Teachers : Profs. Pedro Reis and Sangwoo Kim
ME-104 Introduction to Structural Mechanics
Spring 2025
(Practice) Quiz I
12^{th} March 2025
Duration : 60 minutes

1

Student name

SCIPER : **999000**

Do not turn the page before the start of the exam. This document is double-sided, has 16 pages, the last ones possibly blank. Do not unstaple.

- Place your student card on your table.
- Please read all questions carefully and write directly onto this test.
- You can use one A5 sheet of handwritten notes (both sides) and a calculator.
- Phones, laptops and any other device capable of communication are **not allowed**.
- For the **multiple choice** questions, we give:
  +2.5 points if your answer is correct,
  0 points if you give no answer or more than one.
- For the **open-ended** questions, the maximum number of points is listed next to each question and sub-question. Show your work clearly and put a box around your final answer.
- Use a **black or dark blue ballpen** and clearly erase with **correction fluid** if necessary.
- If there is an erratum (corrections page), an announcement will be made at the start of the exam. **In case there is an erratum, please take note of any necessary corrections to the questions.**

Part I	Part II	Part III
/40	/20	/40
Total: /100		

Respectez les consignes suivantes | Observe this guidelines | Beachten Sie bitte die unten stehenden Richtlinien

choisir une réponse | select an answer Antwort auswählen

ne PAS choisir une réponse | NOT select an answer NICHT Antwort auswählen

Corriger une réponse | Correct an answer Antwort korrigieren

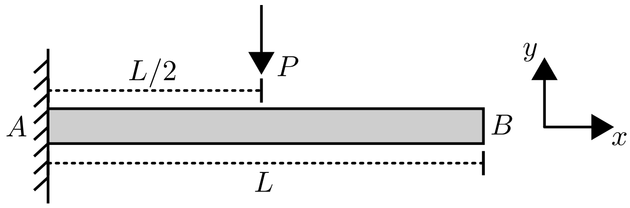
ce qu'il ne faut **PAS** faire | what should **NOT** be done | was man **NICHT** tun sollte

PART I: CONCEPT QUESTIONS (MULTIPLE-CHOICE)

For each question, mark the box corresponding to the correct answer. Each question has **exactly one** correct answer.

1. Equilibrium and boundary conditions of a beam I

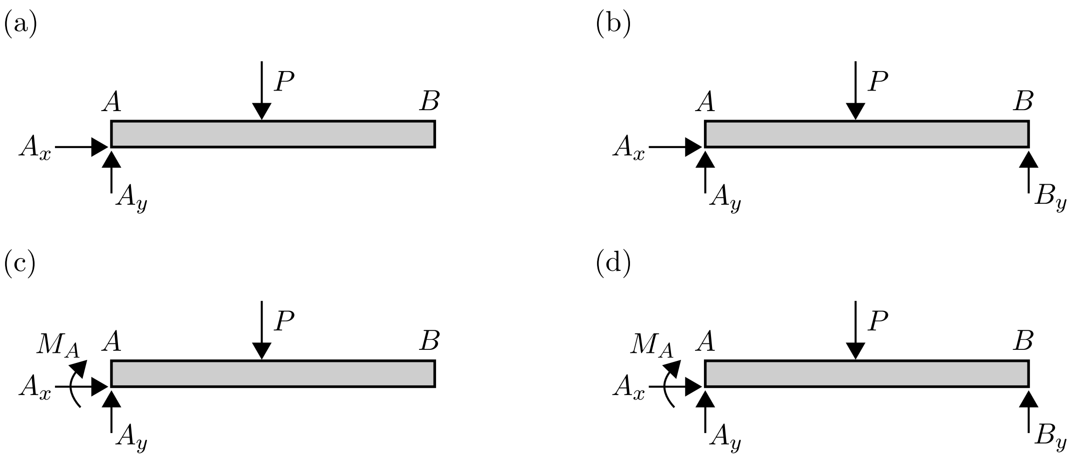
[7.5 points] Consider the beam system shown below.



