Exercise 0. Let G be an amenable group. Prove that there is $\mu \in \mathscr{M}(G)$ which is invariant under both the left and the right multiplication. (Start off by writing out explicitly this statement…)

Exercise 1. Let G be a group. Prove that G contains a *maximal* normal amenable subgroup, and that it is unique. We call it the *amenable radical* of G and denote it by $\mathrm{Ramen}(G)$. Show that the amenable radical of $G/\mathrm{Ramen}(G)$ is trivial.

Exercise 2. Make sure that you understand the definition of $\bigoplus_{i\in I} G_i$ for a family $\{G_i\}_{i\in I}$ of groups G_i. Prove that $\bigoplus_{i\in I} G_i$ is amenable if all of the G_i are amenable.

Compare to Ex. 2 from last week and meditate.

Exercise 3. (i) Let G be a finite group. Prove that $G^{\mathbf{N}} := \prod_{n\in\mathbf{N}} G$ is amenable.

Hint: try to use Ex. 2 above.
Compare again to Ex. 2 of last week and meditate again.

(ii) Give an example of an amenable group G such that $G^{\mathbf{N}}$ is non-amenable.

Hint: combine the above meditations.

Other Exercise. We have emphasized the difference between a group being a *directed* union of subgroups — and being simply a union of subgroups.

Can a group G be the union of *two* subgroups $G_1, G_2 < G$? (Of course, we assume $G_i \neq G$.)
How about three?