Time Series Exercise Sheet 4

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Exercise 4.1

Determine the sample autocovariances for the Wolfer sunspot numbers posted on Moodle for lags $0, 1, 2, 3$. Determine the Yule-Walker estimators of ϕ_1, ϕ_2 and σ^2 in the model

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \varepsilon_t$$

where Y_t is the mean corrected series, and ε_t is assumed to be mean-zero white noise.

Exercise 4.2

Let $\{Y_t\}$ be a mean-zero stationary process with autocovariance γ. Show that the values $\phi_1, \ldots, \phi_k$ which minimise

$$\mathbb{E}\left[(Y_t - \phi_1 Y_{t-1} - \cdots - \phi_k Y_{t-k})^2\right]$$

satisfy the Yule-Walker equations

$$\gamma_j = \phi_1 \gamma_{j-1} + \cdots + \phi_k \gamma_{j-k}$$

for $j = 1, \ldots, k$.

Exercise 4.3

We assume that we have observations from a mean-zero MA(1). We observe $X_1 = 0$, $X_2 = 1$ and $X_3 = 0.5$. Estimate θ.

Exercise 4.4

From a series of length 100, we have computed that the ACF at lag 1 is 0.8, at lag 2 is 0.5, and at lag 3 is 0.4. If we assume that an AR(3) model with zero mean is appropriate, obtain estimates of ϕ_1 and ϕ_2 and ϕ_3?

Exercise 4.5

Consider the Wolfer sunspot data. Compute the forwards least squares estimator for an AR(2) model. Do the same for the backwards estimator. Remember you need to remove the mean if it is significantly non-zero.
Hint: do not do this by hand!