

**Modern Algebraic Geometry (MATH-510) – Final**

12.01.2021, 16:15-19:15

**4. exercise**

Let  $k$  be a field. Consider  $S = k[x_0, \dots, x_n]$  as a graded ring with the standard grading given by  $\deg x_i = 1$ .

A closed subscheme  $X$  of  $\mathbb{P}_k^n$  is called a *degree  $d$  hypersurface* if  $X = \text{Proj} \left( S/(f) \right)$ , where  $f \in S$  is an irreducible homogeneous polynomial of degree  $d$  (note that  $\text{Proj} \left( S/(f) \right)$  is a closed subscheme of  $\mathbb{P}_k^n$  by exercise Exercise II.3.12.a.).

We note that there was a sample exam question stating that exact sequences of graded modules yield exact sequences of sheaves by applying the  $(\_)$  construction to them. You can use this statement freely during your solution.

- (a) Show that if  $X$  is a degree  $d > 0$  hypersurface in  $\mathbb{P}_k^n$ , then there is an exact sequence of sheaves on  $\mathbb{P}_k^n$  as follows

$$0 \longrightarrow \mathcal{O}_{\mathbb{P}_k^n}(-d) \longrightarrow \mathcal{O}_{\mathbb{P}_k^n} \longrightarrow \iota_* \mathcal{O}_X \longrightarrow 0$$

where  $\iota : X \rightarrow \mathbb{P}_k^n$  is the closed embedding.

- (b) It is a fact shown in Hartshorne, Thm III.5.1 that

$$\forall 0 < i < n, \forall r \in \mathbb{Z} : H^i \left( \mathbb{P}_k^n, \mathcal{O}_{\mathbb{P}_k^n}(r) \right) = 0.$$

Using this fact, show that if  $X$  is a degree  $d > 0$  hypersurface in  $\mathbb{P}_k^n$ , then  $H^i(X, \mathcal{O}_X(s)) = 0$  for every  $0 < i < n - 1$  and every  $s \in \mathbb{Z}$ .

GENERAL INFORMATION: 4 exercises: Please make sure that you do not miss any of the exercises. The exam contains 4 exercises.

Contacting Zsolt Patakfalvi: Show all your computations and justify/explain your answers. If you are not sure what you are allowed to assume, ask Zsolt Patakfalvi whom you can reach on the Zoom link of the lectures of the course. He is online there throughout the entire exam, as well as for 15 minutes before and for 30 minutes after. If you have problems with your internet connection, you can also reach Zsolt Patakfalvi on his office phone +41 21 693 55 20.

Aids that can be used: This is an open book exam. So you can use every *off-line* written aid, but no communication with anyone and no use of any kind of online help is allowed during the

exam. In particular statements of the exercises from Hartshorne can be used (even the ones we did not solve). On the other hand, you have to have available off-line all the sources that you plan to use during the exam, and the internet should be limited to use for

- downloading the exam,
- uploading the solution, and
- asking questions to Zsolt Patakfalvi.

If the suspicion of breaking the above rules arises, we will ask you to explain in person on Zoom your solution after the grading of the exam is done.

**Separate points can be solved separately:** You get maximum credit for solving second or third points of exercises assuming the statements of the previous points, even if you did not solve (all of) those previous points.

**End of the exam & uploading:** Put down the pencil at 19:15, and start photocopying and uploading the solutions. You can find information on how to scan and convert into pdf in the online student guide:

[https://moodle.epfl.ch/pluginfile.php/2870624/mod\\_resource/content/1/QUICKSTART\\_MoodleExamsForStudents-EN-Dec-17-2020.pdf](https://moodle.epfl.ch/pluginfile.php/2870624/mod_resource/content/1/QUICKSTART_MoodleExamsForStudents-EN-Dec-17-2020.pdf). Please make a test scan and a test upload before the exam, as explained in the document.

You should upload the scan of each exercise to the separate corresponding assignment gadgets on [moodleexam.epfl.ch](https://moodle.epfl.ch). You should upload everything in pdf format. We expect that you can do the uploading in at most 15 minutes. If it takes more time than that please log into the Zoom session where Zsolt Patakfalvi is online (see above), and please explain why it takes more time. As you have to upload the pdfs for separate exercises to separate places, uploading takes time. So, please do put down the pencil at 19:15, and start the uploading right then.

**Citing the results of Hartshorne:** Whenever you are using anything particular from Hartshorne, please cite it precisely to avoid confusion. The best is to use a format such as Ex II.x.y, Prop III.z.v, ...

**E-mail open all the time:** we suggest that you keep your e-mail open all the time, as all information about the exam will be sent out via e-mail during the exam.