Teachers : Profs. Pedro M. Reis and Sangwoo Kim
ME-104 Introduction to Structural Mechanics
Spring 2025
(Practice) Quiz II
30th April 2025
Duration : 60 minutes

1

Student name

SCIPER : **999000**

Do not turn the page before the start of the exam. This document is double-sided, has 16 pages, the last ones possibly blank. Do not unstaple.

- Place your student card on your table.
- Please read all questions carefully and write directly onto this test.
- You can use one A5 sheet of handwritten notes (both sides) and a calculator.
- Phones, laptops and any other device capable of communication are **not allowed**.
- For the **multiple choice** questions, we give:
  +3 points if your answer is correct,
  0 points if you give no answer or more than one.
- For the **open-ended** questions, the maximum number of points is listed next to each question and sub-question. Show your work clearly and put a box around your final answer.
- Use a **black or dark blue ballpen** and clearly erase with **correction fluid** if necessary.
- If there is an erratum (corrections page), an announcement will be made at the start of the exam. **In case there is an erratum, please take note of any necessary corrections to the questions.**

Part I	Part II	Part III
/30	/36	/34
Total: /100		

Respectez les consignes suivantes | Observe this guidelines | Beachten Sie bitte die unten stehenden Richtlinien

choisir une réponse | select an answer Antwort auswählen

ne PAS choisir une réponse | NOT select an answer NICHT Antwort auswählen

Corriger une réponse | Correct an answer Antwort korrigieren

ce qu'il ne faut **PAS** faire | what should **NOT** be done | was man **NICHT** tun sollte

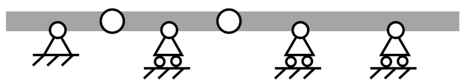
PART I: CONCEPT QUESTIONS (MULTIPLE-CHOICE)

For each question, mark the box corresponding to the correct answer. Each question has **exactly one** correct answer.

1. Static and kinematic determinacy

[12 points] For each of the four questions below (1.1–1.4), is the structure (comprising bars, beams, joints, and supports) statically and/or kinematically determinate? Choose the correct answers.

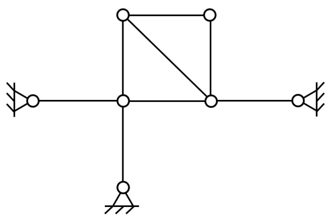
Question 1.1



The system is

☐ **hyperstatic-hypostatic** and kinematically **indeterminate**.
☐ **hyperstatic** and kinematically **determinate**.
☐ **hypostatic** and kinematically **indeterminate**.
■ **isostatic** and kinematically **determinate**.

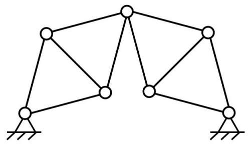
Question 1.2



The system is

☐ **hyperstatic** and kinematically **determinate**.
☐ **isostatic** and kinematically **determinate**.
☐ **hypostatic** and kinematically **indeterminate**.
■ **hyperstatic-hypostatic** and kinematically **indeterminate**.

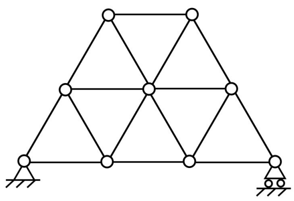
Question 1.3



The system is

- [ ] **hyperstatic-hypostatic** and kinematically **indeterminate**.
- [ ] **hypostatic** and kinematically **indeterminate**.
- [ ] **hyperstatic** and kinematically **determinate**.
- [x] **isostatic** and kinematically **determinate**.

Question 1.4



The system is

- [ ] **isostatic** and kinematically **determinate**.
- [ ] **hypostatic** and kinematically **indeterminate**.
- [x] **hyperstatic** and kinematically **determinate**.
- [ ] **hyperstatic-hypostatic** and kinematically **indeterminate**.

