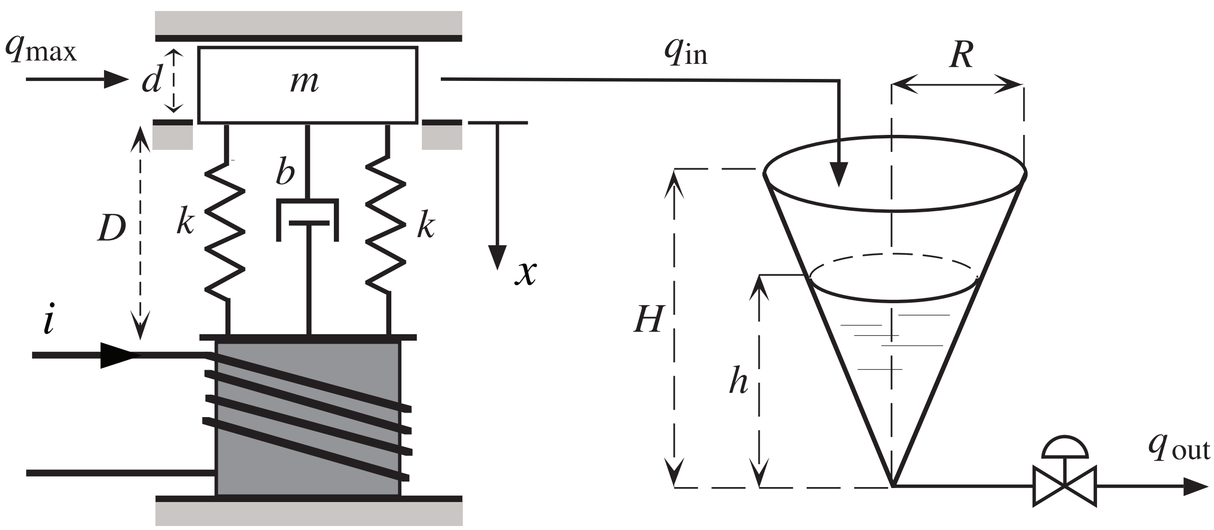
ME-221 Final Exam —— Version A

Problem 1 (20 points)

Consider the liquid-level system shown below. The input flow q_{in} is controlled using a solenoid valve. The ferromagnetic plunger with mass m is pulled towards the electromagnetic coil by the magnetic force F. This opens the orifice so that the liquid can flow through. F depends on the displacement x and the current i that flows through the coil according to the following formula:

$$F(x,i) = \frac{L}{2}\frac{i^2}{(D-x)^2}$$

where L is an inductance term. The input flow is proportional to the opening of the valve, $q_{in} = q_{max}(x/d)$. The liquid level $h(t)$ of the conical tank with radius R and height H is determined by the input flow q_{in} and the output flow q_{out} where $q_{out} = c\sqrt{h(t)}$ The system is initially at rest ($x(0) = 0$, $i(0) = 0$, and $h(0) = 0$).

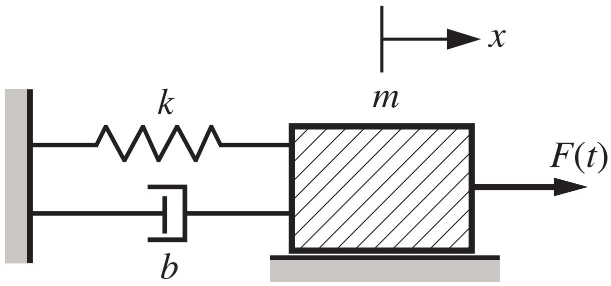


1. (5 points) Derive the equations of motion. Ignore the mg term for the plunger.

2. (15 points) The system parameters are given as $m = 2$ kg, $k = 25$ N/cm, $b = 10$ N/cm^2, $d = 2$ cm, $D = 5$ cm, $L = 400$ N.cm^2/A^2, $H = 9$ cm, $R = 3$ cm, $q_{max} = 0.02$ cm^3/s, and $c = 0.01$ m^2/s. Linearize the system around the equilibrium point $\bar{x} = d/2$. Formulate a state-space representation by taking the current i as the input and the output flow q_{out} as the output.

Problem 2 (25 points)

Consider the mechanical mass-spring-damper system shown below. The system is initially at rest.



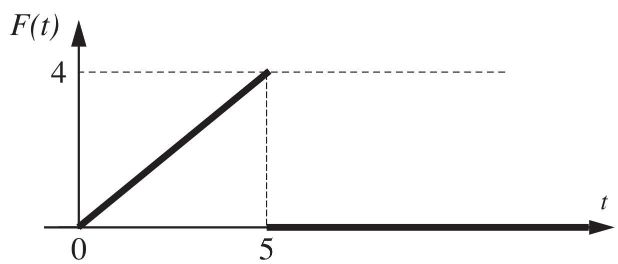
1. (10 points) Determine the numerical values of m, k, and b so that when the input force is $F(t) = 100\varepsilon(t)$ N

   - the mass resides at $x = 2$ cm at the steady-state
   - the mass settles to 2% of its final value within 2 seconds
   - the maximum percent overshoot of $x(t)$ from its steady-state value is 50%

   To ensure you understand the design specifications, first make a rough sketch of the displacement $x(t)$ for $0 \leq t \leq 2$ sec and clearly identify peak time, maximum overshoot, and settling time on the graph. Note that $\varepsilon(t)$ is the unit step function.

2. (5 points) We would like to decrease the maximum percent overshoot without changing the natural frequency. Which parameters (m, k, and/or b) shall we play with? How would settling time change as a result?

