Time Series Exercise Sheet 5

Sofia Olhede

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Exercise 5.1

Suppose that $\{X_t\}$ is an ARIMA(p, d, q) process satisfying the equations

$$\phi(B)(\mathrm{I} - \mathrm{B})^d X_t = \theta(B) Z_t$$

where Z_t is white noise. Show that the same equations are also satisfied by $W_t = X_t + A_0 + A_1 t + \cdots + A_{d-1} t^{d-1}$, where A_j are independent random variables.

Exercise 5.2

You observe U_t that follows a SARMA $(1, 2) \times (0, 1)_4$ model. Write down the equations that specify U_t in terms of the past. Determine the weights of the process in its infinite MA representation

$$U_t = \sum_{j=0}^{\infty} \psi_j \epsilon_{t-j}.$$

You may find it useful to define ψ_j in terms of ψ_{j-1} as this will simplify algebra.

Exercise 5.3

Let us study the seasonal process $\left(1 - 0.7B^2\right) X_t = \left(1 - 0.3B^2\right) \epsilon_t$ for ϵ_t white noise with unit variance.

1. Find the coefficients $\{a_j\}$ in the representation $X_t = \sum_{j=0}^{\infty} a_j \epsilon_{t-j}$
2. Find the coefficients $\{b_j\}$ in the representation $\epsilon_t = \sum_{j=0}^{\infty} b_j X_{t-j}$
3. Find and visualise the auto-correlation of X_t.