Mathematical Modeling of Behavior
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Michel Bierlaire.
TAs: Cloe Cortes, Pavel Ilinov, Evangelos Paschalidis, Anne-Valérie Preto, Negar Rezvany

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Question 1: Error component model

Consider a mixture model with utility function U_{in} of individual n choosing alternative $i \in \mathcal{C}_n$ written as:

$$U_{in} = V_{in} + \xi_{in} + \nu_{in},$$

where V_{in} is the systematic component of the utility, $\nu_{in} \overset{\text{iid}}{\sim} \text{EV}(0, \mu)$, and ξ_n follows a distribution characterited by the probability density function $f(\xi)$. The conditional probability $P_n(i|\xi_n)$ on ξ_n is given as:

$$P_n(i|\xi_n) = \frac{e^{\mu(V_{in}+\xi_{in})}}{\sum_{j\in\mathcal{C}_n} e^{\mu(V_{jn}+\xi_{jn})}}.$$

The choice probability $P_n(i)$ that n chooses i is formulated as:

$$P_n(i) = \int_{\xi} P_n(i|\xi) f(\xi) d\xi, \qquad (1)$$

where $\xi \in \mathbb{R}^{J_n}$, and J_n is the number of alternatives in $\mathcal{C}_n$. The integral is calculated on the domain where each component ξ_i of ξ ranges from $-\infty$ to $+\infty$.

1. What are the advantages of the mixture model compared to the logit model?

2. The integral involved in (1) has no closed form and is usually difficult to calculate with numerical integration (quadrature), especially when J_n is large. In that case, $P_n(i)$ must be approximated. How? Explain the approach, and provide the formula for the approximation.

Question 2: Heteroscedasticity and random parameters

Consider a transportation mode choice model with the choice set $\mathcal{C}_n = \{\text{car, bus, rail}\}, \forall n$. The specification of the logit model is the following:

$$\begin{aligned} U_{\text{car},n} &= \text{ASC}_{\text{car}} + \beta_{\text{time}} \text{time}_{\text{car},n} + \varepsilon_{\text{car},n}, \\ U_{\text{bus},n} &= \text{ASC}_{\text{bus}} + \beta_{\text{time}} \text{time}_{\text{bus},n} + \varepsilon_{\text{bus},n}, \\ U_{\text{rail},n} &= \beta_{\text{time}} \text{time}_{\text{rail},n} + \varepsilon_{\text{rail},n}, \end{aligned}$$

where ASC_i is the alternative specific constant of mode i, $\text{time}_{i,n}$ is the travel time of mode i for individual n, and $\varepsilon_{i,n} \overset{\text{iid}}{\sim} \text{EV}(0, \mu)$. We call this model the **base model**, that involves three unknown parameters: ASC_{car}, ASC_{bus} and β_{time}.

1. Because of the i.i.d. assumption, the variances of the utility functions are identical across alternatives. We would like to relax this assumption, and allow the variance of the utility functions to vary across alternatives. Modify the base model to achieve this objective. Provide the specification of the new model and enumerate the unknown parameters involved.

2. The taste parameter for travel time is captured by the parameter β_{time}, that is the same for all individuals in the population. We would like to relax this assumption, and capture the heterogeneity of taste within the population. But we do not have access to relevant socio-economic characteristics. Modify the base model to achieve this objective. Provide the specification of the new model and enumerate the unknown parameters involved.

3. Because of the i.i.d. assumption, the error terms of the utility functions are independent across alternatives. We would like to relax this assumption, and capture a possible correlation between bus and rail (because they are both public transportation alternatives) and a possible correlation between car and bus (because the two modes use the road network and are both subject to congestion). Modify the base model to achieve this objective. Provide the specification of the new model and enumerate the unknown parameters involved.