Appendix, W5, Ex. 1

S_3	133.1
U_3	0
Z_3	0

strike	95
1-p*	0.6
interest r.	0.02

u=(1+b)	1.1
d=(1+a)	0.9

S_2	121
U_2	0
Z_2	0
X_2	0

S_1	110
U_1	0.907
Z_1	0
X_1	0.907

S_3	108.9
U_3	0
Z_3	0

S_0	100
U_0	3.2205
Z_0	0
X_0	3.2205

S_2	99
U_2	2.31373
Z_2	0
X_2	2.31373

S_1	90
U_1	6.851
Z_1	5
X_1	6.851

S_3	89.1
U_3	5.9
Z_3	5.9

S_2	81
U_2	14
Z_2	14
X_2	12.1373

$$X_n = \frac{E(U_n \mid \xi_{n-1})}{1+r}$$

S_3	72.9
U_3	22.1
Z_3	22.1

$n = 0, 1, 2$

Appendix: Optimal stopping

From the tree we immediately obtain the values of ν. The values of ν_{max} can be (honestly) derived by explicitely computing the (unique) Doob decomposition for $\tilde{U}$ (some work).

Doob

$$\tilde U_n = \tilde U_0 + \sum_{k=1}^{n} \Delta \tilde U_k$$

$$= \underbrace{\tilde U_0}_{=\tilde M_0} + \underbrace{\sum_{k=1}^{n} E_Q(\Delta \tilde U_k \mid \mathcal{F}_{k-1})}_{-\tilde A_n} + \underbrace{\sum_{k=1}^{n} \left(\Delta \tilde U_k - E_Q(\Delta \tilde U_k \mid \mathcal{F}_{k-1})\right)}_{\tilde M_n - \tilde M_0}$$

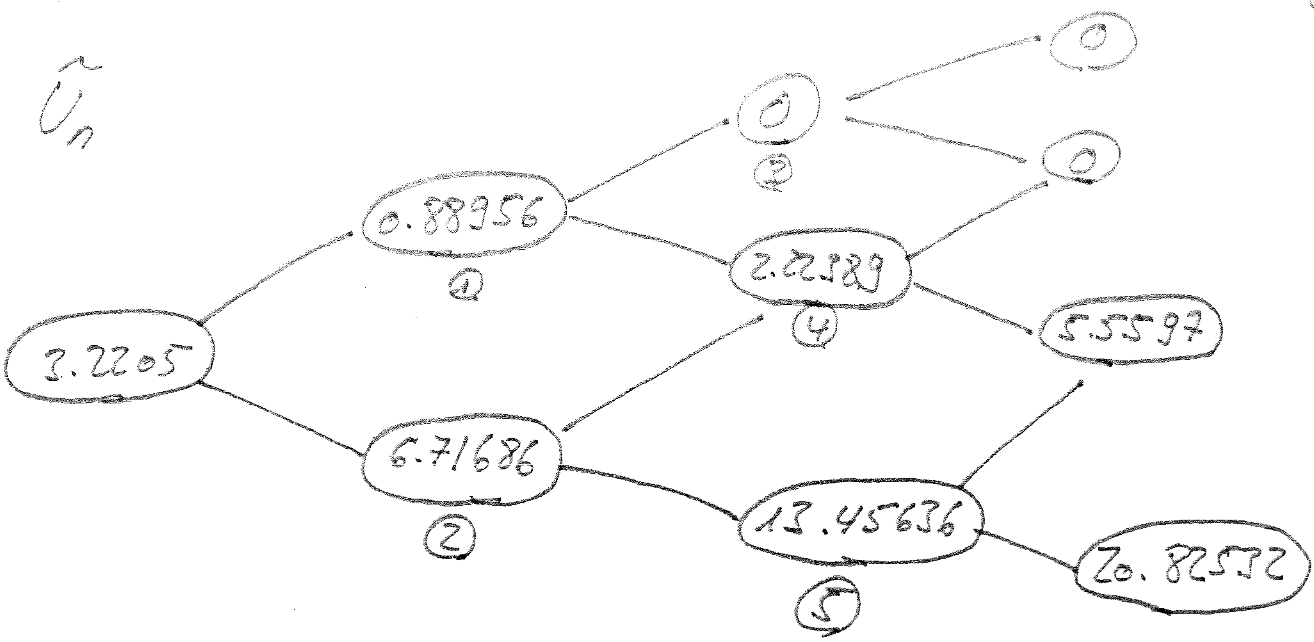
$\left(= \hat M_n \text{ being a mart. with } M_0 = 0\right)$

