



Teachers : Profs. Pedro M. Reis and Sangwoo Kim  
ME-104 Introduction to Structural Mechanics  
Spring 2025  
(Practice) Quiz II  
30<sup>th</sup> April 2025  
Duration : 60 minutes

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








Student name

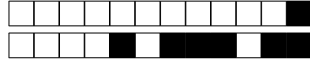
SCIPER : 999000

Do not turn the page before the start of the exam. This document is double-sided, has 16 pages, the last ones possibly blank. Do not unstaple.

- Place your student card on your table.
- Please read all questions carefully and write directly onto this test.
- You can use one A5 sheet of handwritten notes (both sides) and a calculator.
- Phones, laptops and any other device capable of communication are **not allowed**.
- For the **multiple choice** questions, we give:
  - +3 points if your answer is correct,
  - 0 points if you give no answer or more than one.
- For the **open-ended** questions, the maximum number of points is listed next to each question and sub-question. Show your work clearly and put a box around your final answer.
- Use a **black or dark blue ballpen** and clearly erase with **correction fluid** if necessary.
- If there is an erratum (corrections page), an announcement will be made at the start of the exam. **In case there is an erratum, please take note of any necessary corrections to the questions.**

Part I	Part II	Part III
<input type="text"/> /30	<input type="text"/> /36	<input type="text"/> /34
Total: <input type="text"/> /100		

Respectez les consignes suivantes   Observe this guidelines   Beachten Sie bitte die unten stehenden Richtlinien		
choisir une réponse   select an answer Antwort auswählen	ne PAS choisir une réponse   NOT select an answer NICHT Antwort auswählen	Corriger une réponse   Correct an answer Antwort korrigieren
  		 
ce qu'il ne faut <b>PAS</b> faire   what should <b>NOT</b> be done   was man <b>NICHT</b> tun sollte		
     		



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## PART I: CONCEPT QUESTIONS (MULTIPLE-CHOICE)

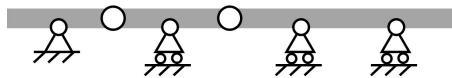
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For each question, mark the box corresponding to the correct answer. Each question has **exactly one** correct answer.

### 1. Static and kinematic determinacy

**[12 points]** For each of the four questions below (1.1–1.4), is the structure (comprising bars, beams, joints, and supports) statically and/or kinematically determinate? Choose the correct answers.

#### Question 1.1

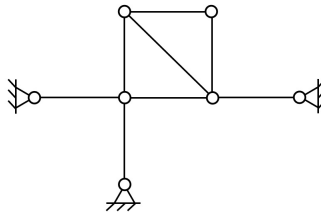


The system is

- ☐ **hyperstatic-hypostatic** and kinematically **indeterminate**.
- ☐ **hyperstatic** and kinematically **determinate**.
- ☐ **hypostatic** and kinematically **indeterminate**.
- ☐ **isostatic** and kinematically **determinate**.

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#### Question 1.2

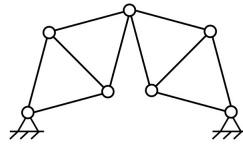


The system is

- ☐ **hyperstatic** and kinematically **determinate**.
  - ☐ **isostatic** and kinematically **determinate**.
  - ☐ **hypostatic** and kinematically **indeterminate**.
  - ☐ **hyperstatic-hypostatic** and kinematically **indeterminate**.
-



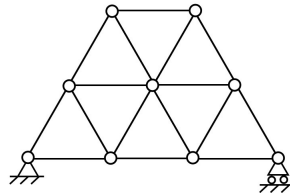
### Question 1.3



The system is

- ☐ **hyperstatic-hypostatic** and kinematically **indeterminate**.
- ☐ **hypostatic** and kinematically **indeterminate**.
- ☐ **hyperstatic** and kinematically **determinate**.
- ☐ **isostatic** and kinematically **determinate**.