Question 1.1 What are the correct boundary conditions for this beam, at points A and B, respectively?

- [ ] Clamped, simply supported
- [x] Clamped, free
- [ ] Pinned, free
- [ ] Pinned, simply supported

Question 1.2 Choose the correct free-body diagram for the system above.



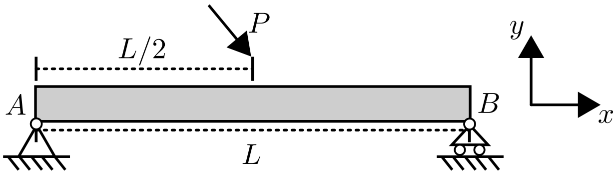
- [ ] (b)
- [ ] (d)
- [ ] (a)
- [x] (c)

Question 1.3 Choose the correct set of solutions for the reaction forces and moment for the system above.

- [x] $A_x = 0,\ A_y = P,\ M_A = -PL/2$
- [ ] $A_x = 0,\ A_y = P,\ M_A = 0$
- [ ] $A_x = 0,\ A_y = B_y = P/2,\ M_A = 0$
- [ ] $A_x = 0,\ A_y = B_y = P/2,\ M_A = -PL/2$

2. Equilibrium and boundary conditions of a beam II

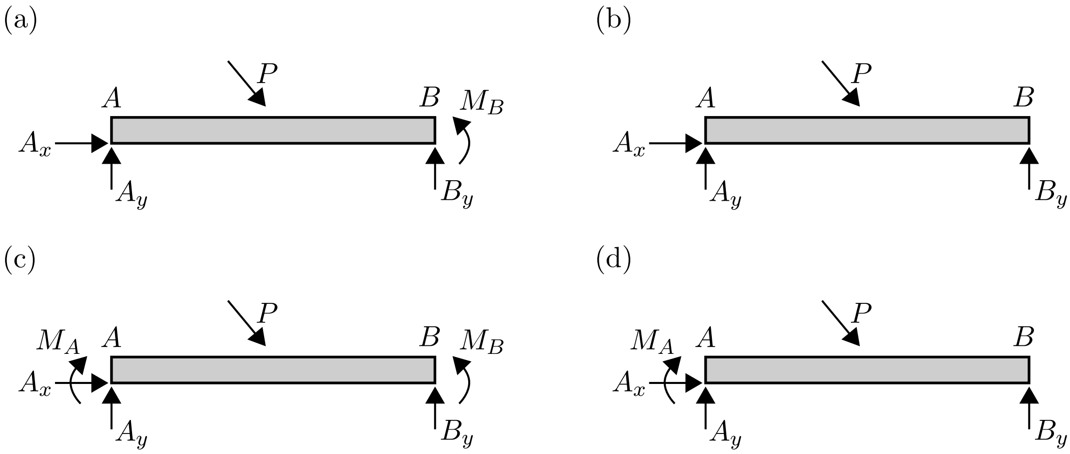
[7.5 points] Consider the beam system shown below.



Question 2.1 What are the correct boundary conditions for this beam, at points A and B, respectively?

■ Pinned, roller
☐ Pinned, slider
☐ Clamped, slider
☐ Clamped, roller

Question 2.2 Choose the correct free-body diagram for the system.



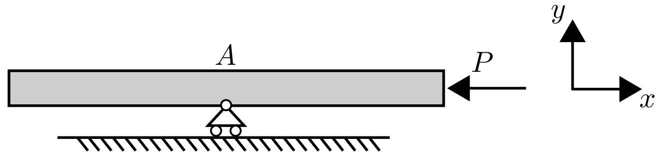
☐ (c)
☐ (d)
☐ (a)
■ (b)

Question 2.3 Choose the correct set of solutions for the reaction forces and moment for the system above.