$$\Rightarrow \tilde M_n = \tilde U_n + \tilde A_n$$

After some work (you are not expected to do that) we obtain the missing information for:

$\omega_1 = [up, up, up]$	$\nu(\omega_1) = 2$,	$\nu_{max}(\omega_1) = 3$
$\omega_2 = [up, up, down]$	$\nu(\omega_2) = 2$,	$\nu_{max}(\omega_2) = 3$
$\omega_3 = \{up, down, up\}$	$\nu(\omega_3) = 3$,	$\nu_{max}(\omega_3) = 3$
$\omega_4 = \{up, down, down\}$	$\nu(\omega_4) = 3$,	$\nu_{max}(\omega_4) = 3$
$\omega_5 = \{down, up, up\}$	$\nu(\omega_5) = 3$,	$\nu_{max}(\omega_5) = 3$
$\omega_6 = \{down, up, down\}$	$\nu(\omega_6) = 3$,	$\nu_{max}(\omega_6) = 3$
$\omega_7 = \{down, down, up\}$	$\nu(\omega_7) = 2$,	$\nu_{max}(\omega_7) = 2$
$\omega_8 = [down, down, down]$	$\nu(\omega_8) = 2$	$\nu_{max}(\omega_8) = 2$

$\tilde{U}_n$



Doob

$$\tilde{U}_n = \tilde{U}_0 + \sum_{k=1}^{n} \Delta \tilde{U}_k$$

$$= \tilde{U}_0 + \underbrace{\sum_{k=1}^{n} E_Q(\Delta \tilde{U}_k \mid \mathcal{F}_{k-1})}_{-\tilde{A}_n} + \underbrace{\sum_{k=1}^{n} \left(\Delta \tilde{U}_k - E_Q(\Delta \tilde{U}_k \mid \mathcal{F}_{k-1})\right)}_{\tilde{M}_n - \tilde{M}_0 = \tilde{M}_n}$$

$$\tilde{M}_0 = \tilde{U}_0$$

$$\tilde{U}_n + \tilde{A}_n = \tilde{M}_n$$

That these are the cond. exp.: See Shreve p. 66 f.

(2)

$$E(\Delta \tilde{U}_3 | \mathcal{F}_2)(③) = 0$$

$$E(\Delta \tilde{U}_3 | \mathcal{F}_2)(④) = 0.6(0 - 2.22389) + 0.4(5.5597 - 2.22389) = 0$$

$$E(\Delta \tilde{U}_3 | \mathcal{F}_2)(⑤) = 0.6(5.5597 - 13.45636) + 0.4(20.82532 - 13.45636) = -1.79041$$

$$E(\Delta \tilde{U}_2 | \mathcal{F}_1)(①) = 0.6(0 - 0.88956) + 0.4(2.22389 - 0.88956) = 0$$

$$E(\Delta \tilde{U}_2 | \mathcal{F}_1)(②) = 0.6(2.22389 - 6.71686) + 0.4(13.45636 - 6.71686) = 0$$

$$E(\Delta \tilde{U}_1 | \mathcal{F}_0) = 0.6(0.88956 - 3.2205) + 0.4(6.71686 - 3.2205) = 0$$

