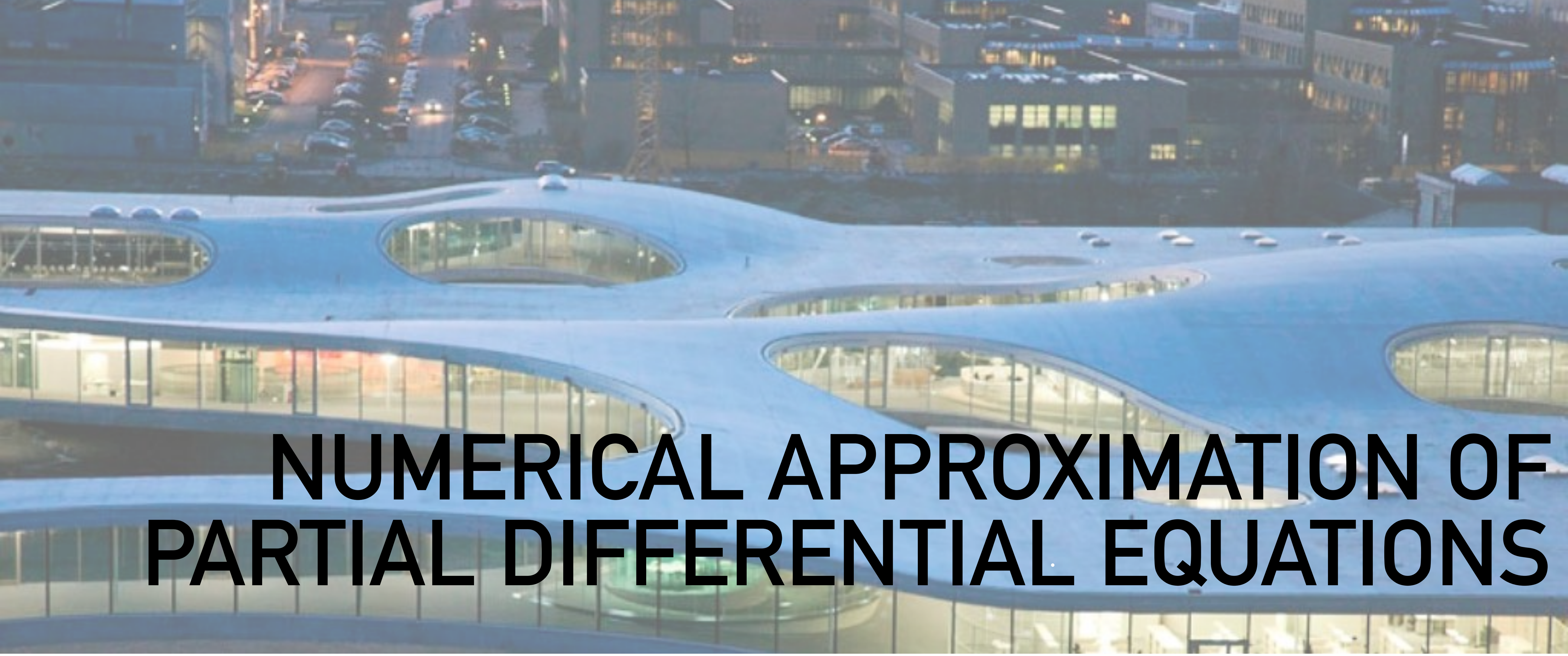


NUMERICAL APPROXIMATION OF PARTIAL DIFFERENTIAL EQUATIONS

Annalisa Buffa

Assistant:

Mohammed Ben Abdelouhab

mohamed.benabdelouahab@epfl.ch

EPFL

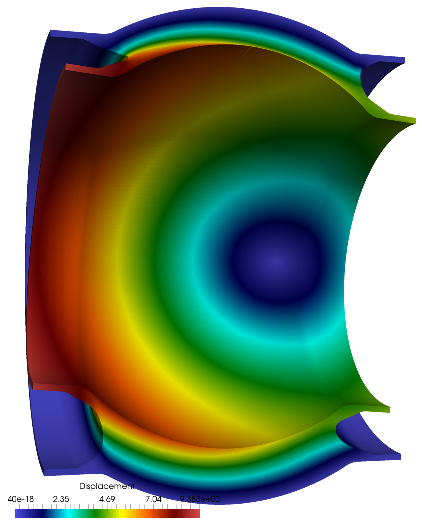
To make prediction

WHY ?