### Question 1.4



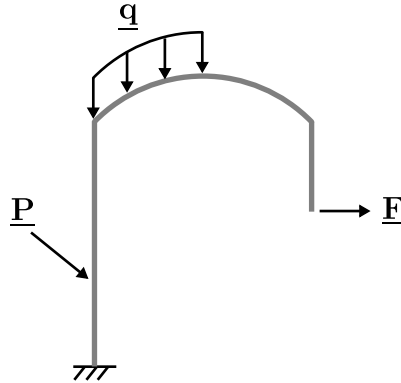
The system is

- ☐ **isostatic** and kinematically **determinate**.
- ☐ **hypostatic** and kinematically **indeterminate**.
- ☐ **hyperstatic** and kinematically **determinate**.
- ☐ **hyperstatic-hypostatic** and kinematically **indeterminate**.

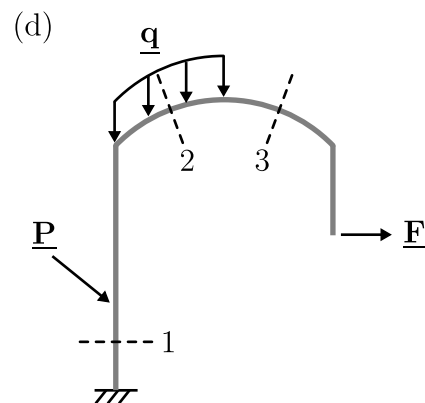
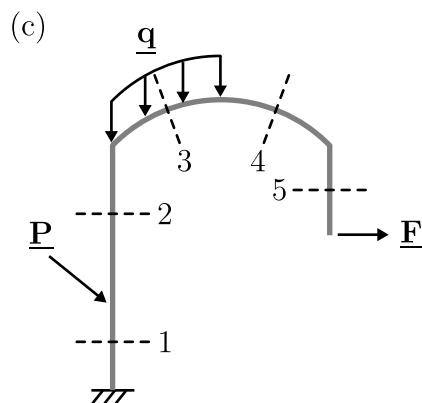
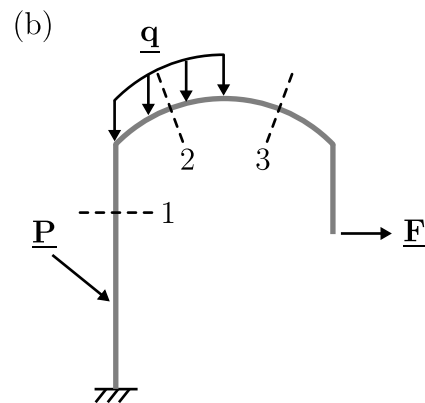
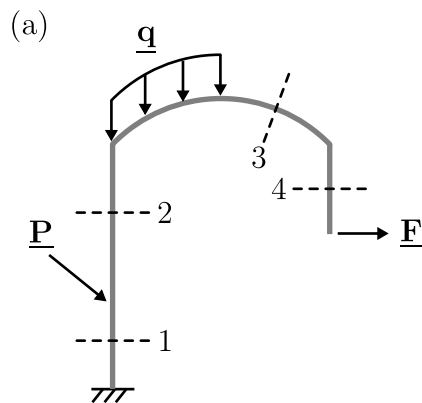


## 2. Frames and segmenting

[3 points] A rigid frame comprises two straight (vertical) elements connected by a circular arch, as shown below. A distributed force  $\underline{q}$  and two concentrated forces  $\underline{F}$  and  $\underline{P}$  are applied to the frame.



**Question 2.1** Choose the correct system of cuts that would be necessary to determine **all** internal load diagrams of the loaded frame.



- ☐ (b)  
☐ (c)  
☐ (a)  
☐ (d)



### 3. Internal load diagrams for beams

[15 points] Consider a straight beam subjected to concentrated forces  $\underline{F}_i$ , distributed forces  $\underline{q}_i$ , and/or concentrated couples  $\underline{M}_i$ . In each of the five questions below (3.1—3.5), choose the answer that, when inserted into the blank space (\_\_\_\_), ensures that the sentence is correct.

