Time Series Exercise Sheet 7

Sofia Olhede

April 3, 2025

Exercise 7.1

Let $\{\varepsilon_t\}_{t\in\mathbb{Z}}$ be a white noise process with variance σ_ε^2. Determine the spectral density function.

Exercise 7.2

Consider the sequence

$$\gamma_\tau = \begin{cases} 1 & \text{if } |\tau| \le K \\ 0 & \text{if } |\tau| > K \end{cases}$$

where K is a positive integer. Is γ_τ an ACVS for some discrete parameter stationary process $\{Y_t\}$? What would be the SDF?

Exercise 7.3

Determine the auto-covariance function of $\{X_t\}_{t\in\mathbb{Z}}$ if it has spectrum

$$S(f) = \sigma^2 \frac{(1 - 2|f|)}{\pi} \mathbb{1}(|f| \le 1/2)$$

Exercise 7.4

What is the spectral density function of an MA(q) process?

Exercise 7.5

Consider a harmonic process with a time varying amplitude

$$X_t = \epsilon_t \cos(\nu t + \Theta)$$

where we assume that ν is fixed, Θ is a random phase, and ϵ_t is not constrained to be positive, but that its a mean zero white noise process and its variance σ^2. Moreover, assume that ϵ_t is independent of Θ. The harmonic process in the lecture stipulated that Θ had to be uniformly distributed on $[-\pi, \pi]$. Suppose (instead) that Θ has a pdf given by

$$f_\Theta(\theta) = \frac{1}{2\pi}(1 + \cos(\theta)), \quad |\theta| \le \pi$$

Determine the first two moments of X_t. Are they still independent of t?