$$\boldsymbol{\sigma} = 2\,\mu\,\boldsymbol{\varepsilon} + \lambda\,\nabla\cdot\mathbf{u}\,\mathbf{1}$$
$$\boldsymbol{\varepsilon} = \nabla^s\mathbf{u}$$
$$\mathrm{div}\,(\boldsymbol{\sigma}) = \mathbf{f}$$

$\lambda \to \infty$ incompressible limit

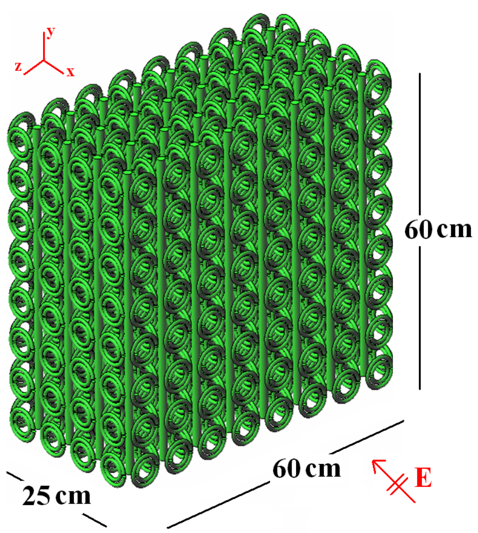
Elasticity



$$\begin{aligned} \nabla\cdot(\mathbf{u}\otimes\mathbf{u}) - \nu\Delta\mathbf{u} + \nabla\,p &= \mathbf{f} \\ \mathrm{div}\,\mathbf{u} &= 0 \end{aligned}$$

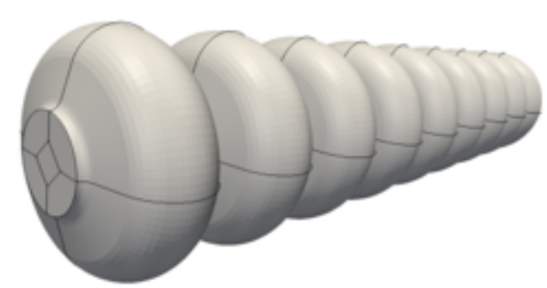
fluids

$$\mathrm{div}\,(\boldsymbol{\sigma}) = \mathbf{f} \qquad \boldsymbol{\sigma} = \mathbb{C}\big[\mathbf{F}(\mathbf{u})\big]$$
$$\mathrm{div}\,(\mathbf{u}) = 0$$
$$\mathfrak{g}(\mathbf{u}) \geq 0\,,\;\; \boldsymbol{\sigma}_{\mathbf{n}} \leq 0\,,\;\; \boldsymbol{\sigma}_{\mathbf{n}}\mathfrak{g}(\mathbf{u}) = 0 \quad \textbf{contact}$$