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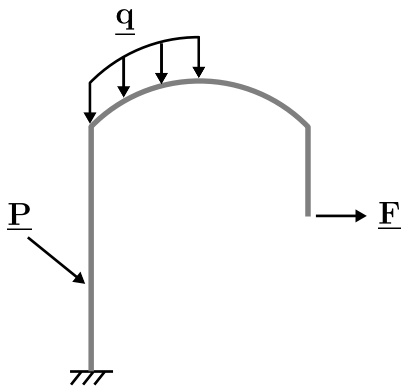
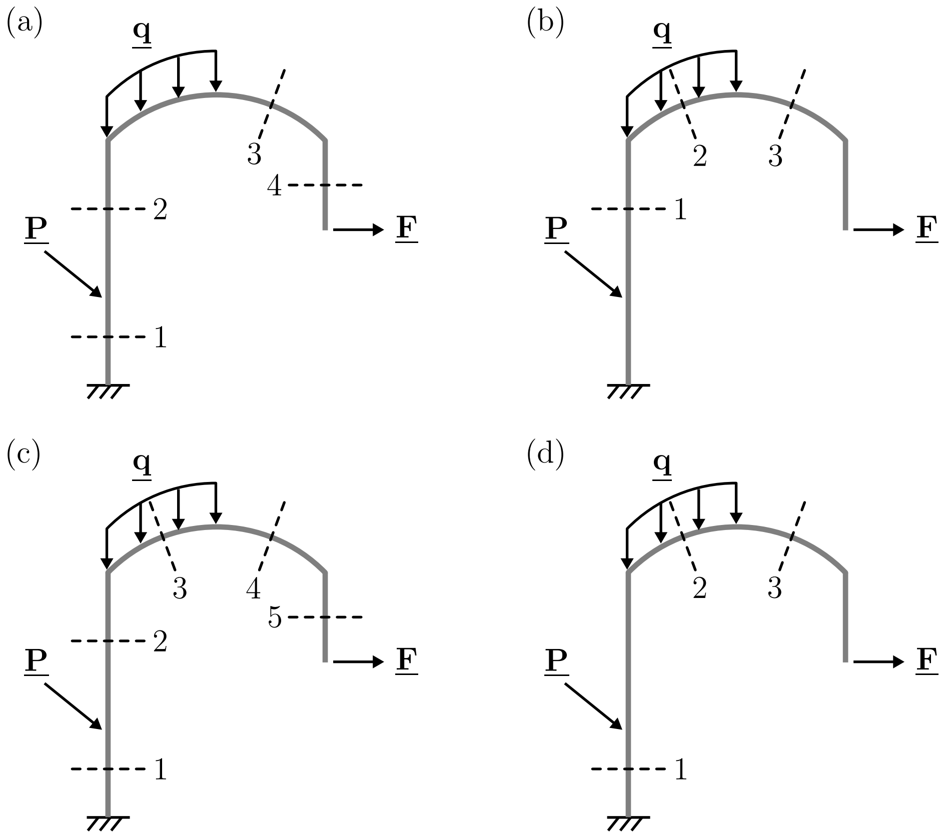
2. Frames and segmenting

[3 points] A rigid frame comprises two straight (vertical) elements connected by a circular arch, as shown below. A distributed force $\underline{\mathbf{q}}$ and two concentrated forces $\underline{\mathbf{F}}$ and $\underline{\mathbf{P}}$ are applied to the frame.



Question 2.1 Choose the correct system of cuts that would be necessary to determine **all** internal load diagrams of the loaded frame.



- ☐ (b)
- ☒ (c)
- ☐ (a)
- ☐ (d)

3. Internal load diagrams for beams

[15 points] Consider a straight beam subjected to concentrated forces $\underline{\mathbf{F}}_i$, distributed forces $\underline{\mathbf{q}}_i$, and/or concentrated couples $\underline{\mathbf{M}}_i$. In each of the five questions below (3.1—3.5), choose the answer that, when inserted into the blank space (______), ensures that the sentence is correct.

