Mathematical Modeling of Behavior (2024-2025)

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Introduction — September 10, 2024

Question 1: Electric vehicles

We want to build a model that predicts the market penetration of electric vehicles (EV) as a function of the income level. We have a sample of 1000 individuals. The data is summarized in Table 1.

	Income			
EV	low	medium	high	
yes	15	50	135	200
no	200	450	150	800
	215	500	285	1000

Table 1: Contingency table of EV ownership conditional on income level

1. Estimate the parameters π_1, π_2 and π_3 using maximum likelihood estimation, where:

$$\begin{aligned} \Pr(\text{EV} = \text{yes} \mid \text{income} = \text{low}) &= \pi_1, \\ \Pr(\text{EV} = \text{yes} \mid \text{income} = \text{medium}) &= \pi_2, \\ \Pr(\text{EV} = \text{yes} \mid \text{income} = \text{high}) &= \pi_3. \end{aligned} \tag{1}$$

   Hint: write the likelihood function and find its maximum.

2. Perform an informal test on the values of the parameters.

3. Calculate the final log likelihood of the model.

4. We would like to test the hypotheses that $\pi_1 = \pi_2$. Construct the corresponding restricted model, and use a likelihood ratio test to determine which model should be preferred.

5. We would like to test the hypotheses that $\pi_1 = \pi_2 = \pi_3$. Construct the corresponding restricted models, and use a likelihood ratio test to determine which model should be preferred.

6. Suppose now that after some economical growth, the income distribution is as follows: 7.5% of the population with low income, 40% of the population with medium income and 52.5% of the population with high income. Use the preferred model to forecast the market share of EV for this scenario.

7. Could we have used linear regression instead of discrete choice models in the context above? If so, what would have been the dependent variable?