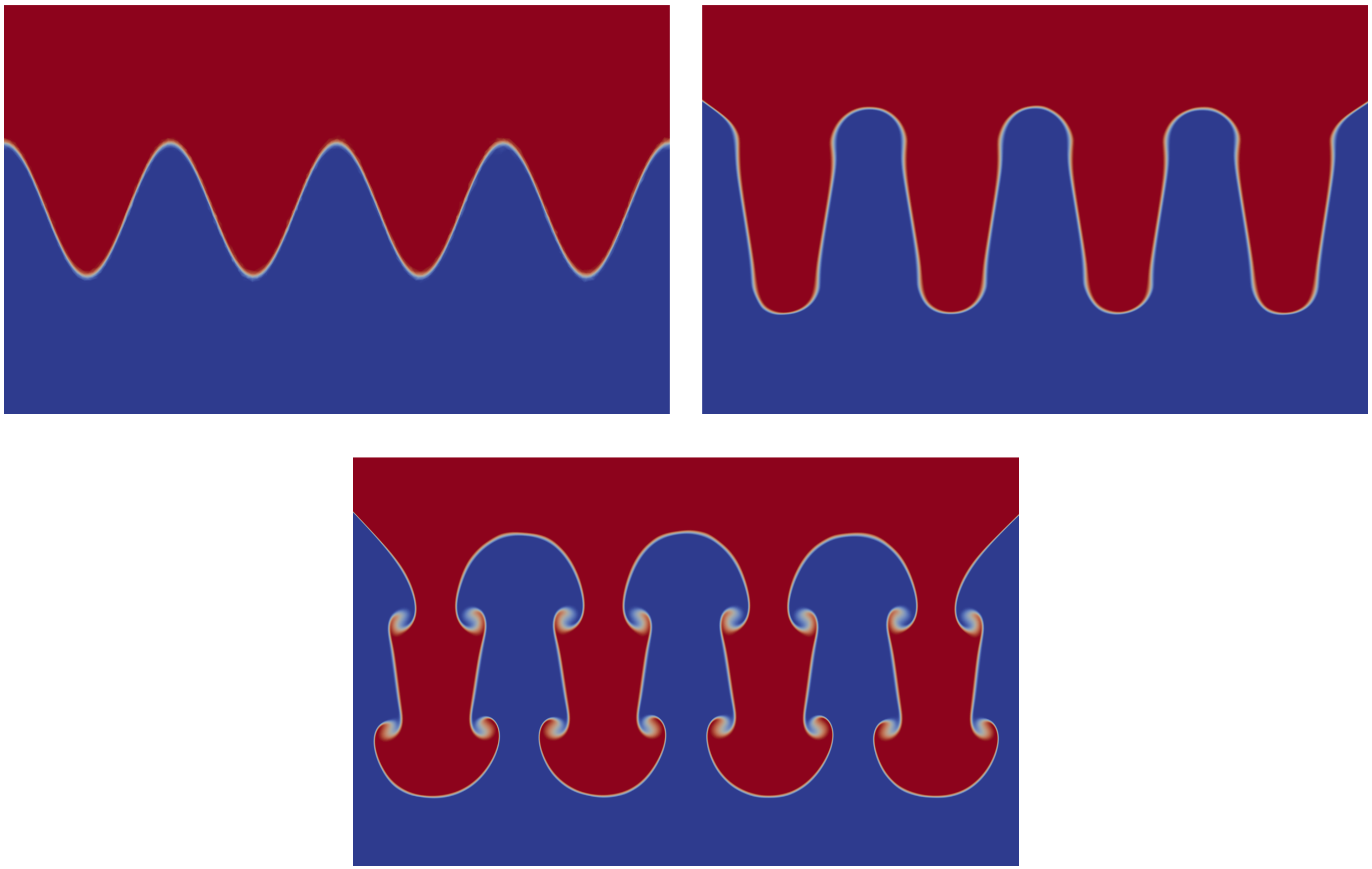
electromagnetics

$$\mathbf{curl}\,\mathbf{H} = i\,\omega\,\mathbf{D} + \mathbf{J} \qquad \mathbf{curl}\,\mathbf{E} = -i\,\omega\,\mathbf{B}$$
$$\mathbf{B} = \mu\mathbf{H} \qquad \mathbf{D} = \varepsilon\mathbf{E}$$
$$\mathrm{div}\,(\mathbf{B}) = 0 \qquad \mathrm{div}\,(\mathbf{D}) = \rho$$



