Time Series Exercise Sheet 3

Sofia Olhede

March 6, 2025

Exercise 3.1

Find the coefficients ψ_j $j = 0, 1, 2, 3...$ in the representation of

$$X_t = \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j},$$

of the ARMA process

$$(1 - 0.5\mathrm{B} + 0.04\mathrm{B}^2)X_t = (1 + 0.25\mathrm{B})\varepsilon_t$$

when ε_t is white noise.

Exercise 3.2

Assume that Y_t is a causal and invertible ARMA process $\phi(\mathrm{B})Y_t = \theta(\mathrm{B})\epsilon_t$ define $\tilde{\phi}(\mathrm{B}) = \phi(\mathrm{B})\theta^{-1}(\mathrm{B})$ and take $a(\mathrm{B}) = (\tilde{\phi}(\mathrm{B}))^{-1} = \sum_{j=0}^{\infty} a_j \mathrm{B}^j$. Determine the representation of Y_t in terms of ϵ_t, firstly when Y_t is a stationary AR(1) and secondly when Y_t is a stationary AR(2).

Exercise 3.3

Assume we study an AR(2) process given by (where ϵ_t is a Gaussian process):

$$\frac{15}{16}Y_t = \frac{1}{4}Y_{t-1} - \frac{1}{16}Y_{t-2} + \epsilon_t, \quad t = 0, 1, 2, 3\ldots$$

Write down the characteristic AR polynomial associated with this process. Is this a stationary process? Is it invertible?

Exercise 3.4

Assume we study the ARMA(2, 1) process generated according to

$$Y_t - \frac{1}{4}Y_{t-1} + \frac{1}{16}Y_{t-2} = \epsilon_t - 0.5\epsilon_{t-1}, \quad t = 0, 1, 2, 3\ldots$$

Is this a stationary process? Is it invertible?

Exercise 3.5

Let $\{U_t\}$ be a stationary zero-mean time series. Define

$$X_t = (1 - 0.4\mathrm{B})U_t$$

and

$$W_t = (1 - 2.5\mathrm{B})U_t.$$

(a) Express the autocorrelation functions of $\{X_t\}$ and $\{W_t\}$ in terms of that for $\{U_t\}$.

(b) Show that $\{X_t\}$ and $\{W_t\}$ have the same autocorrelation functions.

(c) Show that the process $\varepsilon_t = -\sum_{j=1}^{\infty} (0.4)^j X_{t+j}$ satisfies the difference equations

$$\varepsilon_t - 2.5\varepsilon_{t-1} = X_t.$$