Question 3.1 In the absence of ______, the bending moment diagram is always linear or piecewise-linear.

- [ ] $\underline{\mathbf{F}}_i$
- [x] $\underline{\mathbf{q}}_i$
- [ ] $\underline{\mathbf{M}}_i$
- [ ] both $\underline{\mathbf{q}}_i$ and $\underline{\mathbf{M}}_i$

Question 3.2 The shear force diagram can have a non-zero slope only when ______ are applied.

- [ ] $\underline{\mathbf{M}}_i$
- [ ] $\underline{\mathbf{F}}_i$
- [x] $\underline{\mathbf{q}}_i$
- [ ] $\underline{\mathbf{M}}_i$ or $\underline{\mathbf{F}}_i$

Question 3.3 The slope of the bending moment diagram depends on $\underline{\mathbf{F}}_i$ ______.

- [ ] and both $\underline{\mathbf{q}}_i$ and $\underline{\mathbf{M}}_i$
- [ ] only
- [ ] and $\underline{\mathbf{M}}_i$
- [x] and $\underline{\mathbf{q}}_i$

Question 3.4 The bending moment diagram is discontinuous at the points where ______ are applied.

- [ ] $\underline{\mathbf{q}}_i$
- [x] $\underline{\mathbf{M}}_i$
- [ ] $\underline{\mathbf{M}}_i$ or $\underline{\mathbf{F}}_i$
- [ ] $\underline{\mathbf{F}}_i$

Question 3.5 The shear force diagram is discontinuous at the points where ______ are applied.

- [ ] $\underline{\mathbf{q}}_i$
- [x] $\underline{\mathbf{F}}_i$
- [ ] $\underline{\mathbf{M}}_i$
- [ ] $\underline{\mathbf{F}}_i$ or $\underline{\mathbf{q}}_i$

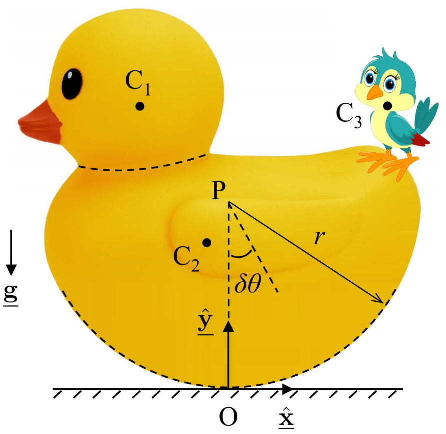
PART II: NUMERICAL QUESTIONS (OPEN-ENDED)

Provide your answer to each question in the empty space provided. Your answer should be carefully justified, and all the steps of your argument should be discussed in detail. Leave the checkboxes empty; they are used for grading.

4. Equilibrium states and stability

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

The rubber duck shown below is composed of three parts: part (1) is the duck head, part (2) is the duck body (excluding the head), and part (3) is an additional bird glued to the duck's tail[1]. The bottom of the duck is a circular arc of radius r, which is in frictionless contact with the flat ground at origin O. Point P is the center of the circle containing the arc. The three parts of the duck are described by their masses $m_1 = m/4$, $m_2 = m$, and $m_3 = m/4$ and their corresponding centers of mass: $(x_{C1}, y_{C1}) = (-r/2, 3r/2)$, $(x_{C2}, y_{C2}) = (-r/8, 3r/4)$, and $(x_{C3}, y_{C3}) = (r, 3r/2)$. The constant gravitational field is $\underline{\mathbf{g}} = -g\underline{\hat{\mathbf{y}}}$.



Question 4.1 [12 points] **Compute** the coordinates (x_C, y_C) of the center of mass of the entire system (comprising all three parts). Use this to **justify** why the system is in equilibrium.