☐ $A_x = -P_x$, $A_y = B_y = P_y/2$, $-M_A + M_B = P_yL/2$
☐ $A_x = -P_x$, $A_y = B_y = P_y/2$, $M_A = -P_yL/2$
☐ $A_x = -P$, $A_y = B_y = P/2$, $M_A = M_B = 0$
■ $A_x = -P_x$, $A_y = B_y = P_y/2$, $M_A = M_B = 0$

3. Equilibrium and constraints

[7.5 points] Consider the beam system shown below.

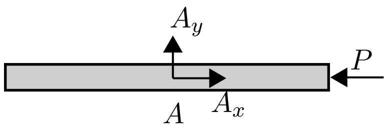


Question 3.1 Regarding classification according to statical determinacy, which of these statement(s) is true?

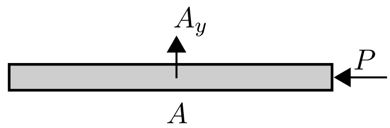
- [ ] None of the other three options is correct.
- [x] The system is hypostatically constrained.
- [ ] The system is hyper-hypostatically constrained.
- [ ] The system is hyperstatically constrained.

Question 3.2 Choose the correct free-body diagram for the system above.

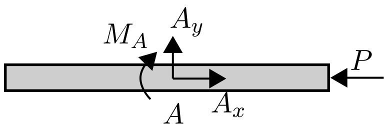
(a)



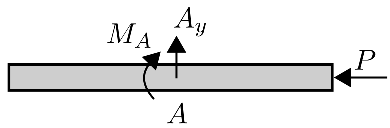
(b)



(c)



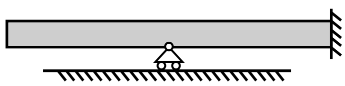
(d)



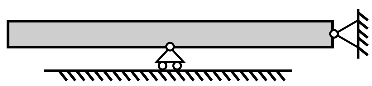
- [ ] (a)
- [x] (b)
- [ ] (c)
- [ ] (d)

Question 3.3 The above system is not in static equilibrium. Choose the diagram with changes that can bring it to static equilibrium and **without redundant constraints**.

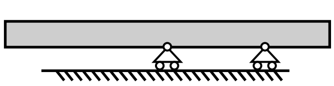
(a)



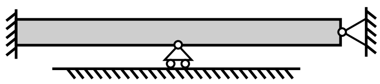
(b)



(c)



(d)

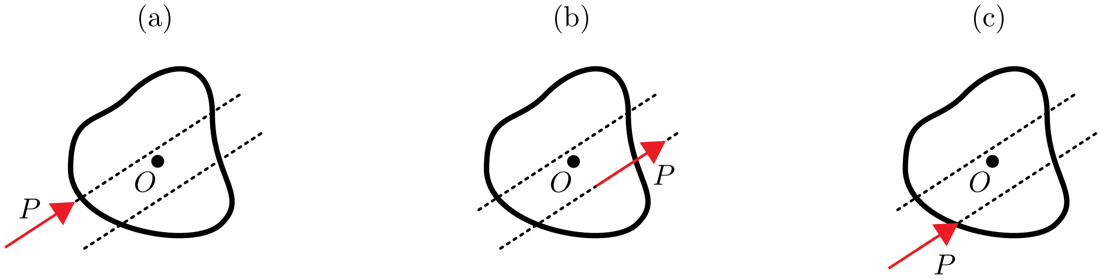


- [x] (b)
- [ ] (d)
- [ ] (a)
- [ ] (c)

4. Equivalent systems of forces and moments

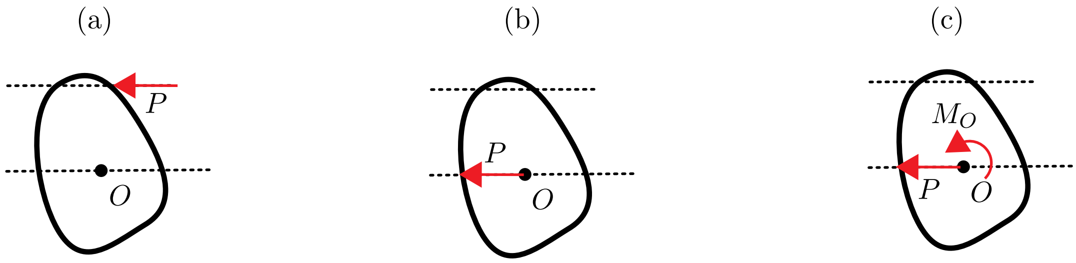
[7.5 points] For each of the three questions below (4.1, 4.2 and 4.3), **identify** which system of forces and moments is unique (*i.e.*, not mechanically equivalent to the other two choices).

