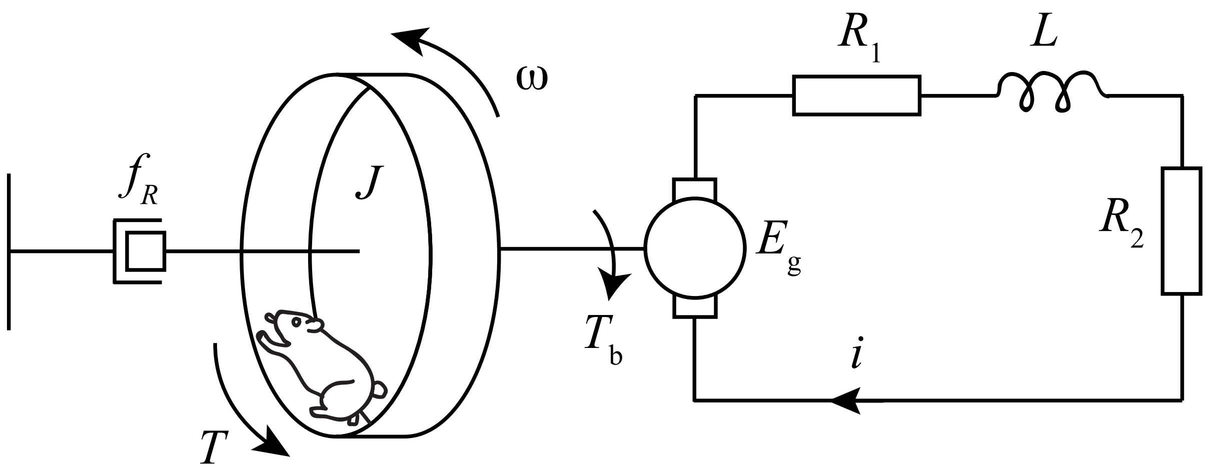
ME-221 Final Exam —— Spring 2019

Problem 1 (30 points)

Consider the electromechanical system depicted below. A running rodent provides the input torque T for the electric generator by spinning a wheel. The rotation of the wheel generates voltage E_g that is linearly proportional to the angular velocity ω. A current i starts to flow through the load circuit with resistors of resistances R_1 and R_2 as well as an inductor with inductance L. The current, in return, induces a back-torque denoted by T_b that is linearly proportional to the current and resists the motion of the wheel. The generator and back-torque constants are given by K_g and K_b, respectively. The wheel has an inertia denoted by J while f_R represents the rotational viscous damping coefficient of the shaft.



1. (7 points) Write down the equations of motion.

2. (6 points) Derive a state-space representation of the system.

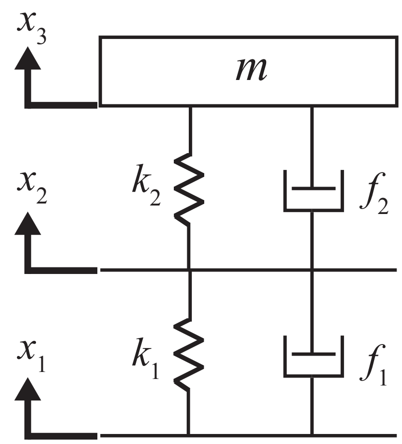
3. (6 points) Find the transfer function $G(s) = I(s)/T(s)$ from the input torque T to the output current i. What is the order of this system?

4. (4 points) The parameter values are given as $J = 0.1$ kg·m^2, $f_R = 0.4$ kg·m^2/rad·sec, $R_1 = 2\ \Omega$, $R_2 = 6\ \Omega$, $L = 2$ H, $K_g = 9$ V·sec/rad , and $K_b = 0.2$ N·m/A. What is the steady-state value of the unit step response? Does this system display a resonant peak? If yes, find the resonant frequency. Otherwise, explain why not.

5. (7 points) We would like to design a first order filter $F(s) = \dfrac{K}{\tau s + 1}$ in a way that the new system with the transfer function $G'(s) = G(s) \times F(s)$ has magnitude $|G'(j\omega)| = 45$ and phase angle $\phi = -3\pi/4$ at frequency $\omega = 5$ rad/sec. What are the values of K and τ?

Problem 2 (20 points)

Consider a simplified model of a mountain bike suspension system shown on the right. The input is the position $x_1(t)$ of the rocky terrain and the output is the position $x_3(t)$ of the person with mass m. The spring and damping coefficients for the tire and fork are given by k_1, f_1, k_2 and f_2, respectively. The masses of the bike components are negligible. Note that the variable $x_2(t)$ at the interface between the tire and fork captures the motion of the fork with respect to the tire. **Ignore the effect of gravity**.



1. (10 points) Derive the transfer function $G(s) = X_3(s)/X_1(s)$ of the system.
2. (3 points) Propose an analogous electrical circuit.
3. (7 points) Make the approximation that $k_1 = \infty$ and $f_1 = 0$ and derive the new transfer function G'(s) given that $k_2 = 10$ N/m, $f_2 = 0.2$ N·s/m and $m = 0.1$ kg. Explain why we can ignore the zero and analyze the transfer function in the standard second-order form. Is this system overdamped, critically damped, or underdamped? Determine the settling time and sketch the output for a unit step input.