Solution:
The coordinates of the center of mass of the system:

$$x_C = \frac{\Sigma_{i=1}^{3} x_{Ci} m_i}{\Sigma_{i=1}^{3} m_i} = 0 \,\textbf{[4 points]},\; y_C = \frac{\Sigma_{i=1}^{3} y_{Ci} m_i}{\Sigma_{i=1}^{3} m_i} = r \,\textbf{[4 points]}.$$

Since the contact between the duck and the ground is frictionless, the only component of the force exerted by the ground at the contact point is the reaction force, which passes through the center of gravity of the system. Therefore, the moment about point P is zero, and the system is in equilibrium. **[4 points]**

[1] Adapted from Example 8.9 in D. Gross *et al.*, *Engineering Mechanics 1: Statics* (2009). Image credits: amazon.com/Liberty-Imports-Jumbo-Rubber-Duck/dp/B000DBPB1U, pixabay.com/vectors/bird-cute-feather-robin-small-5112854

Question 4.2 [6 points] **Determine** the type of stability of the current equilibrium configuration (stable, unstable, or neutral) when a small rotational perturbation $\delta\theta$ about the center of rotation point P is imposed.

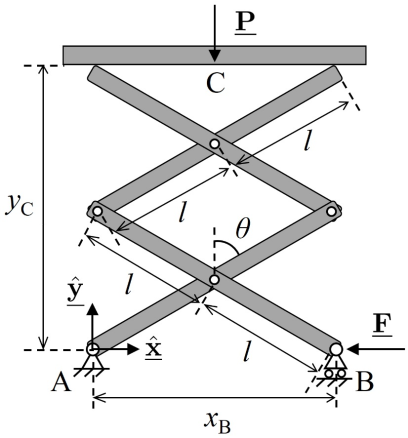
Solution:
The center of gravity of the system coincides with the center of rotation P, thus independent of rotation. **[3 points]** For any sufficiently small $\delta\theta$, the line of action of the reaction force exerted by the ground always passes through the center of gravity, resulting in zero moment about point P. Therefore, the system is in neutral equilibrium. **[3 points]**

Alternatively, the potential energy of the entire system is $V = 3/2mgr$ for any rotation about point P. Hence, the first and second derivatives of V will be zero (and also higher-order derivatives), so it is a neutral equilibrium.

5. Principal of virtual work

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

Consider a scissor lift with the geometry and dimensions shown in the diagram below. The origin of the coordinate system is at the pinned support A. A vertical force $\underline{\mathbf{P}} = -P\hat{\underline{\mathbf{y}}}$ is applied to the horizontal platform at point C. A horizontal force $\underline{\mathbf{F}} = -F\hat{\underline{\mathbf{x}}}$ is imposed at roller B to keep the lift in equilibrium. Assume that all joints and contacts in the structure are frictionless, and neglect its self-weight.



To derive the equilibrium equation of the system using the principle of virtual work (PVW), we choose the angle θ as the single degree of freedom to describe the kinematics of the lift, as shown in the diagram. Then, by geometry, the horizontal position of point B is $x_B = 2l\sin\theta$, and $y_C = 4l\cos\theta$ is the vertical position of point C.

Question 5.1 [6 points] Taking the above expressions for x_B and y_C as given (you do not need to derive them), **compute** the corresponding virtual displacements δx_B and δy_C as functions of l, θ, and the virtual rotation $\delta\theta$.

Solution:
The virtual displacements are given by

$$\delta x_B = \frac{\mathrm{d}x_B}{\mathrm{d}\theta}\delta\theta = 2l\cos\theta\delta\theta\,\textbf{[3 points]},$$
$$\delta y_C = \frac{\mathrm{d}y_C}{\mathrm{d}\theta}\delta\theta = -4l\sin\theta\delta\theta\,\textbf{[3 points]}.$$

Question 5.2 [12 points] Using the above results, **write** an expression for the virtual work of the system, δU, and **derive** the equilibrium equation.

Solution:
We present two equivalent methods to determine the virtual work of the system.

