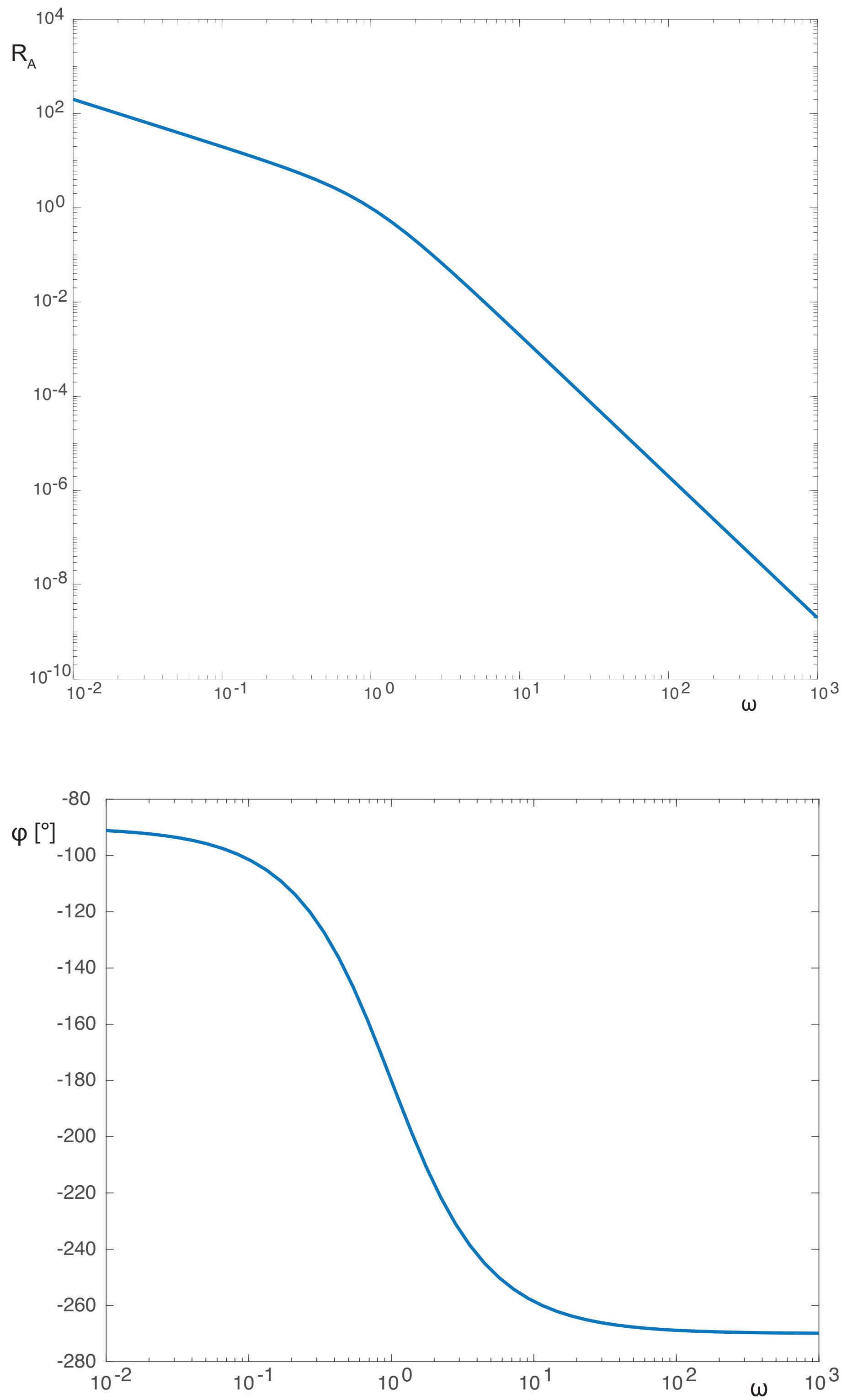
c) Find the output $y(t)$ of a system described by the following differential equation:

$$\dddot{y}(t) + 8\ddot{y}(t) + 17\dot{y}(t) + 10y(t) = 0$$

The initial conditions are given by $y(0) = 2$, $\dot{y}(0) = 1$, $\ddot{y}(0) = 0.5$.

Problem 4

Consider a dynamical system with the following Bode Diagram.



a) Calculate the transfer function of the system $G(s)$. What is the order of the system and where are the poles located? Find the static gain and the time constant of the system.

b) Imagine that we want to design a first order filter (denoted by the transfer function $F(s)$) with a zero at -1. The filtered system is expected to have a magnitude of 1 and phase shift of -145 at $\omega = 1$. Find the gain and the time constant of $F(s)$ that would lead to the desired specifications. Note that, the transfer function of the filtered system $G_f(s)$ is simply the product of the transfer function of the original system and the transfer function of the first order filter (i.e. $G_f(s) = G(s)F(s)$).