Exercise 1. Make sure that you remember (or learn) the definition of a **totally bounded** metric space: for every $\epsilon > 0$, the space can be covered by finitely many balls of radius ϵ.

Let p be a prime number and consider the p-adic distance defined last week.
(i) Prove that $\mathbf{Z}$ is totally bounded.
(ii) Prove that $\mathbf{Z}$ is not complete.

Explain why this implies that the completion of $\mathbf{Z}$ is compact. This completion is denoted by $\mathbf{Z}_p$ and called the **p-adic integers**.

Exercise 2. Let $\mathscr{G} = (V, E)$ be a **simple graph**, which means that V is a set (the "vertices") and $E \subseteq V \times V$ is a symmetric subset not meeting the diagonal (the "oriented edges"). A bijection of V is an **automorphism** of this graph if the induced map on V^2 preserves E. Thus the automorphism group $\mathrm{Aut}(\mathscr{G})$ is a subgroup of the group of bijections $\mathrm{Bij}(V)$ and therefore we can endow it with the topology of pointwise convergence.

(i) To develop some intuition, draw a connected locally finite graph for which $\mathrm{Aut}(\mathscr{G})$ is non-discrete (there is a trivial example if you don't impose connectedness. . .).

(ii) Check that $\mathrm{Aut}(\mathscr{G})$ is a closed subgroup of $\mathrm{Bij}(V)$ and verify by an exemple that it is in general not open.

(iii) Suppose that $\mathscr{G}$ is connected and **locally finite**: every vertex belongs to finitely many edges. Prove that $\mathrm{Aut}(\mathscr{G})$ is locally compact.

(iv) For $\mathscr{G}$ locally finite, verify that $\mathrm{Aut}(\mathscr{G})$ is 0-dimensional (without using the theorem about totally disconnected compact spaces).