Method of using scalar distance along force direction:
We see that the virtual displacement $\delta x_{\mathrm{B}} = 2l\cos\theta\delta\theta$ is *positive* for $\theta \in (0, \pi)$ and $\delta\theta > 0$, while the virtual displacement $\delta y_{\mathrm{C}} = -4l\sin\theta\delta\theta$ is *negative*. Hence, under a virtual rotation $\delta\theta > 0$,

- Point B moves along the positive x-direction (i.e., x_{B} increases), which is in the *opposite* direction to the force F (recall $\underline{\mathbf{F}} = -F\hat{\underline{\mathbf{x}}}$). The horizontal distance travelled is $|\delta x_{\mathrm{B}}| = \delta x_{\mathrm{B}}$.
- Point C moves along the *negative* y-direction (i.e., y_{C} decreases), which is in the *same* direction as the force P (recall $\underline{\mathbf{P}} = -P\hat{\underline{\mathbf{y}}}$). The vertical distance travelled is $|\delta y_{\mathrm{C}}| = -\delta y_{\mathrm{C}}$.

The virtual work is therefore

$$\begin{aligned}\delta U &= -F|\delta x_{\mathrm{B}}| + P|\delta y_{\mathrm{C}}| \\ &= -F\delta x_{\mathrm{B}} - P\delta y_{\mathrm{C}} \\ &= -2Fl\cos\theta\delta\theta + 4Pl\sin\theta\delta\theta \,\textbf{[6 points]}.\end{aligned}$$

Invoking PVW, $\delta U = 0$ **[3 points]**, the equilibrium equation

$$-F\cos\theta + 2P\sin\theta = 0 \,\textbf{[3 points]}.$$

Method of using vectors and dot products:
Expressed in vector form, the displacements of the points B and C are

$$\delta\mathbf{r}_{\mathrm{B}} = \delta x_{\mathrm{B}}\hat{\underline{\mathbf{x}}}, \quad \delta\mathbf{r}_{\mathrm{C}} = \delta y_{\mathrm{C}}\hat{\underline{\mathbf{y}}}.$$

The virtual work is therefore

$$\begin{aligned}\delta U &= \underline{\mathbf{F}} \cdot \delta\mathbf{r}_{\mathrm{B}} + \underline{\mathbf{P}} \cdot \delta\mathbf{r}_{\mathrm{C}} \\ &= (-F\hat{\underline{\mathbf{x}}}) \cdot (\delta x_{\mathrm{B}}\hat{\underline{\mathbf{x}}}) + \left(-P\hat{\underline{\mathbf{y}}}\right) \cdot \left(\delta y_{\mathrm{C}}\hat{\underline{\mathbf{y}}}\right) \\ &= -F\delta x_{\mathrm{B}} - P\delta y_{\mathrm{C}}\end{aligned}$$

as above.

PART III: EXTENDED NUMERICAL QUESTION

6. Structural analysis of a platform for cliff diving

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30 31 32 33 34

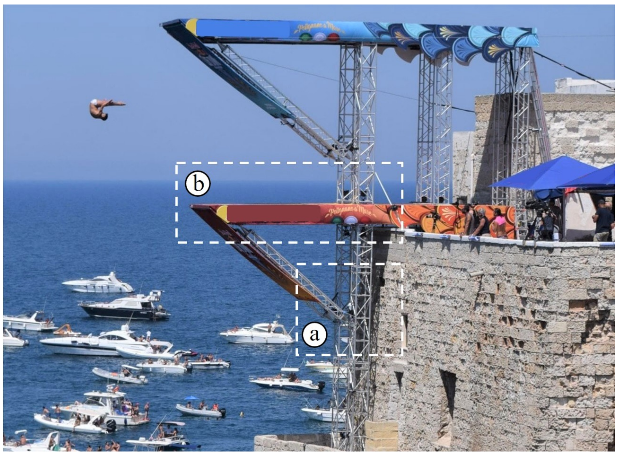


Image credit: bari.repubblica.it/cronaca/2021/05/20/news/red_bull_cliff_diving_polignano-301895183

In cliff diving, an extreme sport, athletes jump acrobatically from platforms built up to 30 m above sea level. The photograph on the left shows the platform constructed out of trusses and beams used in the 2019 Red Bull Cliff Diving World Series (Polignano a Mare, Italy).