Question 4.1



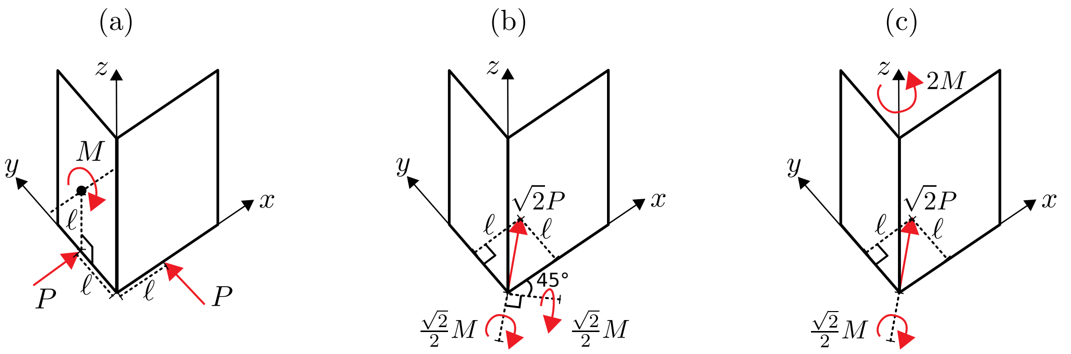
- [ ] (c)
- [ ] (b)
- [x] (a)

Question 4.2



- [x] (b)
- [ ] (c)
- [ ] (a)

Question 4.3



- [x] (c)
- [ ] (a)
- [ ] (b)

5. A beam or a bar, and why?

[10 points] For each of the four questions below (5.1, 5.2, 5.3 and 5.4), is the structural element marked with a dashed curve a *beam* or a *bar* (and why)? Choose the correct answers.

Question 5.1

- [x] Beam, since transverse external forces are applied by the skater.
- [ ] Bar, since transverse external forces are applied by the skater.
- [ ] Beam, since the board is rigid and cannot sustain transverse external force.
- [ ] Bar, since the board is rigid and cannot sustain transverse external force.

Question 5.2

- [ ] Beam, since the structure is under tension.
- [ ] Bar, since the structure is under tension.
- [ ] Bar, since transverse forces are applied by the hanging load.
- [x] Beam, since transverse forces are applied by the hanging load.

Question 5.3

- [x] Beam, since it is loaded under bending due to the weight of the people.
- [ ] Bar, since it is loaded under bending due to the weight of the people.
- [ ] Beam, since the structure is very thick and can sustain a lot of weight.
- [ ] Bar, since the structure is very thick and can sustain a lot of weight.

Question 5.4

- [ ] Beam, since transverse forces are applied by the hanging load.
- [x] Bar, since the structure can only sustain axial tension.
- [ ] Bar, since transverse forces are applied by the hanging load.
- [ ] Beam, since the structure can only sustain axial tension.

