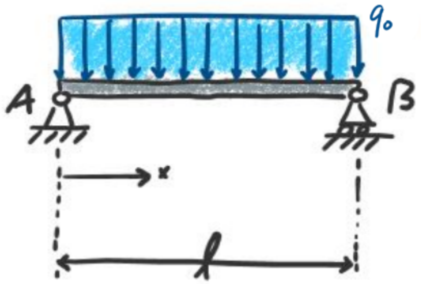
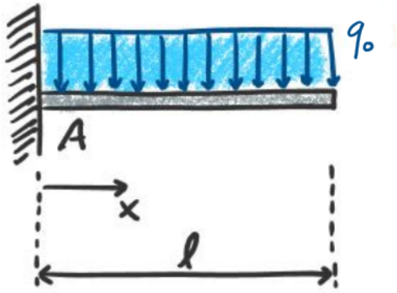
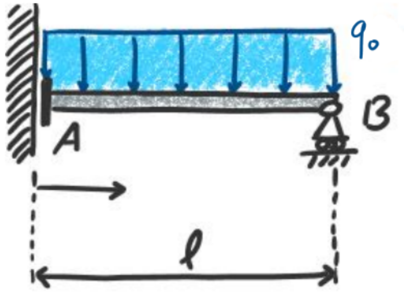
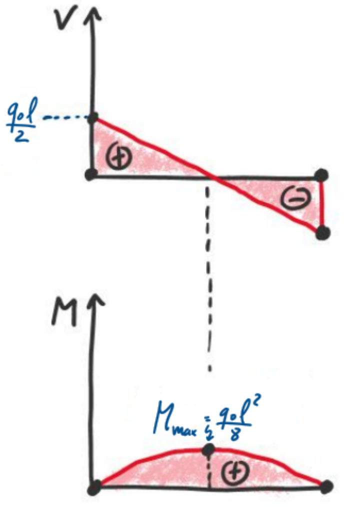
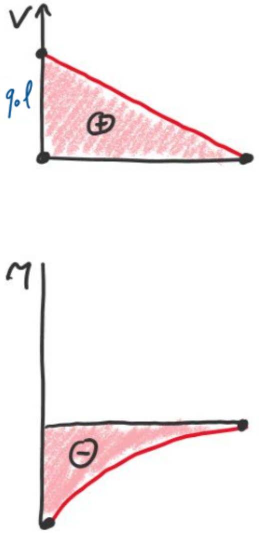
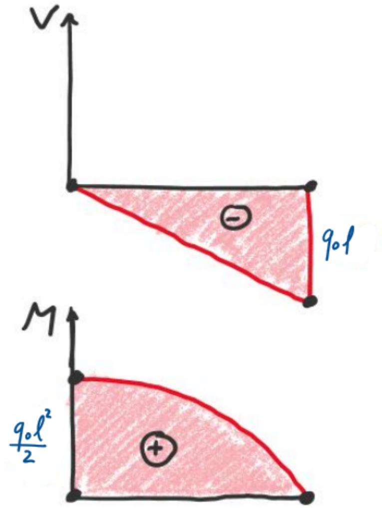
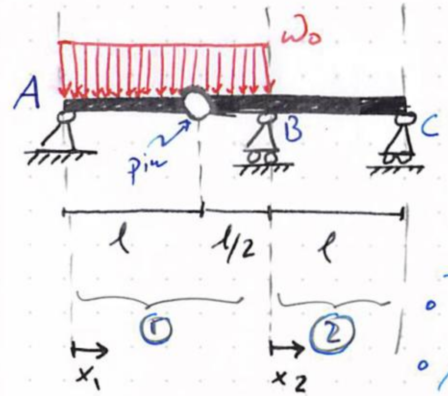
			
Boundary Conditions	$M(0) = 0$ $M(l) = 0$	$V(l) = 0$ $M(l) = 0$	$V(0) = 0$ $M(l) = 0$
Constants of integration	$C_1 = \frac{1}{2}q_o l$ $C_2 = 0$	$C_1 = q_o l$ $C_2 = -\frac{1}{2}q_o l^2$	$C_1 = 0$ $C_2 = \frac{1}{2}q_o l^2$
Stress Resultants	$V = \frac{1}{2}q_o l\left(1 - 2\frac{x}{l}\right)$ $M = \frac{1}{2}q_o l^2 \frac{x}{l}\left(1 - \frac{x}{l}\right)$	$V = q_o l\left(1 - \frac{x}{l}\right)$ $M = -\frac{1}{2}q_o l^2\left(1 - \frac{x}{l}\right)^2$	$V = -q_o x$ $M = \frac{1}{2}q_o l^2\left(1 - \left(\frac{x}{l}\right)^2\right)$
Support Reactions	$A = V(0) = \frac{1}{2}q_o l$ $B = -V(l) = \frac{1}{2}q_o l$	$A = V(0) = q_o l$ $M_A = M(0) = -\frac{1}{2}q_o l^2$	$M_A = M(0) = \frac{1}{2}q_o l$ $B = -V(l) = q_o l$
			

EXAMPLE: (Similar to example 13.1 in notes)



Q: Determine the stress resultants for this compound beam.

