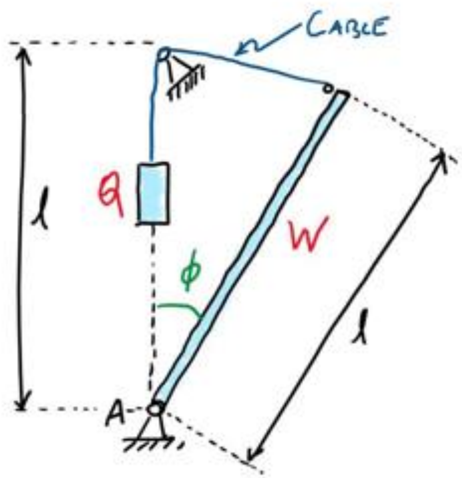
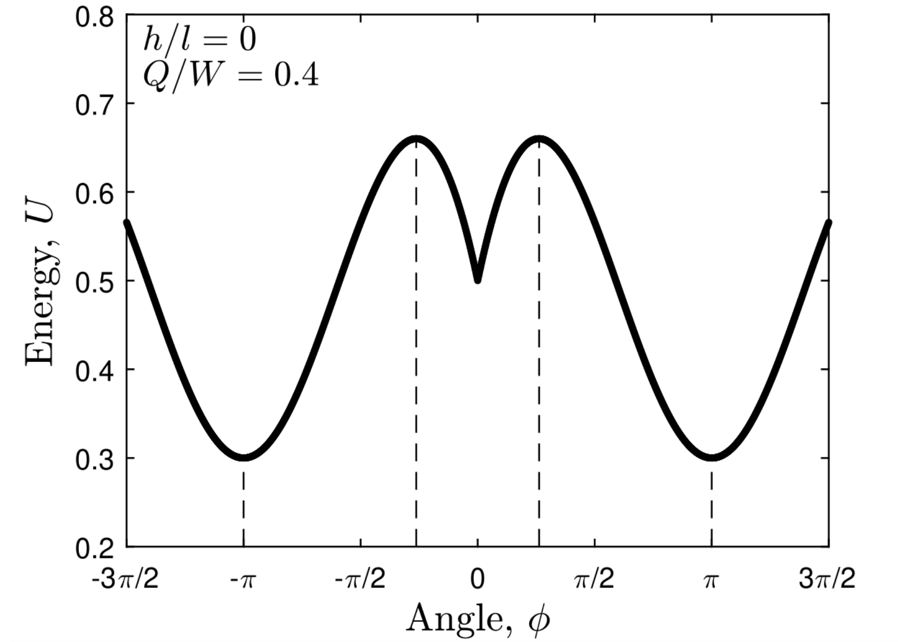
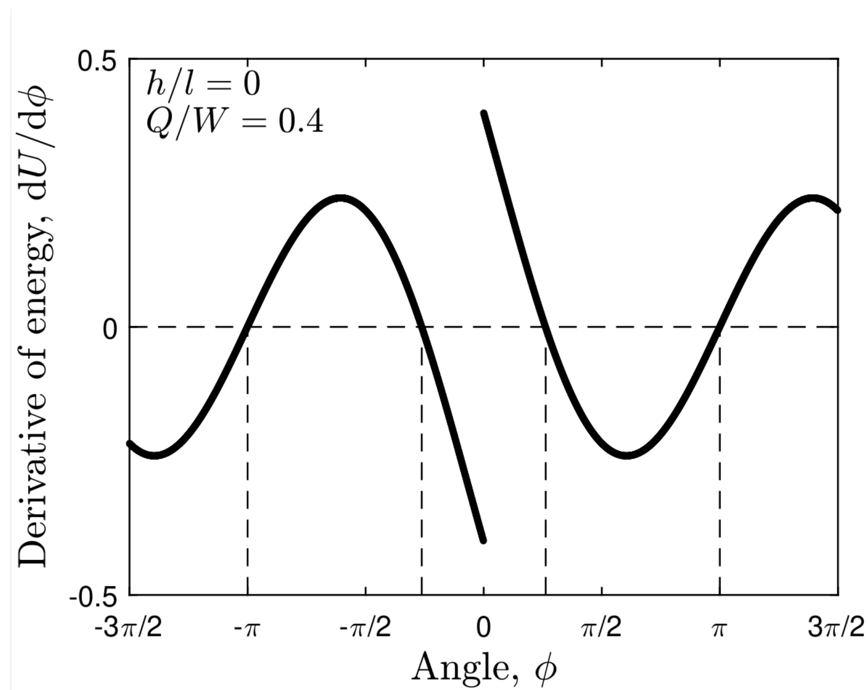
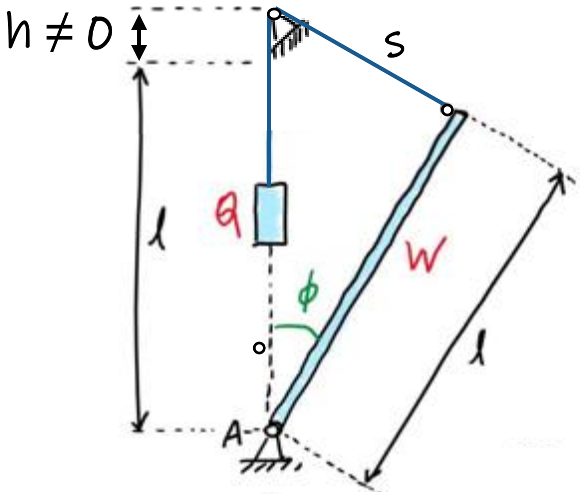
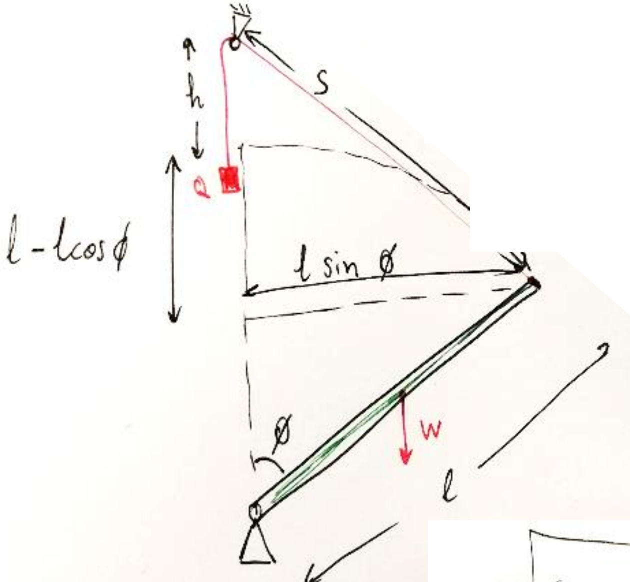