3. (7 points) Given that $m = 4$ kg, $k = 20$ N/m, and $b = 16$ N.s/m, calculate the displacement $x(t)$ in response to the following triangular input function:



4. (3 points) Does this system display a resonant peak in the Bode plot? If yes, find the resonant frequency. Otherwise, explain why not.

Problem 3 (25 points)

a) (6 points) Consider an LTI system described by the following differential equation:

$$\ddot{y}(t) + 3\dot{y}(t) + 2y(t) = 2\dot{u}(t) + 4u(t)$$

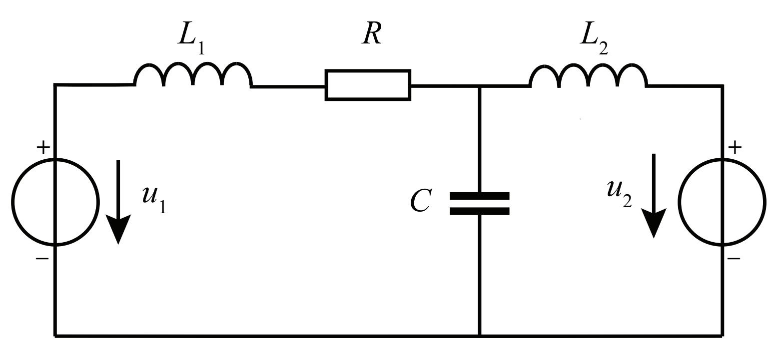
Find the output $y(t)$ in response to an input $u(t) = (1 - e^{-t})\varepsilon(t)$. The initial conditions are given by $y(0) = 2$, $\dot{y}(0) = 0$, $u(0) = 0$.

b) (8 points) Consider an LTI system with the transfer function $G(s) = \frac{4}{(s+2)^2}$. Determine the steady-state (DC) gain of this system. Calculate the output $y(t)$ for an input $u(t) = e^{-3t}\varepsilon(t)$ using convolution operation in time domain. Propose a state-space model for the system. Note that convolution operation is defined as follows:

$$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau) f_2(t - \tau) d\tau$$

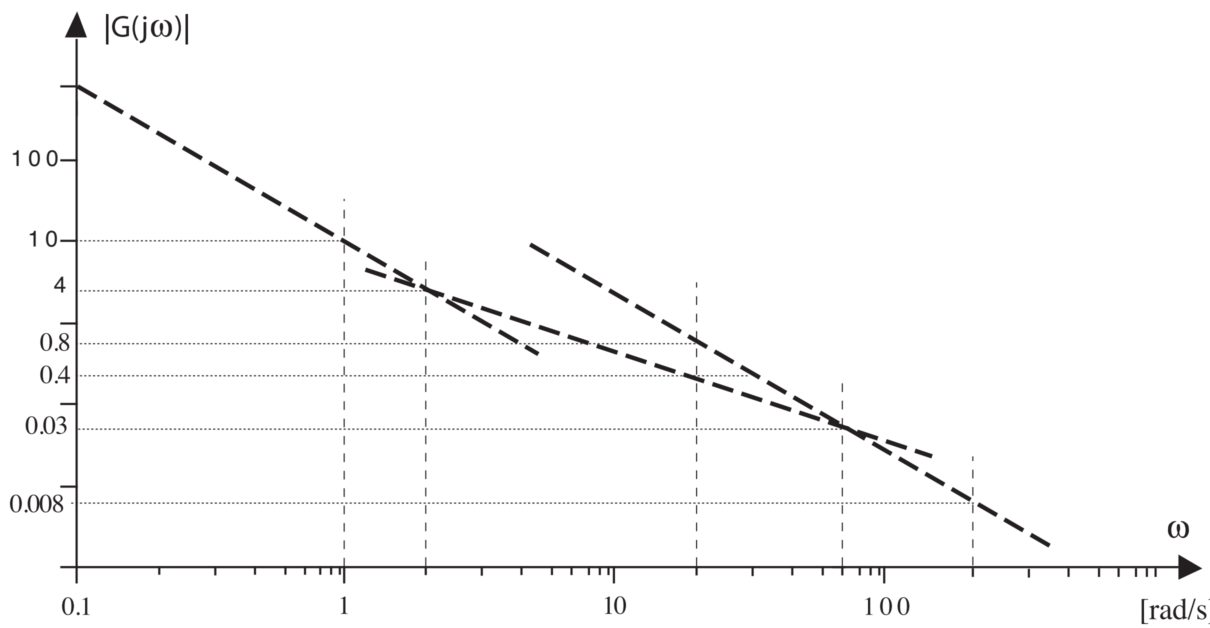
c) (5 points) Consider an LTI system with the transfer function $G(s) = \frac{2}{(s+a)(s^2+20s+4)}$ where a is a positive real number. Find the poles of the system. For what values of a dominant pole approximation is acceptable?

d) (6 points) Develop a mathematical model for the circuit shown below. Propose an analogous mechanical system using the displacement/charge, force/tension analogy.



Problem 4 (30 points)

In this problem, you are expected to sketch the Bode and Nyquist plots and the find the transfer function of a system using the asymptotes of the magnitude plot. See the following diagram.



1. (5 points) Calculate the transfer function of the system given that there are no complex poles or exponential terms. What is the order of the system? Is this system asymptotically stable?

2. (10 points) Sketch the magnitude and phase diagrams (approximated curves). Clearly show the asymptotes of the phase diagram.

3. (5 points) What is the approximate value of the steady-state output if the input is given by $u(t) = 48sin(60t)$? First read from the Bode plot and then directly calculate using the sinusoidal transfer function.

4. (5 points) Sketch the Nyquist plot. Mark the approximate locations of the intersection points with the real and imaginary axes (Hint: Use the Bode plot).

5. (5 points) How would the Bode (magnitude and phase) and Nyquist plots change if we had an additional e^{-s} term in the numerator of the transfer function?