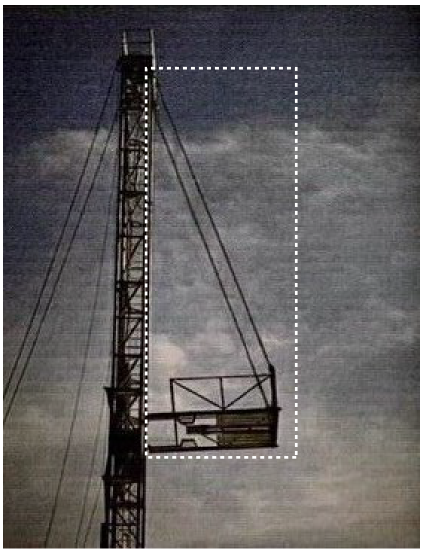
PART II: CONCEPT QUESTIONS (OPEN-ENDED)

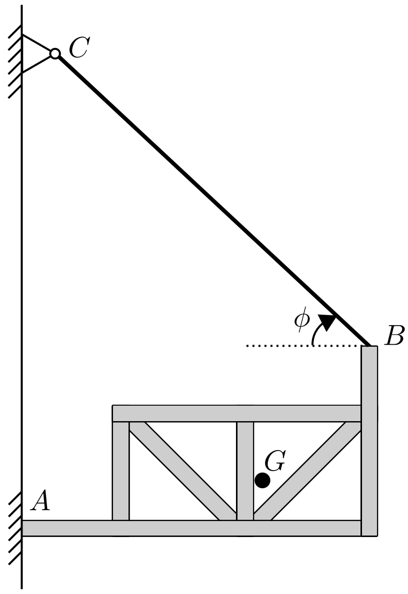
Provide your answer to each question in the empty space provided. Your answer should be carefully justified, and all the steps of your argument should be discussed in detail. Leave the checkboxes empty; they are used for grading.

6. Free-body diagram

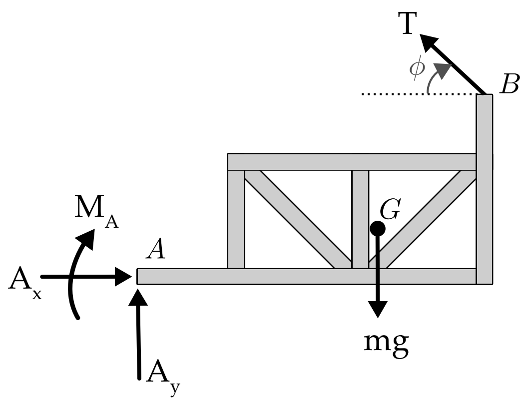
☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ■10

Question 6 [10 points] A platform is suspended off the edge of a large rig, as shown in the photograph below. In the corresponding idealized model shown on the right, the platform is assumed to be clamped at the connection A and is supported by a cable at point B; the cable makes an angle ϕ to the horizontal. The platform has mass m and center of gravity located at point G. **Sketch** the free-body diagram of the platform, including all the applied loads and reaction forces.





Solution:
The FBD is shown below:



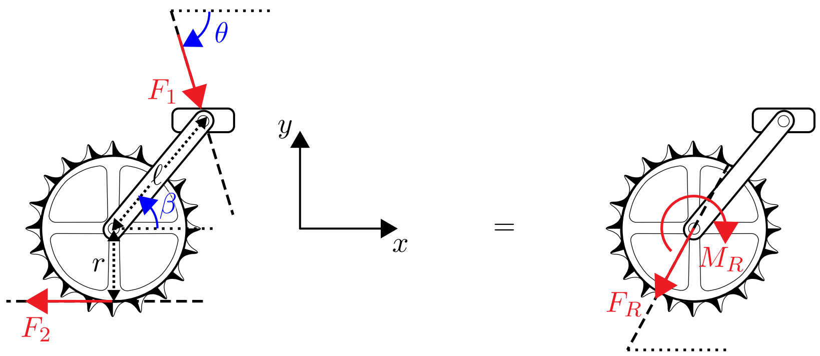
(Pinned support for cable at C is irrelevant for the free body diagram of the platform)