**Question 3.1** In the absence of \_\_\_\_\_, the bending moment diagram is always linear or piecewise-linear.

- ☐  $\underline{F}_i$
- ☐  $\underline{q}_i$
- ☐  $\underline{M}_i$
- ☐ both  $\underline{q}_i$  and  $\underline{M}_i$

**Question 3.2** The shear force diagram can have a non-zero slope only when \_\_\_\_\_ are applied.

- ☐  $\underline{M}_i$
- ☐  $\underline{F}_i$
- ☐  $\underline{q}_i$
- ☐  $\underline{M}_i$  or  $\underline{F}_i$

**Question 3.3** The slope of the bending moment diagram depends on  $\underline{F}_i$  \_\_\_\_\_.

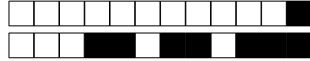
- ☐ and both  $\underline{q}_i$  and  $\underline{M}_i$
- ☐ only
- ☐ and  $\underline{M}_i$
- ☐ and  $\underline{q}_i$

**Question 3.4** The bending moment diagram is discontinuous at the points where \_\_\_\_\_ are applied.

- ☐  $\underline{q}_i$
- ☐  $\underline{M}_i$
- ☐  $\underline{M}_i$  or  $\underline{F}_i$
- ☐  $\underline{F}_i$

**Question 3.5** The shear force diagram is discontinuous at the points where \_\_\_\_\_ are applied.

- ☐  $\underline{q}_i$
- ☐  $\underline{F}_i$
- ☐  $\underline{M}_i$
- ☐  $\underline{F}_i$  or  $\underline{q}_i$



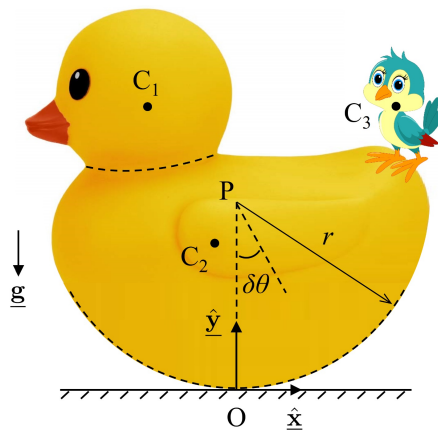
## PART II: NUMERICAL QUESTIONS (OPEN-ENDED)

Provide your answer to each question in the empty space provided. Your answer should be carefully justified, and all the steps of your argument should be discussed in detail. Leave the checkboxes empty; they are used for grading.

### 4. Equilibrium states and stability

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10 ☐ 11 ☐ 12 ☐ 13 ☐ 14 ☐ 15 ☐ 16 ☐ 17 ☐ 18

The rubber duck shown below is composed of three parts: part (1) is the duck head, part (2) is the duck body (excluding the head), and part (3) is an additional bird glued to the duck's tail<sup>1</sup>. The bottom of the duck is a circular arc of radius  $r$ , which is in frictionless contact with the flat ground at origin O. Point P is the center of the circle containing the arc. The three parts of the duck are described by their masses  $m_1 = m/4$ ,  $m_2 = m$ , and  $m_3 = m/4$  and their corresponding centers of mass:  $(x_{C_1}, y_{C_1}) = (-r/2, 3r/2)$ ,  $(x_{C_2}, y_{C_2}) = (-r/8, 3r/4)$ , and  $(x_{C_3}, y_{C_3}) = (r, 3r/2)$ . The constant gravitational field is  $\underline{g} = -g\hat{y}$ .