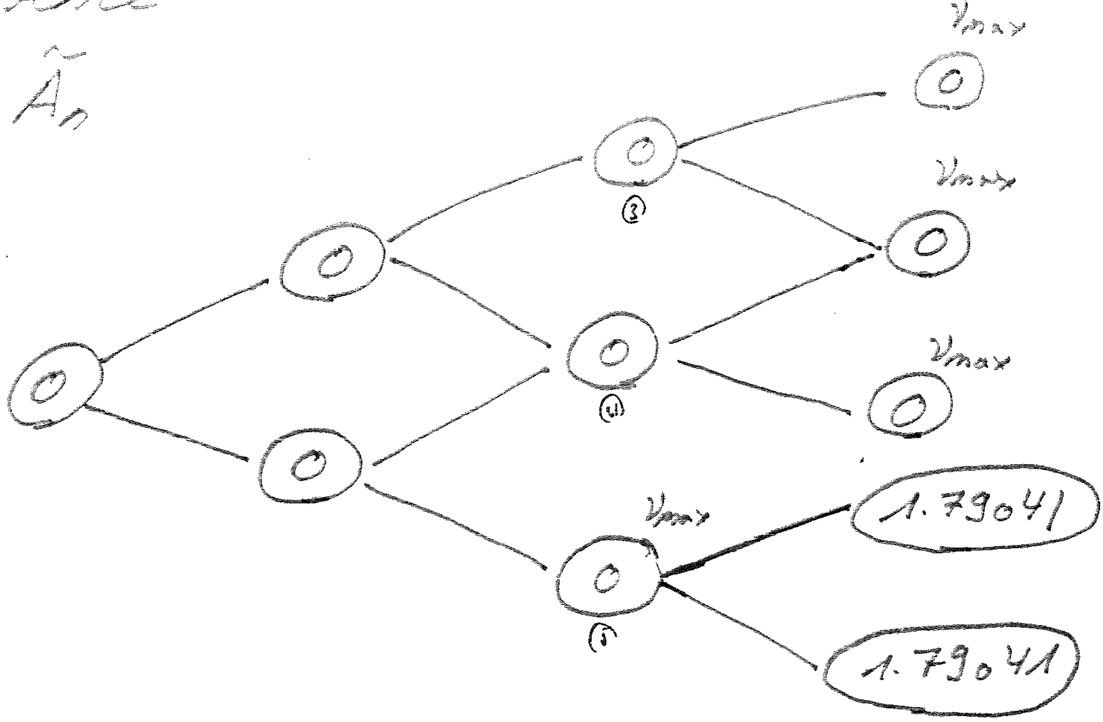
$$\tilde{A}_0 = 0, \quad \tilde{A}_1 = -E_Q(\Delta \tilde{U}_1 | \mathcal{F}_0) = 0$$

$$\tilde{A}_2 = -E_Q(\Delta \tilde{U}_1 | \mathcal{F}_0) - E_Q(\Delta \tilde{U}_2 | \mathcal{F}_1) = 0 - E_Q(\Delta \tilde{U}_2 | \mathcal{F}_1) = 0 - 0 = 0$$