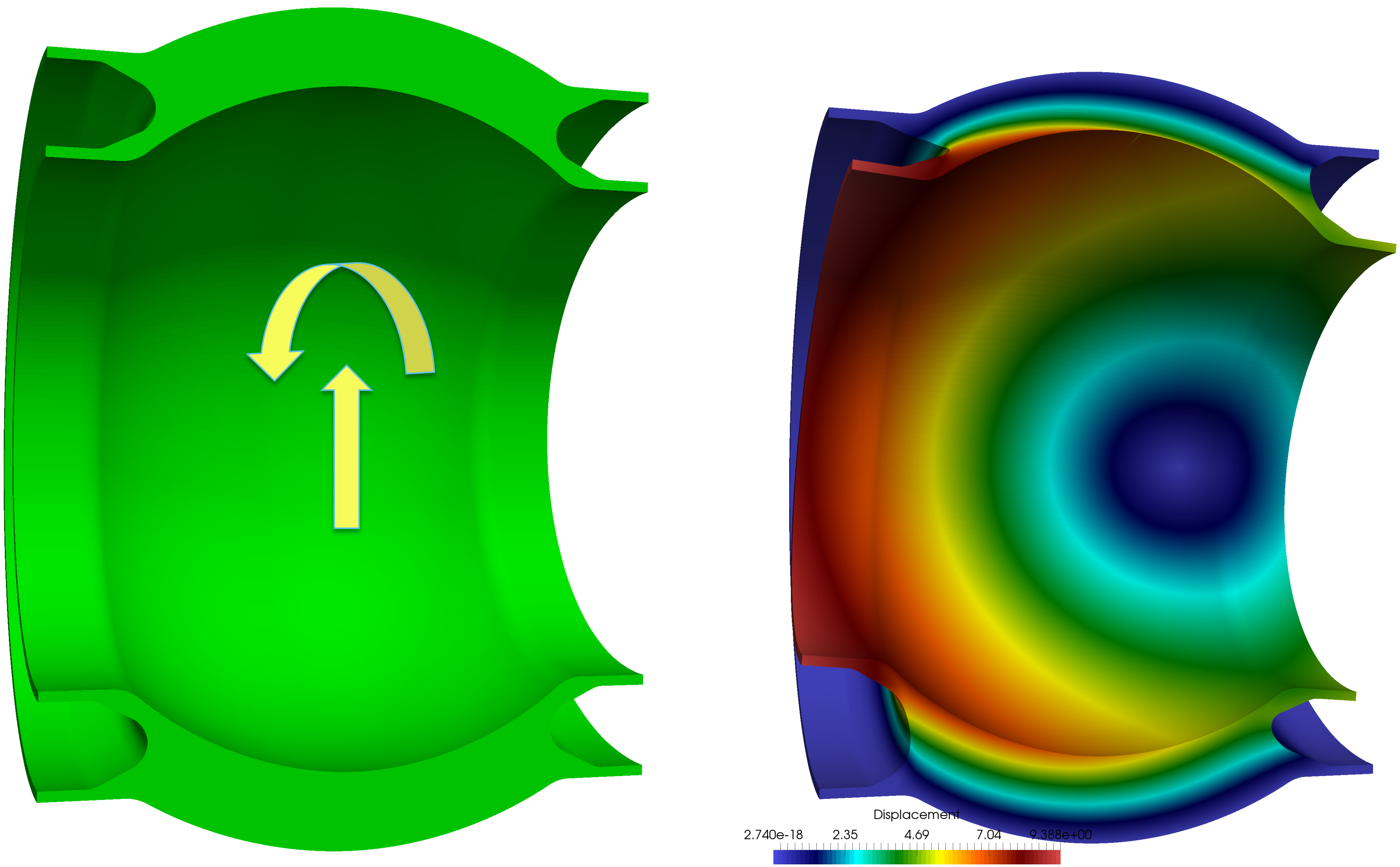
$+$ Radiation condition at ∞

Ethanol - water interaction



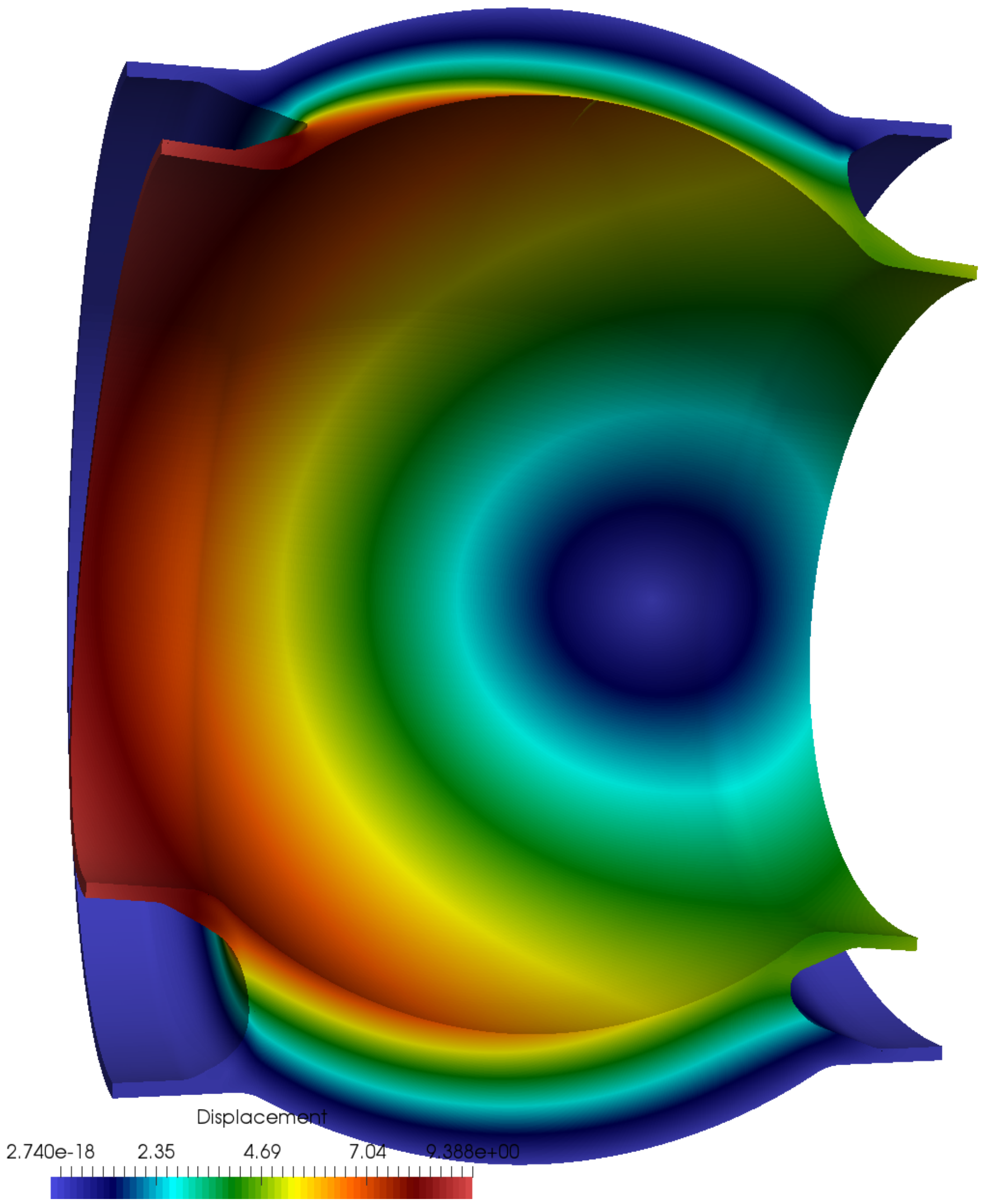
A well known phenomena that reduces the efficiency for the extraction of oil by injecting water in a reservoir !