7. Equivalent system of forces and moments

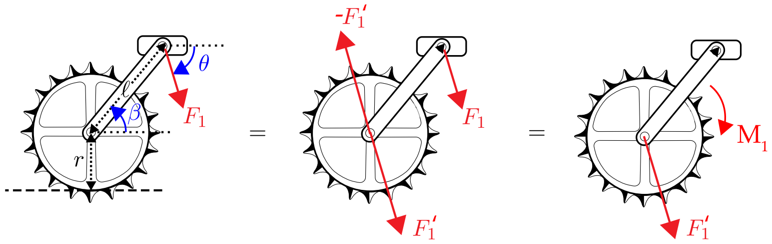
☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ■10

Question 7 [10 points] The crankset of a bicycle consists of a circular chainring, of radius r, connected to a pedal by a rigid crank arm of length ℓ. As shown in the diagram below (left schematic), the crankset is subject to a force of magnitude F_1 applied to the pedal, which makes an angle θ to the horizontal, and a force of magnitude F_2, which acts tangentially at its base. The crank arm makes an angle β to the horizontal.

Compute the magnitudes of the equivalent resultant force, F_R, **and** the resultant moment, M_R, about the center of the chainring (right schematic). Your answers should be functions of F_1, F_2, r, ℓ, β and θ.



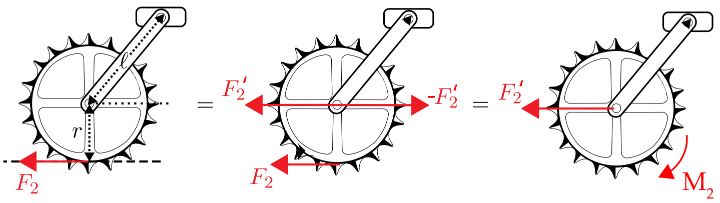
Solution:



Due to F_1:

$$|\mathbf{M_1}| = \mathbf{l} \times \mathbf{F_1} = lF_1 \sin(\pi - \beta - \theta) = lF_1 \sin(\beta + \theta) \quad \text{clockwise}$$

$$\mathbf{F_1}' = F_1(\cos(\theta), -\sin(\theta))$$



Due to F_2:

$$|\mathbf{M_2}| = rF_2 \quad \text{clockwise}$$

$$\mathbf{F_2}' = (-F_2, 0)$$

Finally:

$$\mathbf{F_R} = \mathbf{F_1}' + \mathbf{F_2}' = (F_1 \cos(\theta) - F_2)\hat{x} - F_1 \sin(\theta)\hat{y}$$

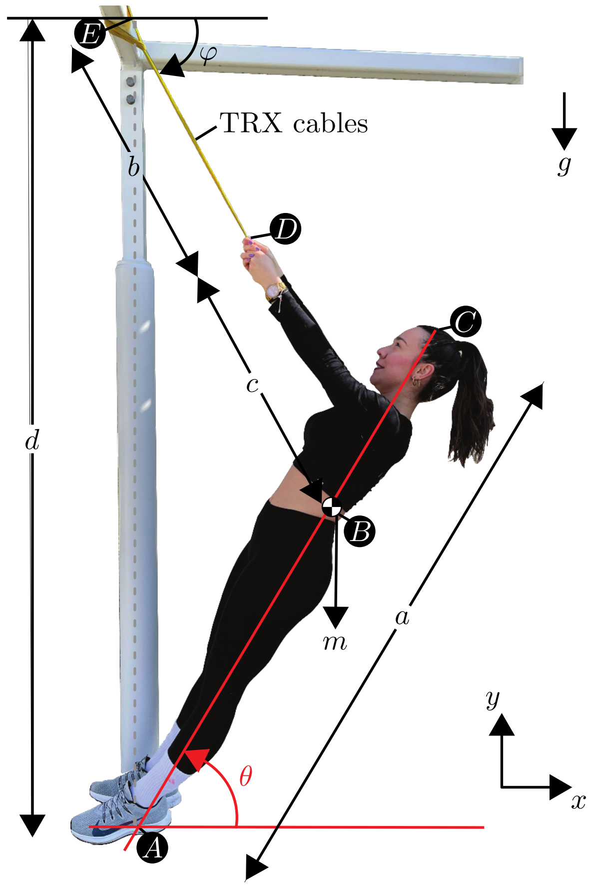
$$\rightarrow F_R = |\mathbf{F_R}| = \sqrt{(F_1 \cos(\theta) - F_2)^2 + (F_1 \sin(\theta))^2}$$