CABLE
Q
W
l
l
φ
A


h/l = 0
Q/W = 0.4
Energy, U
Angle, φ
0.8
0.7
0.6
0.5
0.4
0.3
0.2
-3π/2
-π
-π/2
0
π/2
π
3π/2

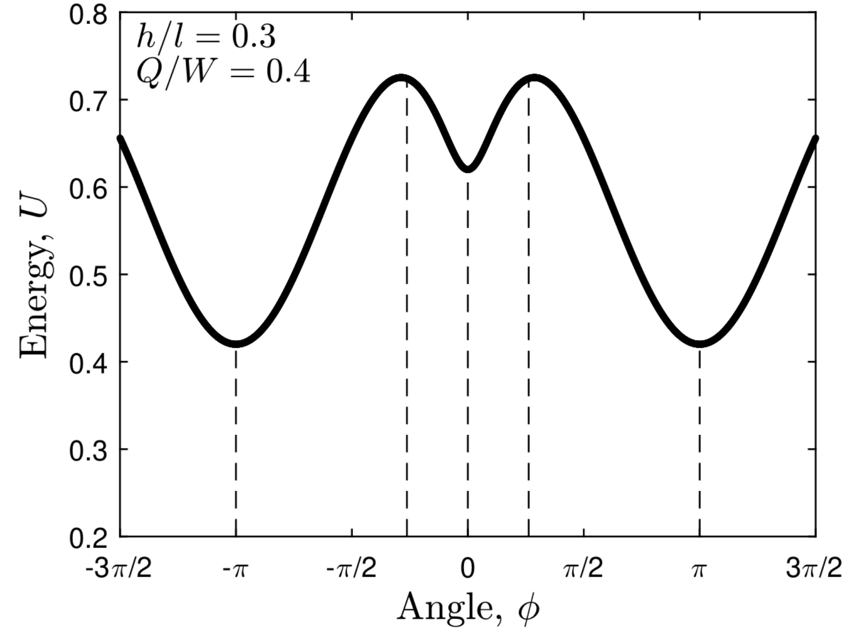

h/l = 0
Q/W = 0.4
Derivative of energy, dU/dφ
Angle, φ
0.5
0
-0.5
-3π/2
-π
-π/2
0
π/2
π
3π/2