- Two regions ① and ②
- Note discontinuity of line load at point B

SOLUTION:

Part ①:

$$w^{①} = w_0$$
$$V^{①} = -w_0 x_1 + C_1$$
$$M^{①} = -\frac{1}{2} w_0 x_1^2 + C_1 x + C_2$$

Part ②:

$$w^{②} = 0$$
$$V^{②} = C_3$$
$$M^{②} = C_3 x_2 + C_4$$

4 Matching/Boundary Conditions

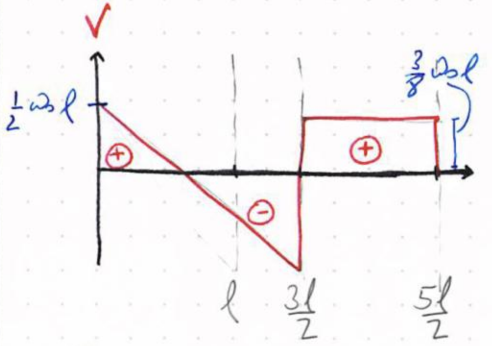
$M^{①}(x_1 = 0) = 0 \qquad M^{②}(x_2 = \ell) = 0$ ...... B.C.

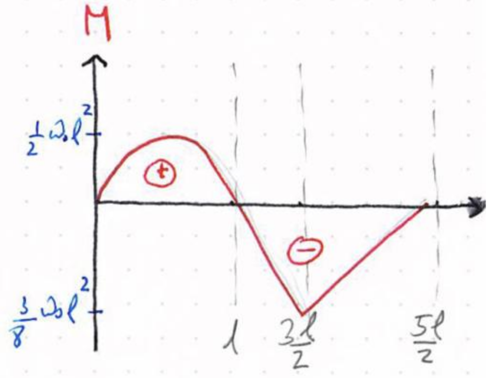
$M^{①}(x_1 = \frac{3}{2}\ell) = M^{②}(x_2 = 0)$ ---------- Matching condition

$M^{①}(x_1 = \ell) = 0$ ---------- Zero moment at internal pin

$$\Longrightarrow C_1 = \frac{1}{2} w_0 \ell \qquad C_2 = 0$$
$$C_3 = \frac{3}{8} w_0 \ell \qquad C_4 = -\frac{3}{8} w_0 \ell^2$$

$$\Longrightarrow V^{①} = -w_0 x_1 + \frac{1}{2} w_0 \ell$$
$$V^{②} = \frac{3}{8} w_0 \ell$$
$$M^{①} = -\frac{1}{2} w_0 x_1^2 + \frac{1}{2} w_0 \ell x_1$$
$$M^{②} = \frac{3}{8} w_0 \ell (x_2 - \ell)$$





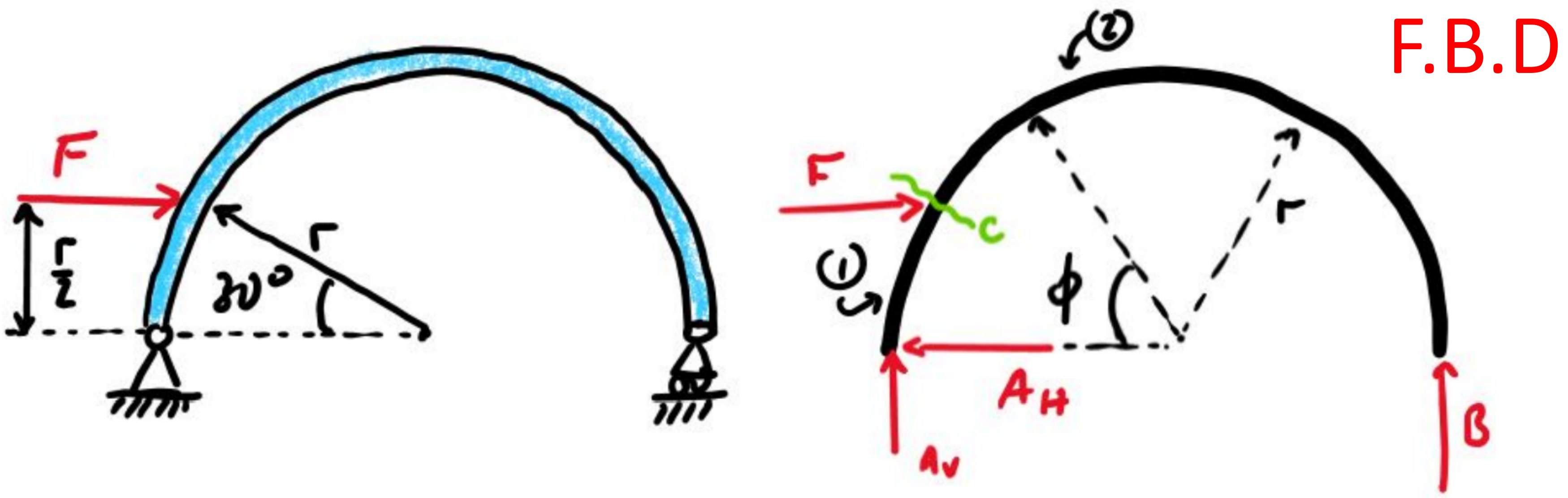
Check support reactions:

$$A_y = V^{①}(x_1 = 0) = \frac{1}{2} w_0 \ell$$
$$B_y = V^{②}(x_2 = 0) - V^{①}(x_1 = \frac{3}{2}\ell) = \frac{11}{8} w_0 \ell$$
$$C_y = -V^{②}(x_2 = \ell) = -\frac{3}{8} w_0 \ell$$

$A_y + B_y + C_y = \frac{3}{2} w_0 \ell \equiv$ Resultant external load.

L14.2 Arches

Example 14.2 — An arch:. We consider an arch subjected to a concentrated force F



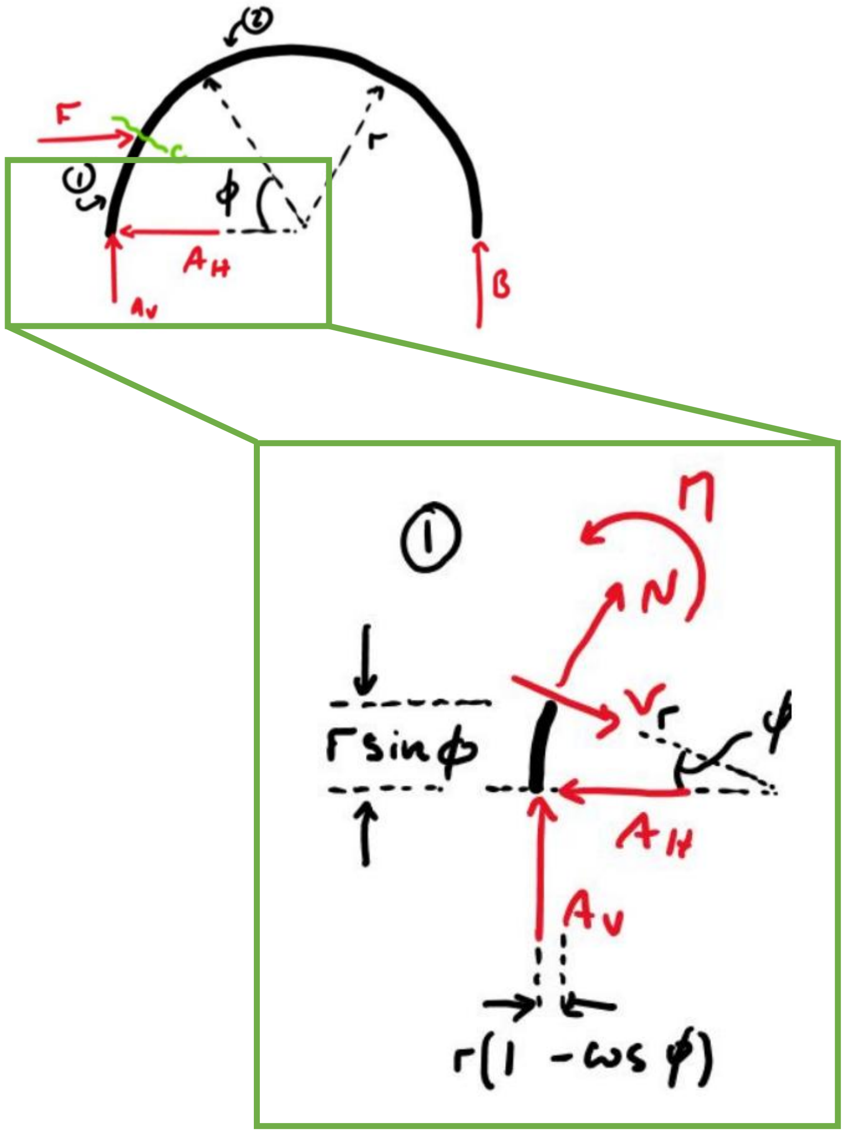
The reactions can be readily obtained from the equilibrium of the whole structure

$$\boxed{B_y = \frac{1}{4}F; \qquad A_V = -B_y = -\frac{1}{4}F; \qquad A_H = F}$$

Global Equilibrium

The internal loads can be determined by segmenting the arch into two parts: part ① for $0 < \phi < 30°$ and part ② for $30° \leq \phi < 180°$.