$$\mathbf{M_R}' = M_1 + M_2 = rF_2 + lF_1 \sin(\beta + \theta)$$

PART III: EXTENDED NUMERICAL QUESTION

8. Structural analysis of a TRX row.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40

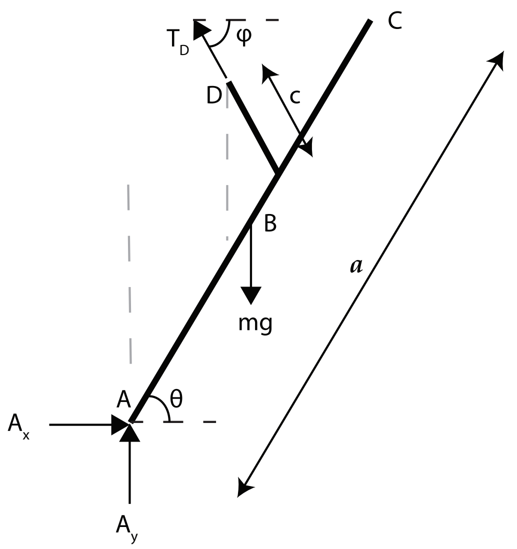


An athlete of height $a = \overline{AC}$ (distance between points A and C) and mass m hangs from TRX cables (used for full-body resistance exercise). These TRX cables are extended to have length $b = \overline{ED}$, as shown in the figure above. The athlete is stable when tilted at an angle θ from the ground, and the extended TRX cables are at an angle φ from the ceiling. Assume that the athlete's body is rigid (ensured by the tensioning of her muscles). The vertical distance between the ground and the ceiling is $d = \overline{AE}$ (points A and E are vertically aligned). For simplicity, assume that the athlete's center of gravity is located at point B ($\overline{AB} \approx a/2$). The gravitational acceleration is g.

Note that the athlete's feet (at point A) can be considered as *pinned* to the ground since her ankles can rotate, but their position is fixed. Also, the motion of the athlete's hands (at point D) is only restricted along the direction of the cables, while rotation and motion perpendicular to the cables are possible; therefore, the cables behave as a connecting rod.

Question 8.1 [8 points] In the box below, **sketch** the free-body diagram of the athlete, clearly indicating all of the relevant reaction forces. (*Use the given coordinate system, x-y*).

Solution:
The FBD is shown below:



Question 8.2 [10 points] **Show** that the reaction force, T_D, exerted by the cables on the athlete at point D is:

$$T_D = \frac{mga\cos\theta}{2d\cos\varphi}$$

(*Hint: For simplicity, you may want to use point A for moment balance.*)

Solution:

$$\sum M_A = 0 \rightarrow -mg(\frac{a}{2}\cos(\theta)) + T_D\cos(\varphi)(d - b\sin(\varphi)) + T_D\sin(\varphi)(b\cos(\varphi)) = 0$$

$$\rightarrow T_D d\cos(\varphi) = \frac{mga\cos(\theta)}{2}$$

$$T_D = \frac{mga\cos(\theta)}{2d\cos(\varphi)}$$

Question 8.3 [10 points] **Compute both** the horizontal **and** the vertical reaction forces, A_x **and** A_y, respectively, exerted by the ground on the athlete at point A, as functions of the angles θ and φ.