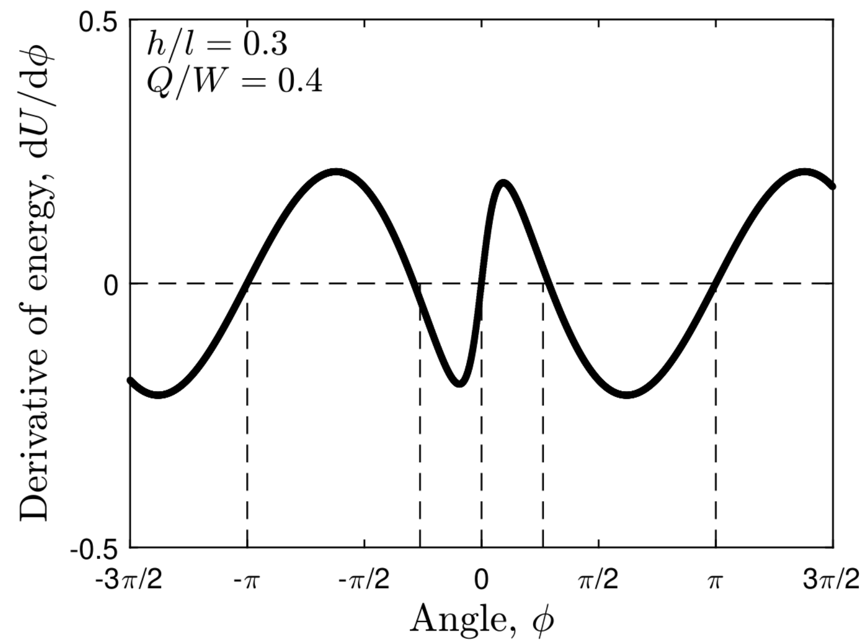




$$s = \sqrt{(l \sin\phi)^2 + (h + l - l\cos\phi)^2}$$

$$s = \sqrt{(l \sin\phi)^2 + (h + l(1-\cos\phi))^2}$$

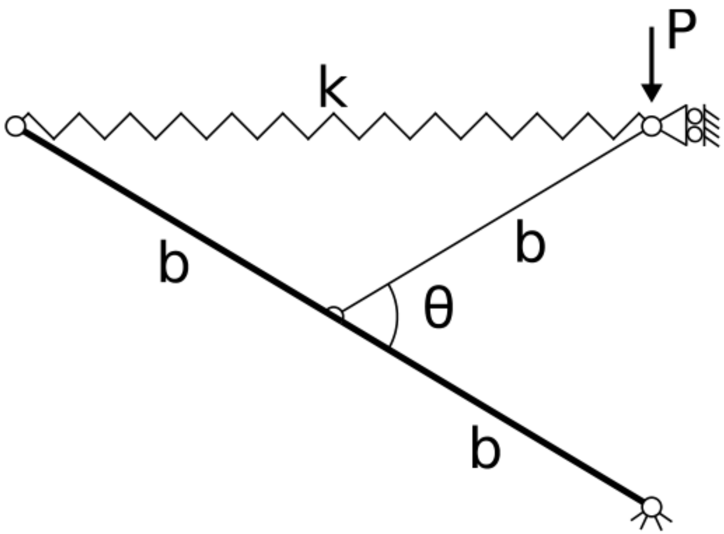


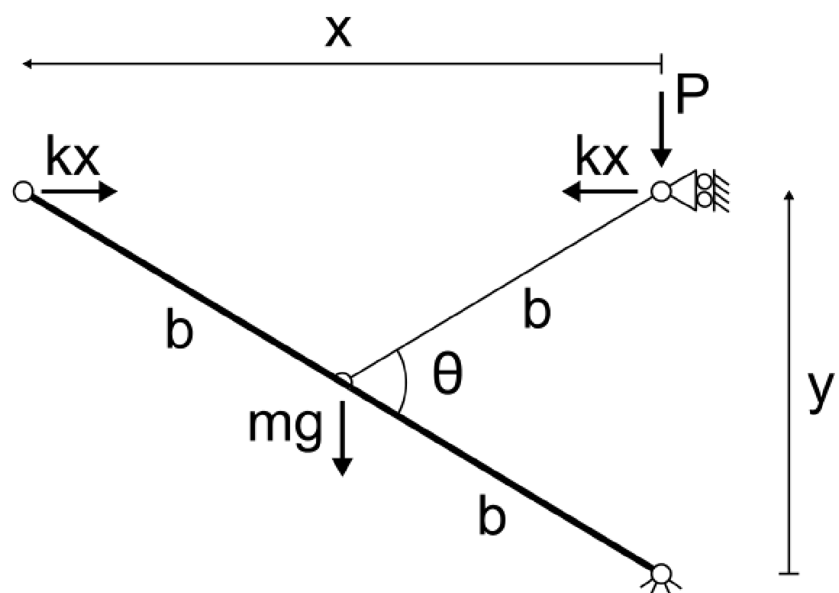


L17.2.1 Cases when the principle of superposition cannot be applied

Example 17.4 — (We have seen this problem in previous lecture).

