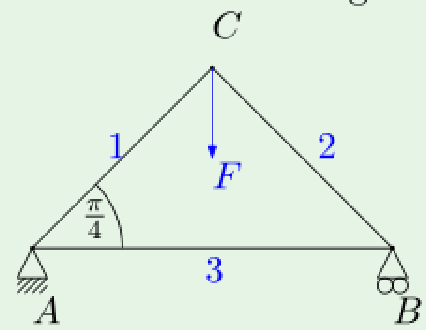
Sample Problem 19.1 Inverse problem with a triangular truss



This triangular truss is made from steel with a Young's modulus of $E = 200\,\text{GPa}$ and is loaded with a vertical force $F = 20\,\text{kN}$ at node C.

a) Determine the minimum cross-sections for the stress never to exceed $\sigma_{\text{max}} = 150\,\text{MPa}$.

b) Determine the minimum cross-sections for the displacement in B not to exceed 0.05% of the length of member 3.

Solutions:

We start by determining the forces in all members as per the recipe.

$$\text{in } C: \quad \sum F_y = 0 = -F - \frac{\sqrt{2}}{2}(N_1 + N_2) \tag{19.12}$$

$$\sum F_x = 0 = \frac{\sqrt{2}}{2}(-N_1 + N_2) \tag{19.13}$$

$$\Rightarrow N_1 = N_2 = -F\frac{\sqrt{2}}{2} \tag{19.14}$$

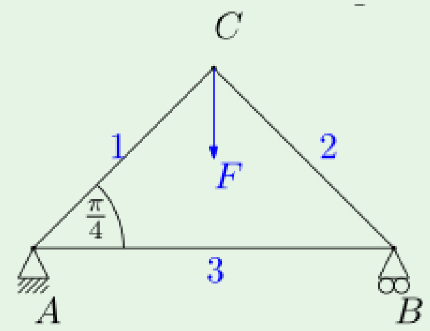
$$\text{in } B: \quad \sum F_x = 0 = -N_3 - \frac{\sqrt{2}}{2}N_2 \Rightarrow N_3 = \frac{1}{2}F. \tag{19.15}$$

- The stress σ in a bar depends on the cross section A and the normal load N:

$$N = \sigma A \Rightarrow N_{\max} = \sigma_{\max} A \quad (19.16)$$

$$\Rightarrow A_1 = A_2 > \frac{||N_1||}{\sigma_{\max}} = 94.3\,\mathrm{mm}^2, \quad (19.17)$$

$$A_3 > \frac{N_3}{\sigma_{\max}} = 66.7\,\mathrm{mm}^2, \quad (19.18)$$



- We now have a set of lower bounds on the cross-sections of all members to satisfy the first condition. The second condition requires us to compute the displacement of joint B. For this, we start by computing elongations as per the recipe. Note that we are only interested in the elongation $\Delta l_3 = \Delta x_B$ of strut 3.

$$\frac{\Delta l_3}{l_3} < \varepsilon_{\max} = 0.05\% \quad \text{and} \quad \frac{\Delta l_3}{l_3} = \varepsilon = \frac{N_3}{EA_3}, \quad (19.20)$$

$$\Rightarrow A_3 > \frac{N_3}{E\varepsilon_{\max}} = \frac{F}{2E\varepsilon_{\max}} = 100\,\mathrm{mm}^2. \quad (19.21)$$

- The minimum cross-sections therefore have to be

$$\boxed{A_1 = A_2 > 94.3\,\mathrm{mm}^2,\, A_3 > 100\,\mathrm{mm}^2} \quad (19.22)$$