Advanced numerical analysis
MATH-351
Marco Picasso - 2022

Intro: why ode's and pde's?

Newton 1687, Euler 1757:

XXI. Nous n'avons donc qu'à égaler ces forces accélératrices avec les accélerations actuelles que nous venons de trouver, & nous obtiendrons les trois équations fuivantes :

$$P - \frac{1}{q}\left(\frac{dp}{dx}\right) = \left(\frac{du}{dt}\right) + u\left(\frac{du}{dx}\right) + v\left(\frac{du}{dy}\right) + w\left(\frac{du}{dz}\right)$$

$$Q - \frac{1}{q}\left(\frac{dp}{dy}\right) = \left(\frac{dv}{dt}\right) + u\left(\frac{dv}{dx}\right) + v\left(\frac{dv}{dy}\right) + w\left(\frac{dv}{dz}\right)$$

$$R - \frac{1}{q}\left(\frac{dp}{dz}\right) = \left(\frac{dw}{dt}\right) + u\left(\frac{dw}{dx}\right) + v\left(\frac{dw}{dy}\right) + w\left(\frac{dw}{dz}\right)$$

Si nous ajoutons à ces trois équations premièrement celle, que nous a fournie la confidération de la continuité du fluide :

$$\left(\frac{dq}{dt}\right)$$

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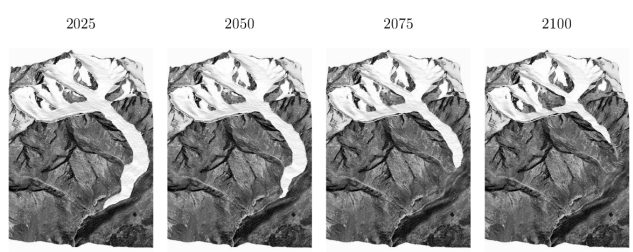
$$\left(\frac{dq}{dt}\right) + \left(\frac{d.qu}{dx}\right) + \left(\frac{d.qv}{dy}\right) + \left(\frac{d.qw}{dz}\right) = 0\,;$$

& enfuite celle que donne le rapport entre l'élafticité p, la denfité q, & l'autre qualité r, qui influë fur l'élafticité p, outre la denfité q, nous aurons cinq équations qui renferment toute la Théorie du mouvement des fluides.

Today, Navier-Stokes equations for airplanes, climate science: retreat of Aletsch's glacier



Open mathematical questions: existence, uniqueness?

1 Runge-Kutta methods for solving ode's (5 lessons)

Lecture notes Hairer, Wanner, Abdulle (see pdf on moodle page).

1.1 Euler explicit and implicit schemes

Existence of solutions, Euler scheme, stability, convergence, implementation with matlab/octave.

1.2 Runge-Kutta schemes

1.3 Order conditions for Runge-Kutta schemes, up to order 4

1.4 Adaptive time stepping

1.5 Convergence of Runge-Kutta schemes

1.6 An optimal control problem

2 Finite Difference Methods (FDM) for solving pde's (8 lessons)

2.1 Origin of pde's, examples

2.2 FDM for solving elliptic pde's in 1,2,3 space dimensions

$-u''(x) = f(x)$, $-\Delta u(x,y,z) = f(x,y,z)$, implementation with matlab/octave.

2.3 FDM for solving parabolic pde's: the heat equation

$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = f$, centered finite differences in space, Euler schemes in time.

2.4 FDM for solving hyperbolic pde's: the transport and wave equations

Transport: $\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$, upwind finite differences in space, Euler scheme in time.
Wave: $\frac{\partial^2 u}{\partial t^2} - c^2\frac{\partial^2 u}{\partial x^2} = f$, Newmark schemes.