Equilibrium for part ① $(0 < \phi < 30^\circ)$:



$$\sum F^{\nearrow} = 0: \qquad N + A_V \cos\phi - A_H \sin\phi = 0 \Rightarrow$$

$$N(x) = \left(\sin\phi + \frac{1}{4}\cos\phi\right) F$$

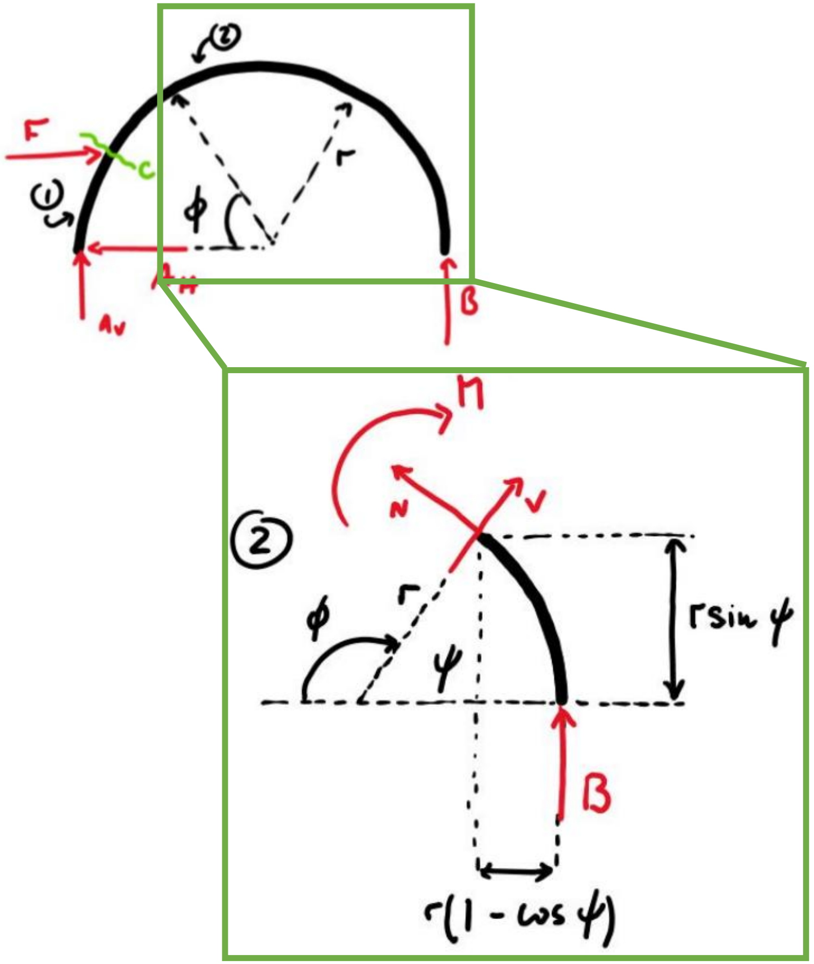
$$\sum F^{\searrow} = 0: \qquad V - A_V \sin\phi - A_H \cos\phi = 0 \Rightarrow$$

$$V(x) = \left(\cos\phi - \frac{1}{4}\sin\phi\right) F$$

$$\sum M_c = 0: \qquad M - r\sin\phi A_H - r(1-\cos\phi)A_V = 0 \Rightarrow$$

$$M(x) = \left(\sin\phi + \frac{1}{4}\cos\phi - \frac{1}{4}\right) rF$$

Equilibrium for part ② ($30^\circ \leq \phi < 180^\circ$): For simplicity, we introduce the angle $\psi = \pi - \phi$ and write the equilibrium conditions for part ②.



$$\sum F^{\nwarrow} = 0 : \qquad N + B\cos\psi = 0 \Rightarrow$$

$$N(x) = -\frac{1}{4}F\cos\psi = \frac{1}{4}F\cos\varphi$$

$$\sum F^{\nearrow} = 0 : \qquad V + B\sin\psi = 0 \Rightarrow$$

$$V(x) = -\frac{1}{4}F\sin\psi = -\frac{1}{4}F\sin\varphi$$

$$\sum M_C = 0 : \qquad -M + r(1-\cos\psi)B = 0 \Rightarrow$$

$$M(x) = \frac{1}{4}(1-\cos\psi)rF = \frac{1}{4}(1+\cos\varphi)rF$$

Note that the jumps $\Delta N = F/2$ in the normal and $\Delta V = \sqrt{3}F/2$ in the shear force, both at $\phi = 30^\circ$ correspond, respectively, to the components of the forces tangential and orthogonal to the arch there.