Mathematical Modeling of Behavior (2024-2025)

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Question 1: Hypothesis testing

For each of the following statements, say if it is true or false, and explain why.

1. If the hypothesis test does not reject the null hypothesis, we can conclude that the null hypothesis is true.
2. When we reject a true null hypothesis, we commit a Type I error.
3. When the null hypothesis is false and is not rejected, we make a type II error.
4. The power of a test is the probability of rejecting the null hypothesis when it is true.
5. If the level of significance α of a test is increased, the power of the test decreases.
6. If a null hypothesis is rejected at the level of significance 0.01, it is also rejected at the level of significance 0.05.
7. For a given level of significance, if the sample size is increased, the power of the test decreases.

Question 2: Likelihood ratio test

For a transportation mode choice model, consider the following utility specifications (where the index n related to the individual has been dropped to simplify the notations):

1. Linear with generic coefficients:

$$
\begin{aligned}
U_{car} &= ASC_{car} + \beta_{tt} \cdot tt_{car} + \beta_{tc} \cdot tc_{car} + \varepsilon_{car}, \\
U_{pt} &= \beta_{tt} \cdot tt_{pt} + \beta_{tc} \cdot tc_{pt} + \varepsilon_{pt}.
\end{aligned}
$$

2. Linear with alternative specific coefficients:

$$
\begin{aligned}
U_{car} &= ASC_{car} + \beta_{tt,car} \cdot tt_{car} + \beta_{tc,car} \cdot tc_{car} + \varepsilon_{car}, \\
U_{pt} &= \beta_{tt,pt} \cdot tt_{pt} + \beta_{tc,pt} \cdot tc_{pt} + \varepsilon_{pt}.
\end{aligned}
$$

3. Power series:

$$
\begin{aligned}
U_{car} &= ASC_{car} + \beta_{tt} \cdot tt_{car} + \beta_{tc} \cdot tc_{car} + \beta_{tc\_squared} \cdot tc^2_{car} + \varepsilon_{car}, \\
U_{pt} &= \beta_{tt} \cdot tt_{pt} + \beta_{tc} \cdot tc_{pt} + \beta_{tc\_squared} \cdot tc^2_{pt} + \varepsilon_{pt}.
\end{aligned}
$$

4. Box-cox:

$$
\begin{aligned}
U_{car} &= ASC_{car} + \beta_{tt\_boxcox} \cdot \frac{(tt_{car} - 1)^{\lambda}}{\lambda} + \beta_{tc} \cdot tc_{car} + \varepsilon_{car}, \\
U_{pt} &= \beta_{tt\_boxcox} \cdot \frac{(tt_{pt} - 1)^{\lambda}}{\lambda} + \beta_{tc} \cdot tc_{pt} + \varepsilon_{pt}.
\end{aligned}
$$

5. Logarithm:

$$
\begin{aligned}
U_{car} &= ASC_{car} + \beta_{tt} \cdot tt_{car} + \beta_{tc\_log} \cdot \log(tc_{car}) + \varepsilon_{car}, \\
U_{pt} &= \beta_{tt} \cdot tt_{pt} + \beta_{tc\_log} \cdot \log(tc_{pt}) + \varepsilon_{pt}.
\end{aligned}
$$

6. Piecewise linear:

$$
\begin{aligned}
U_{car} =& ASC_{car} + \beta_{tt,<15} \cdot tt_{car,<15} + \beta_{tt,[15,60)} \cdot tt_{car,[15,60)} + \beta_{tt,\geq 60} \cdot tt_{car,\geq 60} \\
&+ \beta_{tc} \cdot tc_{car} + \varepsilon_{car}, \\
U_{pt} =& \beta_{tt,<15} \cdot tt_{pt,<15} + \beta_{tt,[15,60)} \cdot tt_{pt,[15,60)} + \beta_{tt,\geq 60} \cdot tt_{pt,\geq 60} + \beta_{tc} \cdot tc_{pt} + \varepsilon_{pt}.
\end{aligned}
$$

where for $i \in \{car, pt\}$

$$tt_{i,<15} = \begin{cases} tt_i, & \text{if} \quad tt_i < 15 \\ 15, & \text{otherwise,} \end{cases}$$

$$tt_{car,[15,60)} = \begin{cases} 0, & \text{if} \quad tt_i < 15 \\ tt_i - 15, & \text{if} \quad tt_i \in [15,60) \\ 60, & \text{otherwise,} \end{cases}$$

$$tt_{i,\geq 60} = \begin{cases} 0, & \text{if} \quad tt_i < 60 \\ tt_i - 60, & \text{otherwise,} \end{cases}$$

where tt_{car} and tt_{pt} are the travel times in minutes by car and public transportation respectively, tc_{car} and tc_{pt} are the travel costs in CHF of car and public transportation respectively; ASC_{car}, β's and λ are parameters to be estimated; and $\varepsilon_{car}, \varepsilon_{pt} \overset{iid}{\sim} EV(0,1)$.
For each pair of models, say if it is possible to apply a likelihood ratio test, and explain why.