DISPERSIVE PDE 23, PROBLEM SET 1

(1) Verify that if $\phi(t,x) \in C^2(\mathbb{R}^{1+3})$, then the function $\psi(t,r)$ defined as in the proof of Prop.1.3 in Lecture1.pdf is of class $C^2(\mathbb{R}^{1+1})$.

(2) Check that if $\phi_0 \in C^3(\mathbb{R}^3), \phi_1 \in C^2(\mathbb{R}^3)$, then the function $\phi(t,x)$ defined by means of Kirchhoff's formula is of class $C^2(\mathbb{R}^{1+3})$.

(3) Let $E : \mathbb{R}^{1+3} \to \mathbb{R}^3$, $B : \mathbb{R}^{1+3} \to \mathbb{R}^3$ vector valued functions which are twice continuously differentiable. Check that if they satisfy the Maxwell's equations in vacuum

$$\nabla \times E = -\partial_t B,\ \nabla \times B = \partial_t E,\ \mathrm{div} E = 0,\ \mathrm{div} B = 0,$$

where $\times$ denotes the vectorial cross product on $\mathbb{R}^3$, then all the components of E_j, B_j of E and B satisfy the wave equation

$$\Box E_j = \Box B_j = 0.$$

(4) For the one dimensional wave equation, check directly that if $\phi_0 \in C_0^2(\mathbb{R}), \phi_1 \in C_0^1(\mathbb{R})$, then the *energy*

$$\int_{\mathbb{R}} [|\partial_x \phi(t,x)|^2 + |\partial_t \phi(t,x)|^2]\, dx = \int_{\mathbb{R}} [|\partial_x \phi_0^2| + |\phi_1|^2]\, dx,$$

and is hence a *conserved quantity*, i. e. independent of time.