Exercise 0 (Functional Analysis)
Prove the *Banach–Alaoglu Theorem*:
Given any Banach space V, the set

$$B = \{\lambda \in V^* : \|\lambda\| \leq 1\}$$

is weak-* compact.

Hint: apply the Tychonoff theorem to some huge product of compact spaces.
Note: B is usually called the "closed unit ball", so maybe a first step is to clarify in your mind in which topologies B is closed...

Exercise 1
For $j \in \mathbf{Z}$, consider the mean $\delta_j \in \mathscr{M}(\mathbf{Z})$.
We define a sequence $\{\mu_n\}$ in B by

$$\mu_n = \frac{1}{n}\sum_{j=1}^{n} \delta_j.$$

Although $\mathscr{M}(\mathbf{Z})$ is compact, prove that $\{\mu_n\}$ has no convergent subsequence.

Exercise 2
Prove that the group $\mathbf{Z}^2$ is amenable.

This will later be a very special case of a theorem, but for now try to imitate the case of $\mathbf{Z}$ *seen in class.*

Exercise 3 (Topology
Read `http://en.wikipedia.org/wiki/Net_(mathematics)` or use the library to understand the concept of *nets*, also known as *generalized sequences*. Understand the connection to open and closed sets, to continuous maps, to compactness. In particular, make sure to understand subnets.

The bottom line is that nets behave like sequences, but indexed by a larger set than $\mathbf{N}$*, in the sense that all first-year ideas connecting sequences and continuity, open/closed sets, etc., hold — except they hold for* general *situations, not just fist-year analysis. This would be false for sequences. Test: is a subnet of a sequence a sequence?*