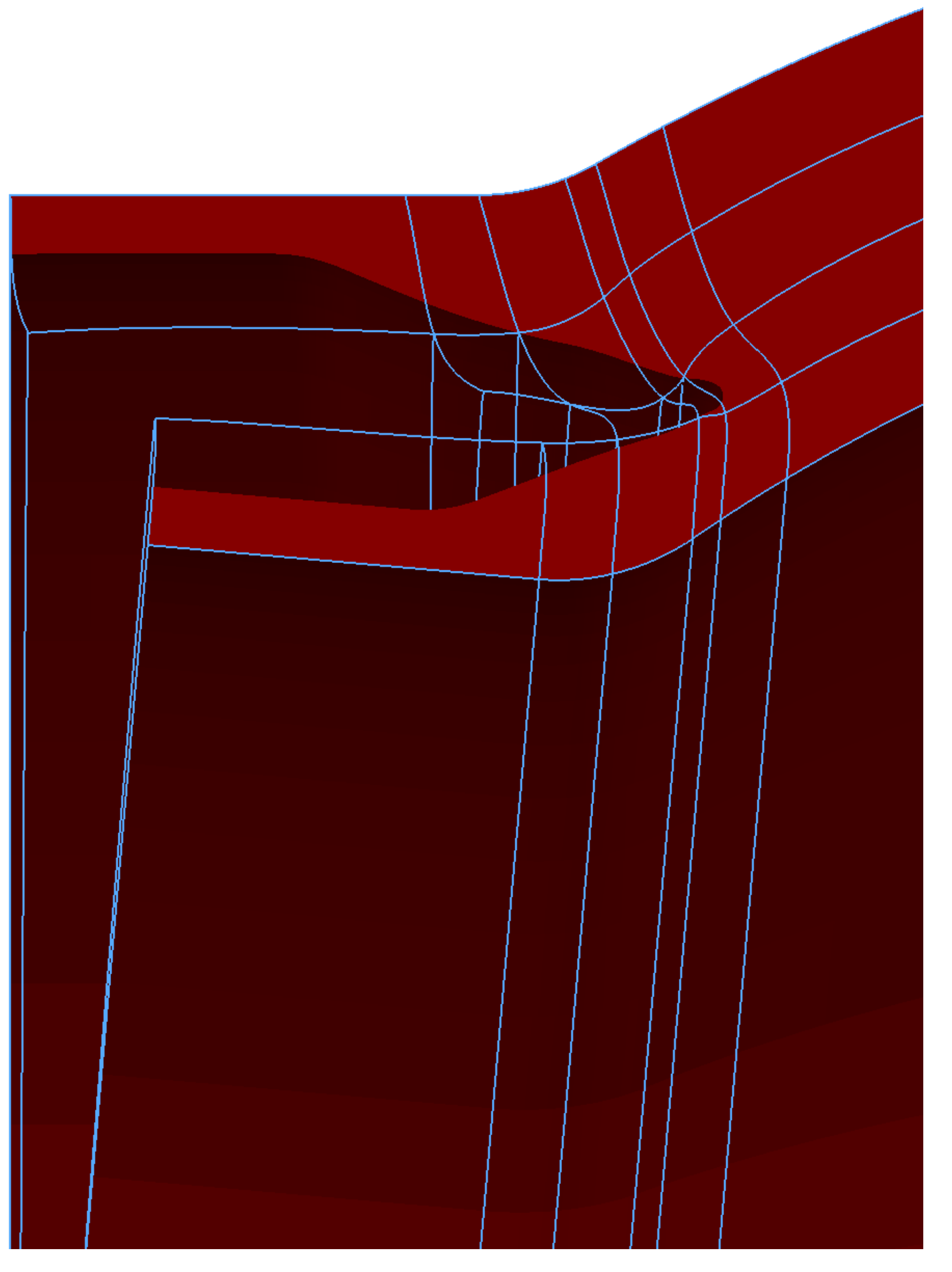
ANOTHER EXAMPLE



Non linear mechanical behaviour: joints made with rubber

ANOTHER EXAMPLE





Non linear mechanical behaviour: joints made with rubber

CONTENTS OF THIS CLASS

Lectures

- Design of numerical methods
- Study of their mathematical properties
- Understand accuracy and its lack thereof
- Theorem proving

Exercises

- Try numerical methods
- Code our own numerical toolbox

Miniprojects will be proposed!
They will count for the final grade – to be defined soon

SOME EQUATIONS

$$-\Delta u = f$$

poisson problem.

$$-\Delta u + \vec{b} \cdot \nabla u + u = f$$

heat equation

$$\frac{\partial u}{\partial t} - \Delta u = f$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} - \Delta \boldsymbol{u} + \nabla p = f$$

$$\mathrm{div}(\boldsymbol{u}) = 0$$

N.S.

$$\frac{\partial u}{\partial t} + \vec{b} \cdot \nabla u = f$$

transport

$$\frac{\partial^2 u}{\partial t^2} - \Delta u = f$$

wave equation.

+ electromagnetics