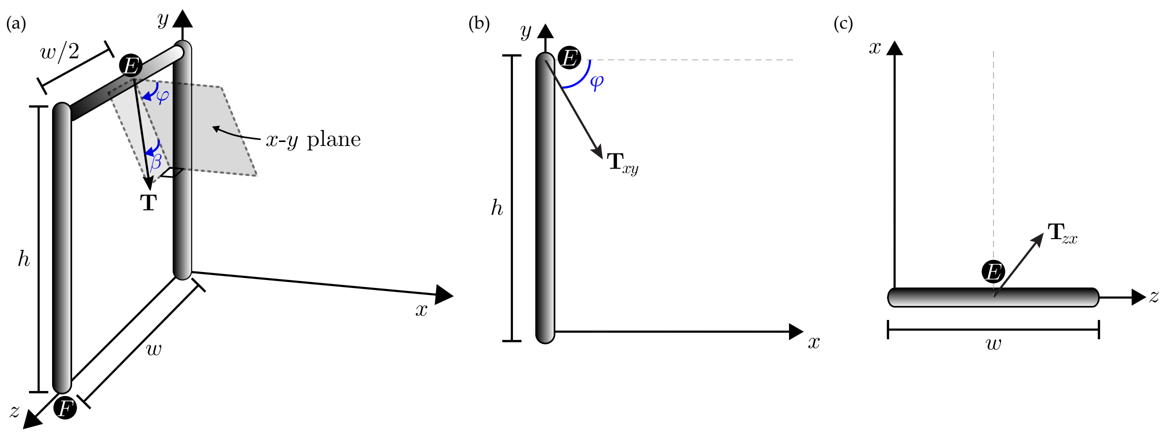
Solution:

$$\sum F_x = 0 \rightarrow A_x - T_D\cos(\varphi) = 0$$
$$\rightarrow A_x = T_D\cos(\varphi)$$
$$A_x = \frac{a\cos(\theta)}{2d}mg$$

$$\sum F_y = 0 \rightarrow A_y - mg + T_D\sin(\varphi) = 0$$
$$\rightarrow A_y = mg - T_D\sin(\varphi)$$
$$A_y = \left(1 - \frac{a\cos(\theta)\sin(\varphi)}{2d\cos(\varphi)}\right)mg$$

Question 8.4 [12 points] Now, consider a new situation where the athlete has rotated around point A so that the tension in the cable between points D and E is no longer parallel to the x-y plate, meaning we need to consider it in three-dimensional (3D) space. As shown in panel (a) below, let us denote this new tension exerted by the athlete on the cable as $\mathbf{T}$, using the shown x-y-z system of axes. In the schematic, we only consider the frame of height h and width w, having omitted the cable and the athlete. We can project the 3D vector $\mathbf{T}$ onto the x-y plane; the angle between the x-axis and the projected vector is φ. The angle between $\mathbf{T}$ and the x-y plane is β. The projections in x-y plane and z-x plane are shown in panels (b) and (c) respectively.

Write a vectorial expression for the moment of the force $\mathbf{T}$, about point F, labelled $\mathbf{M}_F$. Write your answers as functions of the magnitude of $\mathbf{T}$, labeled T, the angles φ and β, and the lengths h and w. (*Hint: You may find it helpful to decompose the tension into components and compute the corresponding moments about F associated with each component.*)



Solution:
Write:

$$\mathbf{T} = (T_x, T_y, T_z) = (T\cos(\beta)\cos(\phi), -T\cos(\beta)\sin(\phi), T\sin(\beta))$$

Moment of $\mathbf{T}$ about F is calculated as:

$$\mathbf{M_F} = \mathbf{FE} \times \mathbf{T},$$

where $\mathbf{FE}$ is the vector going from F to E.

$$\mathbf{FE} = (0, h, -w/2)$$

$$\mathbf{M_F} = \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & h & -w/2 \\ T_x & T_y & T_z \end{pmatrix} = \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & h & -w/2 \\ T\cos(\beta)\cos(\phi) & T\cos(\beta)\sin(\phi) & T\sin(\beta) \end{pmatrix}$$
$$= \begin{pmatrix} hT\sin(\beta) - (w/2)T\cos(\beta)\sin(\phi) \\ -(w/2)T\cos(\beta)\cos(\phi) \\ -hT\cos(\beta)\cos(\phi) \end{pmatrix}$$

Use the empty pages for additional space if necessary.