DISPERSIVE PDE 23, PROBLEM SET 12

(1) Complete the proof of lemma 2.2 in lecture8.pdf by proving the two bounds stated at the beginning of its proof for the contributions of
$$P_k\big(P_{[k-10,k+10]}(u_t) \cdot u_t\big), \ P_k\big(P_{>k+10}(u_t) \cdot u_t\big)$$

(2) Prove the difference bound asserted at the end of the proof of Theorem 2.1 (p. 4 of lecture8.pdf).

(3) Carefully verify the inequality
$$\sum_k 2^{(\frac{5}{2}-\frac{1}{p})\cdot k} \cdot \big\|(P_k f, P_k g)\big\|_{L^2(\mathbb{R}^3) \times \dot{H}^{-1}(\mathbb{R}^3)} \leq C_1 \cdot \big\|(f,g)\big\|_{H^s(\mathbb{R}^3) \times H^{s-1}(\mathbb{R}^3)}$$

provided $2 < \frac{5}{2} - \frac{1}{p} < s$, $s > 2$.