Deterministic interest rates / classical conventions

- r - interest rates p.a., e.g. 1%
- C_0 - initial capital at $t=0$
- If compounded once per annum (at the end)

$$C_1 = C_0 + C_0 r = C_0(1+r).$$

- If m-times compounded per annum

$$C_1 = \underbrace{\underbrace{C_0\left(1+\frac{r_m}{m}\right)}_{C_{1/m}}\left(1+\frac{r_m}{m}\right)\cdots\left(1+\frac{r_m}{m}\right)}_{m\text{-times}} = C_0\left(1+\frac{r_m}{m}\right)^m$$

- $C_n = C_0\left(1+\frac{r_m}{m}\right)^{m\cdot n}$

- $\left(1+\frac{r_c}{m}\right)^m \xrightarrow{m\to\infty} e^{r_c}$ - continuous compounding

- $C_t = C_0 e^{r_c t}$ - solves

$$\frac{dC_t}{dt} = r_c \underbrace{C_0 e^{r_c t}}_{C_t} = r_c C_t$$

- There are also other conventions.
- If the interest rate convention is clear from the context, we often omit the subscript.
- We have the price change factors

$$e^{r_c} = \left(1 + \frac{r_m}{m}\right)^m$$

a) $r_c = m \log\left(1 + \frac{r_m}{m}\right)$

b) $m\left(e^{r_c/m} - 1\right) = r_m$

} - equivalent interest rates

Example: If we use the same nominal interest rate, e.g. 5%

- Annual compounding: $100(1+0.05) = 105$
- Continuous compounding: $\underbrace{100 e^{0.05}}_{\approx 105.127} > 105$