Q: Determine equilibrium position (using PVW)

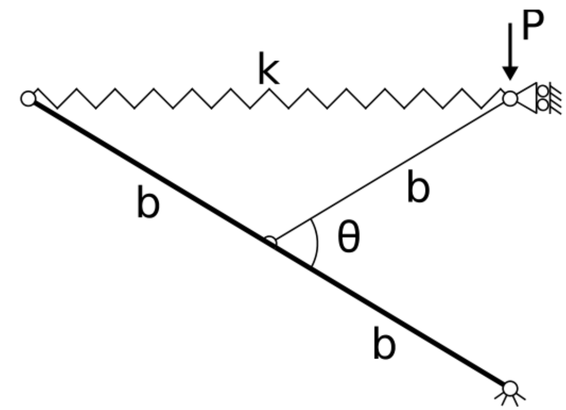




Solution: Active '*external*' forces: P, mg, kx, kx
Corresponding virtual displacements: δy, $\delta y/2$, δx, 0.
Virtual work:

$$\delta U = -P\delta y - mg\frac{\delta y}{2} - kx\delta x$$

Expression for the virtual displacements as a function of the free coordinate θ:



$$y = 2b \sin\frac{\theta}{2} \rightsquigarrow \delta y = \frac{\partial h}{\partial \theta}\delta\theta = b\cos\frac{\theta}{2}\delta\theta$$

$$x = 2b \cos\frac{\theta}{2} \rightsquigarrow \delta x = \frac{\partial x}{\partial \theta}\delta\theta = -b\sin\frac{\theta}{2}\delta\theta$$

Invoke the Principle of Virtual Work, $\delta\mathcal{U} = 0$:

$$\delta\mathcal{U} = 0 \qquad \Rightarrow -P\cos\frac{\theta}{2} - \frac{mg}{2}\cos\frac{\theta}{2} + 2kb\cos\frac{\theta}{2}\sin\frac{\theta}{2} = 0$$

Two possible solutions:

Solution 1

$$\cos\frac{\theta}{2} = 0 \qquad \Rightarrow \qquad \theta = \pi$$

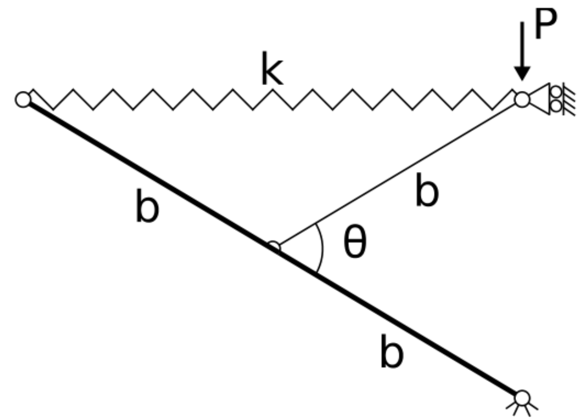
Completely folded

Solution 2

$$\theta = 2\arcsin\left(\frac{P + \frac{mg}{2}}{2kb}\right)$$

Non-trivial

If we had only considered mg:



$$\delta U = 0 \rightsquigarrow -\frac{mg}{2}\cos\frac{\theta}{2} + 2kb\cos\frac{\theta}{2}\sin\frac{\theta}{2} = 0$$

$$\cos\frac{\theta}{2} = 0 \rightsquigarrow \theta^{mg} = \pi \ \text{ completely folded system}$$

$$\theta^{mg} = 2\arcsin\left(\frac{\frac{mg}{2}}{2kb}\right)$$

If we had only considered P:

$$\delta U = 0 \rightsquigarrow -P\cos\frac{\theta}{2} + 2kb\cos\frac{\theta}{2}\sin\frac{\theta}{2} = 0$$

$$\cos\frac{\theta}{2} = 0 \rightsquigarrow \theta^{P} = \pi \ \text{ completely folded system}$$

$$\theta^{P} = 2\arcsin\left(\frac{P}{2kb}\right)$$

Non-trivial solution from above

$$\boxed{\theta = 2\arcsin\left(\frac{P+\frac{mg}{2}}{2kb}\right)}$$

$$2\arcsin\left(\frac{P+\frac{mg}{2}}{2kb}\right) \neq 2\arcsin\left(\frac{\frac{mg}{2}}{2kb}\right) + 2\arcsin\left(\frac{P}{2kb}\right)$$

$$\theta \neq \theta^{mg} + \theta^{P}!$$

The principle of superposition does not apply because the small deformations hypothesis (linear) is not respected