Problem 3 (30 points)

Solve the following three problems. They are not connected to each other.

a) (7 points) Use the Laplace transform table to solve the following differential equation. Indicate which properties you use. Note that $\delta(\cdot)$ is the impulse function.

$$\frac{d^2x}{dt^2} = 4e^{-t}\cos\left(\frac{t}{2}\right)\int_0^t \delta(\tau)\sin\left(\frac{t}{2} - \tau\right)d\tau \qquad x(0) = 2, \dot{x}(0) = 3$$

Remark: You can use the following trigonometric functions.

$$\cos(2\theta) = \cos(\theta)^2 - \sin(\theta)^2, \quad \sin(2\theta) = 2\sin(\theta)\cos(\theta), \quad \tan(2\theta) = 2\tan(\theta)/(1 - \tan(\theta)^2)$$

b) (15 points) Consider the following transfer function:

$$G(s) = \frac{4(2s + 10)}{(s^2 + 52s + 100)(s + 2)}$$

I. Sketch the Bode plot of the system (magnitude and phase). Label all slopes and points.
II. Use the Bode plot to sketch the Nyquist plot. Mark the approximate locations of the intersection points with the real and imaginary axes.
III. How would the Bode (magnitude and phase) and Nyquist plots change if we had an additional e^{-s} term in the numerator of the transfer function?

c) (8 points) The equations of motion of an inverted pendulum system are given as follows:

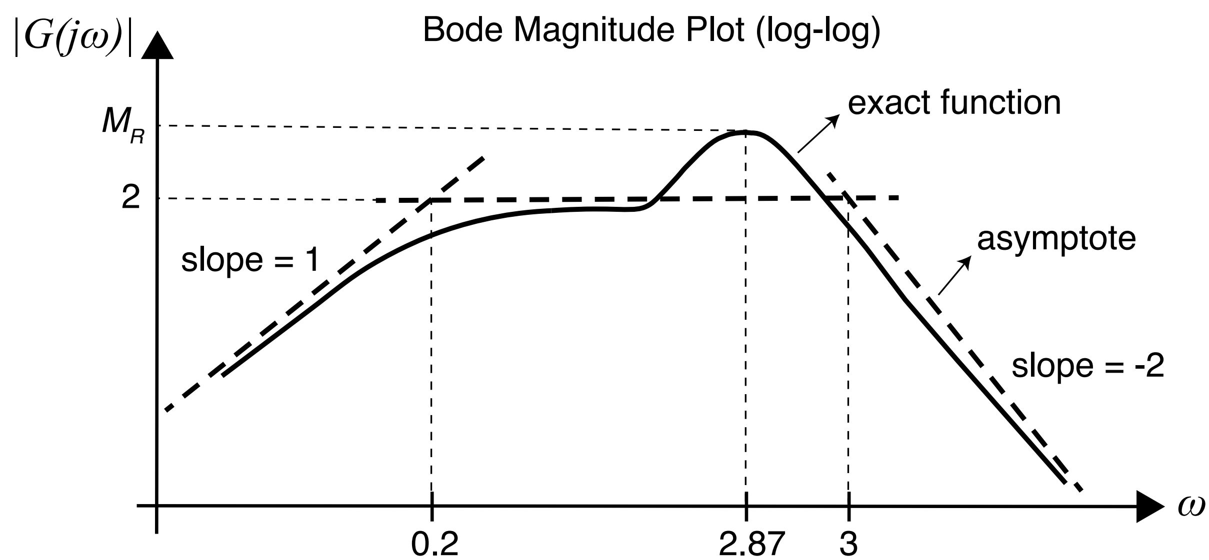
$$\begin{aligned} \ddot{x} &= h(m, M, l, \theta, \dot{\theta}, F), \\ \ddot{\theta} &= \frac{1}{l}\Big(h(m, M, l, \theta, \dot{\theta}, F)\cos(\theta) + g\sin(\theta)\Big) \end{aligned}$$

where $h(\cdot)$ is a non-linear function, F is the input and θ is the output of the system. Linearize the system around the equilibrium point corresponding to $F = 0$, $x = 0$, $\dot{x} = 0$, $\theta = 0$, and $\dot{\theta} = 0$ and find the state-space matrices A, B, C, and D as a function of the parameters m, M, g and l. The following information is provided:

$$\frac{\partial h}{\partial \theta} = \frac{mg}{M}, \quad \frac{\partial h}{\partial \dot{\theta}} = 0, \quad \frac{\partial h}{\partial F} = \frac{1}{M}$$

Problem 4 (20 points)

See the following Bode Diagram. The three bold dashed lines represent the asymptotes of the magnitude plot and the continuous curve is the exact function.



1. (7 points) Calculate the transfer function of the system given that there is no exponential term. What is the order of the system?

2. (2 points) Calculate the magnitude of the resonant peak denoted by M_R in the plot.

3. (4 points) Sketch the Bode phase diagram (semi-log). Clearly mark the asymptotes.

4. (4 points) What is the approximate value of the steady-state output for input $u(t) = 5sin(3t)$?

5. (3 points) Use the Bode plot to sketch the Nyquist plot. Mark the approximate locations of the intersection points with the real and imaginary axes.