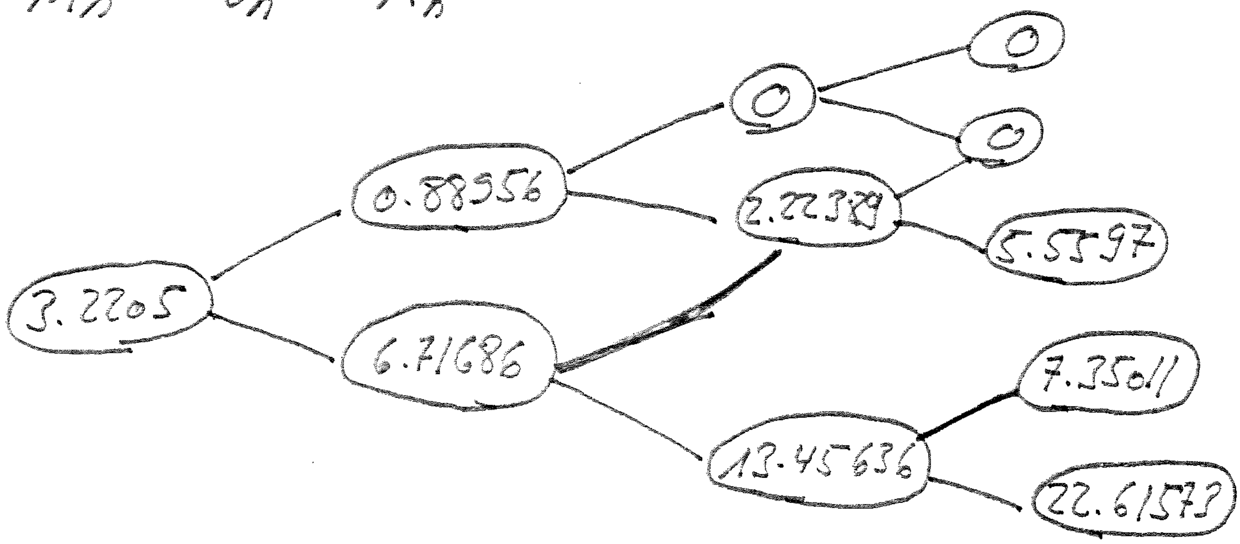
$$\tilde{A}_3 = -E_Q(\Delta \tilde{U}_1 | \mathcal{F}_0) - E_Q(\Delta \tilde{U}_2 | \mathcal{F}_1) - E_Q(\Delta \tilde{U}_3 | \mathcal{F}_2) = -E_Q(\Delta \tilde{U}_3 | \mathcal{F}_2)$$

$$= -E_Q(\Delta \tilde{U}_3 | \mathcal{F}_2)$$

Hence $\tilde{A}_n$



$$\tilde{M}_n = \tilde{U}_n + \tilde{A}_n$$



Mart-Test!