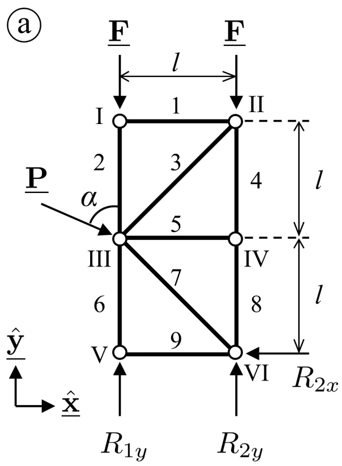
In this question, you will analyze the internal loads in structures similar to the truss (a) and the beam (b) indicated by the dashed boxes in the photograph.

Part (a): Analysis of the truss

For simplicity, we consider two building blocks of the truss, as shown in the schematic on the right. The members and joints are labeled, respectively, by the numbers 1–9 and I–VI. All the vertical and horizontal members have length l. The truss, assumed to be ideal, is subjected to external loads $\underline{\mathbf{F}} = -F\underline{\hat{\mathbf{y}}}$ and $\underline{\mathbf{P}} = P\sin\alpha\underline{\hat{\mathbf{x}}} - P\cos\alpha\underline{\hat{\mathbf{y}}}$ applied at the joints (α is the angle between $\underline{\mathbf{P}}$ and the vertical), as shown in the schematic. The reaction forces at the supports (joints V and VI) are:

$$
\begin{aligned}
R_{1y} &= F + P\cos\alpha - P\sin\alpha, \\
R_{2x} &= P\sin\alpha, \\
R_{2y} &= F + P\sin\alpha.
\end{aligned}
$$

*Note: You do **not** need to show the above results; use them as given.*



Question 6.1 [6 points] **Identify all** the zero-force members in this truss by writing a **list** of their labels (numbers). *Note: You do **not** need to justify your answers but 1.5 points will be **deducted** for each incorrect identification. (Hint: There are 4 zero-force members.)*

Solution:
The zero-force members: 1, 3, 5, and 9 **[6 points]**.

Question 6.2 [12 points] Using either the method of sections or the method of joints, **determine** the internal force, S_7, in member 7 as a function of F, P, and α. (It is not necessary to use all of the variables, F, P, and α). **State** whether member 7 is in tension or in compression.

Solution:
Remember that bars 1, 3, 5, and 9 are zero-force members.

(1) Using method of sections
Perform an imaginary cut that cuts bars 1, 3, 5, 7, and 9 **[2 points]**. This cut separates the truss into two parts. Impose equilibrium to the right part:

$$S_{7x} = -R_{2x} = -P\sin\alpha,$$
$$\text{or } S_{7y} = F - R_{2y} = -P\sin\alpha\,\textbf{[4 points]}.$$

Therefore,

$$S_7 = \sqrt{2}S_{7x} = \sqrt{2}S_{7y} = -\sqrt{2}P\sin\alpha\,\textbf{[4 points]}.$$

Impose equilibrium to the left part gives the same result. Since $S_7 < 0$, the member 7 is in compression **[2 points]**.