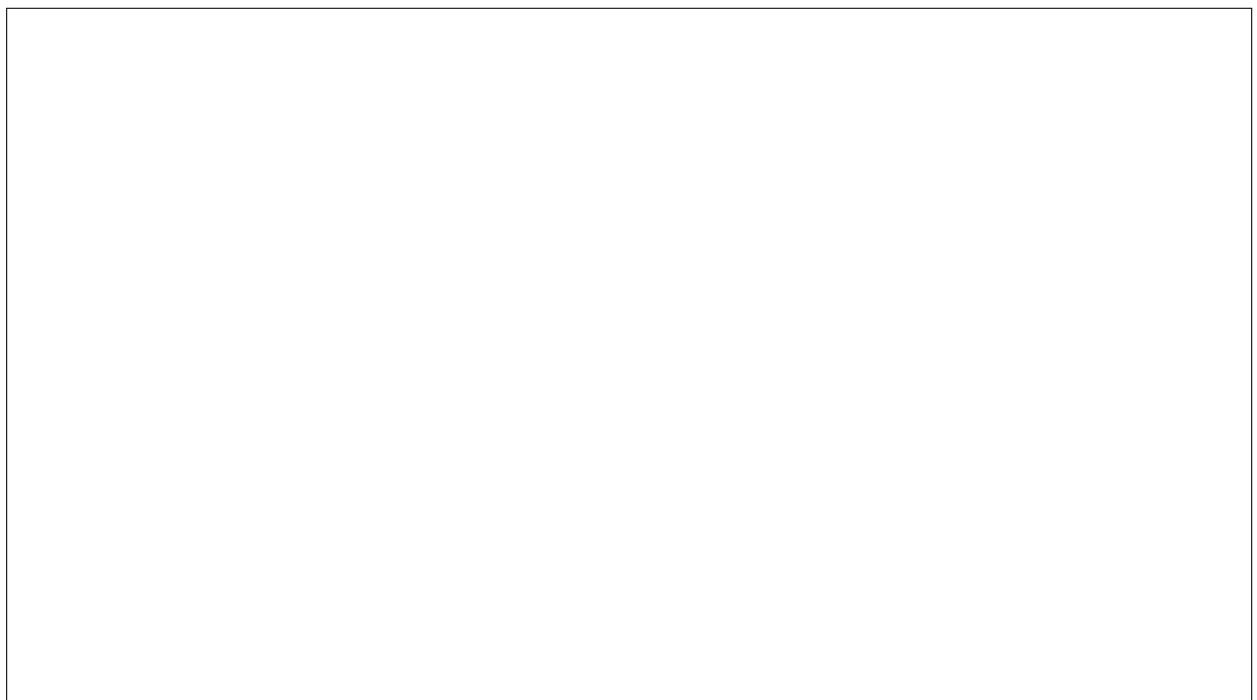


**Question 4.1 [12 points]** Compute the coordinates  $(x_C, y_C)$  of the center of mass of the entire system (comprising all three parts). Use this to **justify** why the system is in equilibrium.

<sup>1</sup>Adapted from Example 8.9 in D. Gross *et al.*, *Engineering Mechanics 1: Statics* (2009). Image credits: [amazon.com/Liberty-Imports-Jumbo-Rubber-Duck/dp/B000DBPB1U](https://amazon.com/Liberty-Imports-Jumbo-Rubber-Duck/dp/B000DBPB1U), [pixabay.com/vectors/bird-cute-feather-robin-small-5112854](https://pixabay.com/vectors/bird-cute-feather-robin-small-5112854)



**Question 4.2 [6 points]** Determine the type of stability of the current equilibrium configuration (stable, unstable, or neutral) when a small rotational perturbation  $\delta\theta$  about the center of rotation point P is imposed.

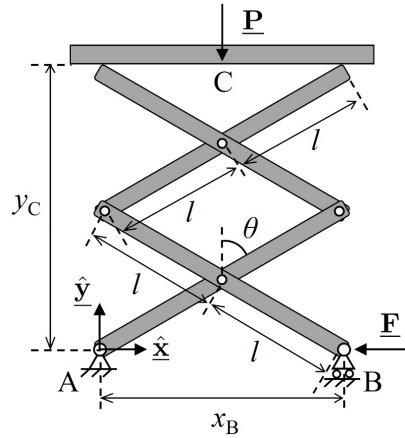




## 5. Principal of virtual work

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

Consider a scissor lift with the geometry and dimensions shown in the diagram below. The origin of the coordinate system is at the pinned support A. A vertical force  $\underline{\mathbf{P}} = -P\hat{\mathbf{y}}$  is applied to the horizontal platform at point C. A horizontal force  $\underline{\mathbf{F}} = -F\hat{\mathbf{x}}$  is imposed at roller B to keep the lift in equilibrium. Assume that all joints and contacts in the structure are frictionless, and neglect its self-weight.



