MATH-562 Statistical Inference

Final Examination

27 January 2025

Instructions: The time allotted for the examination is 180 minutes. You may answer in either English or French. No written material may be brought into the examination, but a simple calculator may be used if necessary. Full marks may be obtained with complete answers to four questions. The final mark will be based on the best four solutions.

First name:

Last name:

SCIPER number:

Exercise	Points	Indicative marks
1		/10 points
2		/10 points
3		/10 points
4		/10 points
5		/10 points
Total:		/40 points

Some formulae

Definition 1 *The moment-generating and cumulant-generating functions of a real-valued random variable X are*

$$M_X(t) = \mathrm{E}\left(e^{tX}\right), \quad K_X(t) = \log M_X(t), \quad t \in \mathcal{T},$$

where $\mathcal{T} = \{t \in \mathbb{R} : M_X(t) < \infty\}$.

Definition 2 *A Bernoulli random variable with parameter $p \in (0,1)$ has probability mass function*

$$f(x;p) = p^x(1-p)^{1-x}, \quad x \in \{0,1\}.$$

Definition 3 *A geometric random variable with parameter $p \in (0,1)$ has probability mass function*

$$f(x;p) = (1-p)^{x-1}p, \quad x \in \{1,2,\ldots\}.$$

Definition 4 *A Poisson variable with parameter $\lambda > 0$ has probability mass function*

$$f(x;\lambda) = \frac{\lambda^x}{x!}e^{-\lambda}, \quad x \in \{0,1,\ldots\}.$$

Definition 5 *A normal (or Gaussian) random variable $X \sim \mathcal{N}(\mu,\sigma^2)$ has probability density function*

$$f(x;\mu,\sigma^2) = \frac{1}{\sigma}\phi\left(\frac{x-\mu}{\sigma}\right), \quad x \in \mathbb{R}, \quad \mu \in \mathbb{R}, \sigma^2 > 0,$$

where $\phi(u) = (2\pi)^{-1/2}e^{-u^2/2}$ for $u \in \mathbb{R}$, and we also define $\Phi(x) = \int_{-\infty}^{x} \phi(u)\,\mathrm{d}u$.

Definition 6 *A gamma random variable with shape parameter $\alpha > 0$ and rate parameter $\beta > 0$, $X \sim \mathrm{Gamma}(\alpha,\beta)$, has mean α/β and probability density function*

$$f(x;\alpha,\beta) = \begin{cases} \dfrac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}, & x \geq 0, \\ 0, & x < 0, \end{cases}$$

where $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$, $\Gamma(\alpha) = (\alpha-1)!$ when α is a positive integer, and $\Gamma(1/2) = \sqrt{\pi}$.

Definition 7 *An exponential random variable X with rate parameter β, $X \sim \exp(\beta)$, has the gamma distribution with $\alpha = 1$.*

Definition 8 *A chi-squared random variable V with ν degrees of freedom, $V \sim \chi^2_\nu$, has the gamma distribution with $\alpha = \nu/2$ and $\beta = 1/2$, and can be expressed as $V \stackrel{\mathrm{D}}{=} Z_1^2 + \cdots + Z_\nu^2$, where $Z_1, \ldots, Z_\nu \stackrel{\mathrm{iid}}{\sim} \mathcal{N}(0,1)$.*