(2) Using method of joints
Isolating joint I, apply the equilibrium condition in the vertical direction:

$$S_2 = -F\,\textbf{[1 point]}.$$

Isolating joint V, apply the equilibrium condition in the vertical direction:

$$S_6 = -R_{1y} = -F - P\cos\alpha + P\sin\alpha\,\textbf{[1 point]}.$$

Isolating joint III, apply the equilibrium conditions in the horizontal or vertical direction:

$$S_{7x} = -P\sin\alpha,$$
$$\text{or } S_{7y} = S_2 - S_6 - P\cos\alpha = -P\sin\alpha\,\textbf{[4 points]}.$$

Therefore,

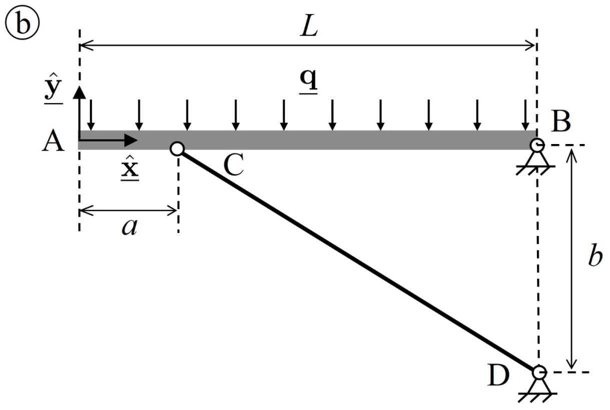
$$S_7 = \sqrt{2}S_{7x} = \sqrt{2}S_{7y} = -\sqrt{2}P\sin\alpha\,\textbf{[4 points]}.$$

Since $S_7 < 0$, the member 7 is in compression **[2 points]**.

Note: Above, we provide two solutions using method of sections and method of joints. The choices of sections and joints are not unique. You can freely use the two methods at different sections and joints to ease the calculation.

Part (b): Analysis of the beam

Now, you are asked to consider an alternative design slightly different from the one in the above photograph, to also include a cantilevered segment of length a (see the schematic on the right). The beam AB of total length L is simply supported at point B and free at point A. A bar CD (connecting rod) is pinned at point C onto the beam with the other extremity D pinned to the ground. The beam is loaded by its self-weight, which imposes a uniformly distributed force $\underline{\mathbf{q}} = -q\underline{\hat{\mathbf{y}}}$. The horizontal distance between points A and C is a, and we assume $0 < a < L/2$. The vertical distance between points B and D is b.



It can be shown that the force $\underline{\mathbf{F}}_{\mathrm{C}}$ exerted by the bar CD on the beam at point C is

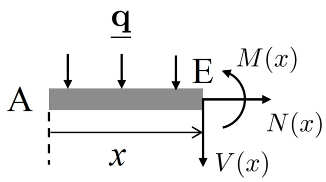
$$\underline{\mathbf{F}}_{\mathrm{C}} = \left(-\frac{1}{2}qL\frac{L}{b}\right)\underline{\hat{\mathbf{x}}} + \left(\frac{1}{2}qL\frac{L}{L-a}\right)\underline{\hat{\mathbf{y}}}.$$

Note: You do ***not*** *need to derive this result. Take it as a given for the remainder of the exercise.*

Question 6.3 [12 points] **Derive** expressions for *both* the internal bending moment, $M(x)$, *and* the shear force, $V(x)$, along the axis of the beam from A to B. Follow the sign convention used in class for internal loads at a positive face: $V(x)$ is assumed positive pointing downward and $M(x)$ is positive in the anticlockwise direction.

Solution:

For $0 \le x < a$, place a cut between points A and C, and consider the left part **[2 point]**:

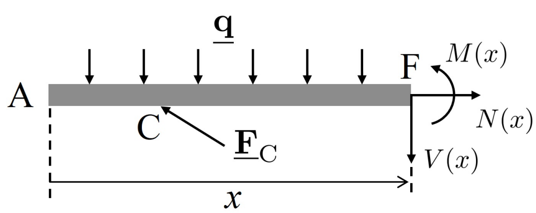


By imposing equilibrium at point E:

$$\Sigma M_{\mathrm{E}} = M(x) + \frac{1}{2}qx^2 = 0 \rightarrow M(x) = -\frac{1}{2}qx^2 \textbf{ [2 points]},$$

$$\Sigma F_{\mathrm{y}} = -V(x) - qx = 0 \rightarrow V(x) = -qx = \frac{\mathrm{d}M}{\mathrm{d}x} \textbf{ [2 points]}.$$