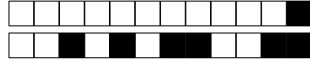
To derive the equilibrium equation of the system using the principle of virtual work (PVW), we choose the angle  $\theta$  as the single degree of freedom to describe the kinematics of the lift, as shown in the diagram. Then, by geometry, the horizontal position of point B is  $x_B = 2l \sin \theta$ , and  $y_C = 4l \cos \theta$  is the vertical position of point C.

**Question 5.1 [6 points]** Taking the above expressions for  $x_B$  and  $y_C$  as given (you do not need to derive them), **compute** the corresponding virtual displacements  $\delta x_B$  and  $\delta y_C$  as functions of  $l$ ,  $\theta$ , and the virtual rotation  $\delta \theta$ .



+1/9/52+

**Question 5.2 [12 points]** Using the above results, **write** an expression for the virtual work of the system,  $\delta U$ , and **derive** the equilibrium equation.



## PART III: EXTENDED NUMERICAL QUESTION

### 6. Structural analysis of a platform for cliff diving

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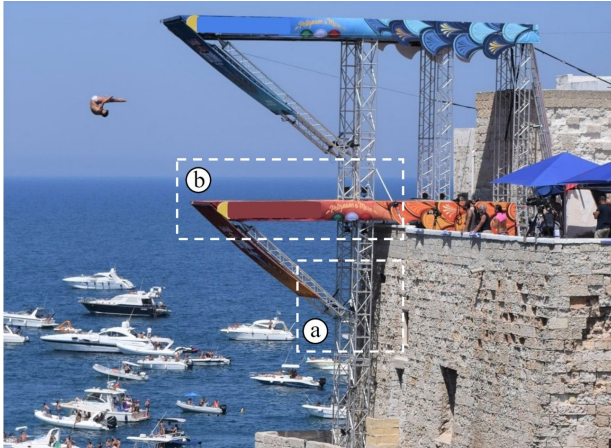


Image credit: [bari.repubblica.it/cronaca/2021/05/20/news/red\\_bull\\_cliff\\_diving\\_polignano-301895183](https://bari.repubblica.it/cronaca/2021/05/20/news/red_bull_cliff_diving_polignano-301895183)

In cliff diving, an extreme sport, athletes jump acrobatically from platforms built up to 30m above sea level. The photograph on the left shows the platform constructed out of trusses and beams used in the 2019 Red Bull Cliff Diving World Series (Polignano a Mare, Italy).

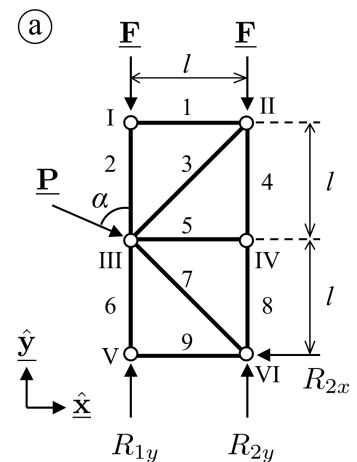
In this question, you will analyze the internal loads in structures similar to the truss (a) and the beam (b) indicated by the dashed boxes in the photograph.

