Pi

Compute π the slow way

$$arctan(x) = \int \frac{1}{1+x^2} dx$$

$$\frac{\pi}{4} = \int_0^1 \frac{1}{1+x^2} dx$$

Riemann version (yes we really said slow)

$$\pi = \sum_{i=0}^{n} \frac{4}{1+(i/n)^2} \frac{1}{n} \quad (1)$$

Poisson equation

Let us consider the 2D Poisson problem:

$$\begin{cases} \Delta u & = f(x,y) \quad \text{in } \Omega \\ u(x,y) & = 0 \quad \text{on } \partial\Omega \end{cases}$$

Finite difference

We define a discrete mesh of points (x_i, y_j) uniformely spaced in both dimensions with

$$x_{i+1} - x_i = h = 1/N$$
$$y_{i+1} - y_i = h = 1/N$$

To approximate the solution at the mesh points, we will use centered finite differences.

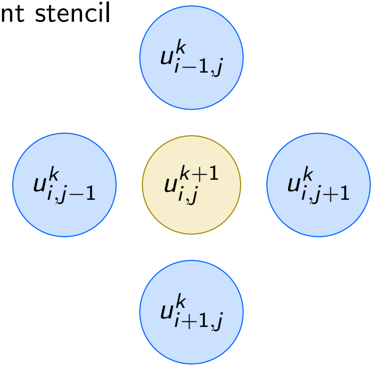
$$(\Delta u)_{i,j} = \frac{u_{i+1,j} + u_{i-1,j} - 2u_{i,j}}{h^2} + \frac{u_{i,j+1} + u_{i,j-1} - 2u_{i,j}}{h^2} = f_{i,j}$$
$$= \frac{1}{h^2}(u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j})$$

Jacobi method and boundary conditions

Solving the system of linear equation with the Jacobi method, we iterate until a given error is reached

$$u_{i,j}^{k+1} = \frac{1}{4}\left(u_{i+1,j}^{k} + u_{i-1,j}^{k} + u_{i,j+1}^{k} + u_{i,j-1}^{k} - f_{i,j}h^2\right)$$

This gives a 4 point stencil



Let solve with $u = 0$ on the boundaries
And $f = -200\pi^2 \sin(10\pi x) sin(10\pi y)$