For $a \le x \le L$, place a cut between points C and B, and consider the left part **[2 point]**:

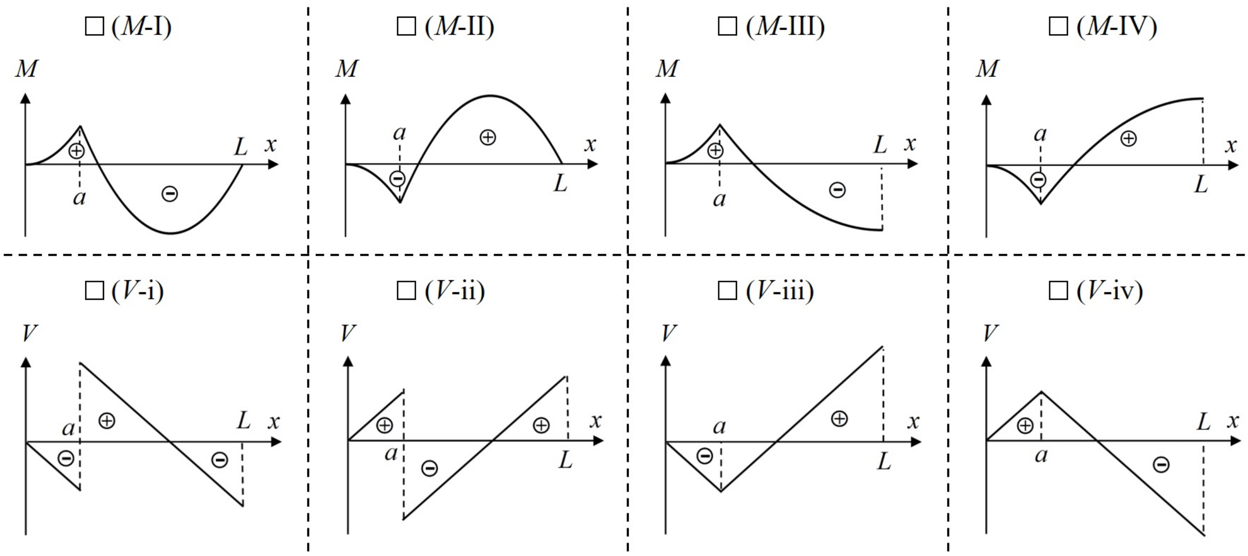


By imposing equilibrium at point F:

$$\Sigma M_{\mathrm{F}} = M(x) + \frac{1}{2}qx^2 - \underline{\mathbf{F}}_{\mathrm{C}} \cdot \underline{\hat{\mathbf{y}}}(x-a) = 0 \rightarrow M(x) = -\frac{1}{2}qx^2 + \frac{1}{2}qL^2\frac{x-a}{L-a} \text{ [2 points]},$$

$$\Sigma F_{\mathrm{y}} = -V(x) - qx + \underline{\mathbf{F}}_{\mathrm{C}} \cdot \underline{\hat{\mathbf{y}}} = 0 \rightarrow V(x) = -qx + \frac{1}{2}qL\frac{L}{L-a} = \frac{\mathrm{d}M}{\mathrm{d}x} \text{ [2 points]}.$$

Question 6.4 [4 points] According to your results in question (i) above, **choose** the correct $M(x)$ diagram from the four choices: M-I, M-II, M-III, and M-IV shown in the table below, and **choose** the correct $V(x)$ diagram from the other four choices: V-i, V-ii, V-iii, and V-iv shown below. *Note: Use the checkboxes in the diagrams to provide your answers.*



Solution:
(M-II) **[2 points]** and (V-i) **[2 points]** show the correct internal bending moment and shear force diagrams, respectively.

Use the empty pages for additional space if necessary.