#### Part (a): Analysis of the truss

For simplicity, we consider two building blocks of the truss, as shown in the schematic on the right. The members and joints are labeled, respectively, by the numbers 1–9 and I–VI. All the vertical and horizontal members have length  $l$ . The truss, assumed to be ideal, is subjected to external loads  $\mathbf{F} = -F\hat{\mathbf{y}}$  and  $\mathbf{P} = P \sin \alpha \hat{\mathbf{x}} - P \cos \alpha \hat{\mathbf{y}}$  applied at the joints ( $\alpha$  is the angle between  $\mathbf{P}$  and the vertical), as shown in the schematic. The reaction forces at the supports (joints V and VI) are:

$$\begin{aligned} R_{1y} &= F + P \cos \alpha - P \sin \alpha, \\ R_{2x} &= P \sin \alpha, \\ R_{2y} &= F + P \sin \alpha. \end{aligned}$$

*Note: You do **not** need to show the above results; use them as given.*



**Question 6.1 [6 points]** Identify all the zero-force members in this truss by writing a list of their labels (numbers). *Note: You do **not** need to justify your answers but 1.5 points will be deducted for each incorrect identification. (Hint: There are 4 zero-force members.)*

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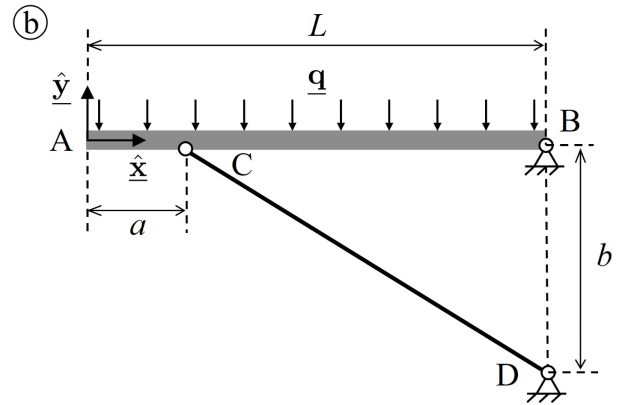


**Question 6.2 [12 points]** Using either the method of sections or the method of joints, **determine** the internal force,  $S_7$ , in member 7 as a function of  $F$ ,  $P$ , and  $\alpha$ . (It is not necessary to use all of the variables,  $F$ ,  $P$ , and  $\alpha$ ). **State** whether member 7 is in tension or in compression.



### Part (b): Analysis of the beam

Now, you are asked to consider an alternative design slightly different from the one in the above photograph, to also include a cantilevered segment of length  $a$  (see the schematic on the right). The beam AB of total length  $L$  is simply supported at point B and free at point A. A bar CD (connecting rod) is pinned at point C onto the beam with the other extremity D pinned to the ground. The beam is loaded by its self-weight, which imposes a uniformly distributed force  $\underline{q} = -q\hat{y}$ . The horizontal distance between points A and C is  $a$ , and we assume  $0 < a < L/2$ . The vertical distance between points B and D is  $b$ .



It can be shown that the force  $\underline{F}_C$  exerted by the bar CD on the beam at point C is

$$\underline{F}_C = \left( -\frac{1}{2}qL\frac{L}{b} \right) \hat{x} + \left( \frac{1}{2}qL\frac{L}{L-a} \right) \hat{y}.$$

*Note: You do **not** need to derive this result. Take it as a given for the remainder of the exercise.*

**Question 6.3 [12 points]** Derive expressions for *both* the internal bending moment,  $M(x)$ , and the shear force,  $V(x)$ , along the axis of the beam from A to B. Follow the sign convention used in class for internal loads at a positive face:  $V(x)$  is assumed positive pointing downward and  $M(x)$  is positive in the anticlockwise direction.

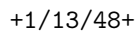
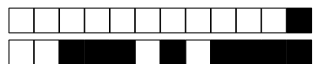


Figure 1 displays eight plots arranged in a 2x4 grid, showing the magnetic field  $M$  (top row) and potential  $V$  (bottom row) as functions of position  $x$  (ranging from 0 to  $L$ ). The plots are labeled (M-I), (M-II), (M-III), (M-IV) in the top row and (V-i), (V-ii), (V-iii), (V-iv) in the bottom row. Each plot includes a vertical axis and a horizontal axis  $x$ . A vertical dashed line is drawn at  $x=a$ . The plots show various shapes and signs (+/-) indicating the field or potential distribution.



Use the empty pages for additional space if necessary.