$0 = 0.6 \cdot 0 + 0.4 \cdot 0$

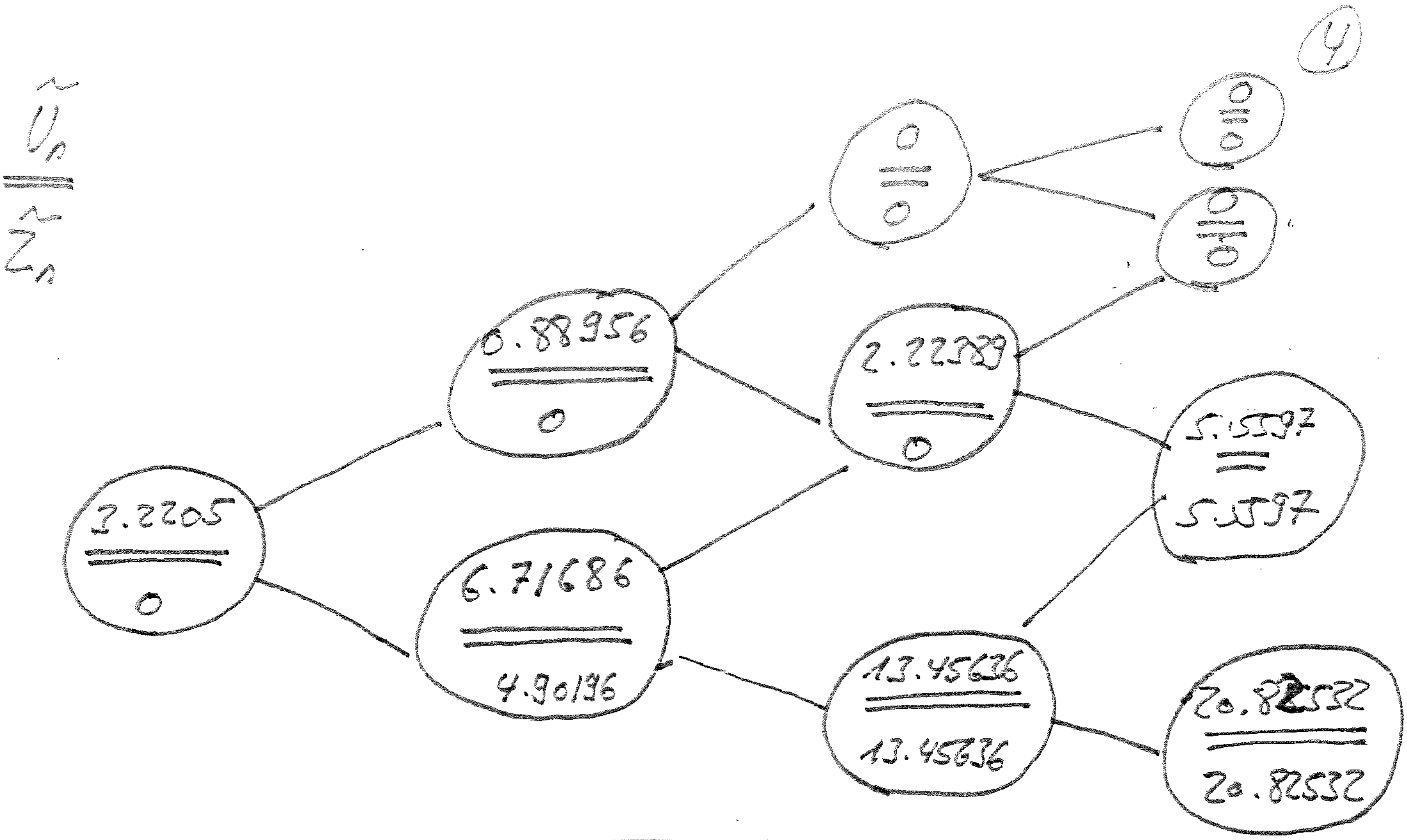
$2.22389 = 0.4 \cdot 5.5597$

$13.45636 = 0.6 \cdot 7.3591 + 0.4 \cdot 22.61573$

$0.88956 = 0.4 \cdot 2.22389$

$6.71686 = 0.6 \cdot 2.22389 + 0.4 \cdot 13.45636$

$3.2205 = 0.6 \cdot 0.88956 + 0.4 \cdot 6.71686$



$\omega_1 = \{up, up, up\}$ $\nu(\omega_1) = 2$, $\nu_{max}(\omega_1) = 3$

$\omega_2 = \{up, up, down\}$ $\nu(\omega_2) = 2$, $\nu_{max}(\omega_2) = 3$

$\omega_3 = \{up, down, up\}$ $\nu(\omega_3) = 3$, $\nu_{max}(\omega_3) = 3$

$\omega_4 = \{up, down, down\}$ $\nu(\omega_4) = 3$, $\nu_{max}(\omega_4) = 3$

$\omega_5 = \{down, up, up\}$ $\nu(\omega_5) = 3$, $\nu_{max}(\omega_5) = 3$

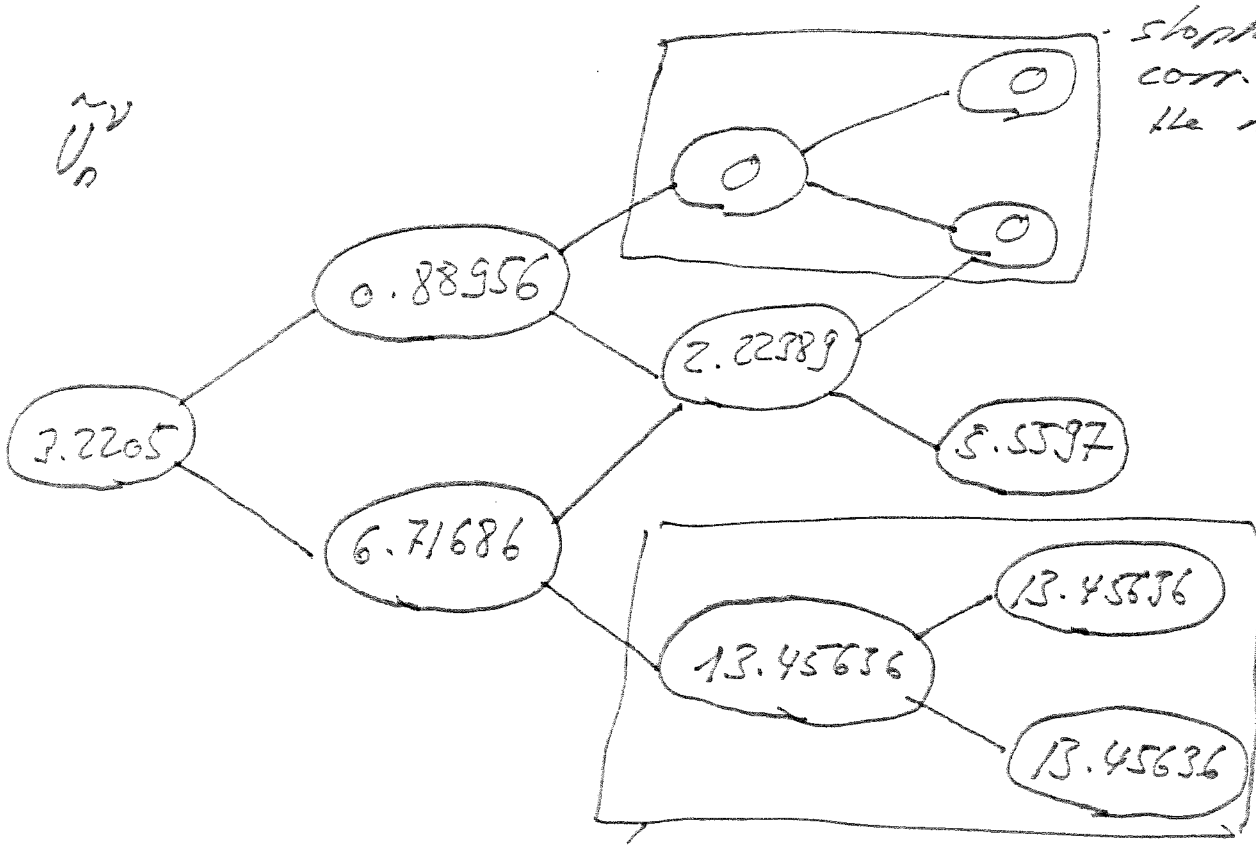
$\omega_6 = \{down, up, down\}$ $\nu(\omega_6) = 3$, $\nu_{max}(\omega_6) = 3$

$\omega_7 = \{down, down, up\}$ $\nu(\omega_7) = 2$, $\nu_{max}(\omega_7)\ 2$

$\omega_8 = \{down, down, down\}$ $\nu(\omega_8) = 2$, $\nu_{max}(\omega_8) = 2$

$\tilde{M}_n$ - price if we follow if we ex. at the last optimal point (and then follow $V_n(\theta)$)

. $\tilde{U}_n^v$ price if we ex. at the first "rational" point and then put money on the bank (discounted it will be a const.)



stopped coin. with the non-st.

difference between stopped and non-stopped

Mart. Test:

$0 = 0 + 0$ ✓

$2.22 = 0.4 \cdot 5.5597$ ✓

$13.45 = 0.6 \cdot 13.45 + 0.4 \cdot 13.45$ ✓

$0.88 = 0.4 \cdot 2.22$ ✓

$6.716 = 0.4 \cdot 13.456 + 0.6 \cdot 2.22$ ✓

$3.2205 = 0.6 \cdot 0.889 + 0.4 \cdot 6.716$ ✓