Exercise 1. Let G be a Hausdorff topological group and $H < G$ a subgroup (not necessarily normal). Verify carefullly that the quotient topology on the set G/H is Hausdorff iff H is closed in G.

Exercise 2. (i) Guess, and then prove, a complete classification of all *closed* subgroups of $\mathbf{R}$.
(ii) Same questions for $\mathbf{R}/\mathbf{Z}$. You can, but don't have to, use part (i).
(iii) How about $\mathbf{R}^2$?

Exercise 3. Fix a prime number p. The **p-adic valuation** $\nu_p(n)$ of an integer $n \neq 0$ is the largest power of p that divides n: thus for instance $\nu_p(p^k) = k$ when $k \in \mathbf{N}$. This extends to a map $\nu_p \colon \mathbf{Q}^* \to \mathbf{Z}$ by the formula $\nu_p(a/b) = \nu_p(a) - \nu_p(b)$. (Why is it well-defined?) Finally, the **p-adic absolute value** $|\cdot|_p$ is defined on $\mathbf{Q}$ by $|x|_p = p^{-\nu_p(x)}$ if $x \neq 0$ and $|0|_p = 0$.

(i) Verify that $d_p(x, y) = |x - y|_p$ is a distance function on $\mathbf{Q}$.
(ii) Prove that $(\mathbf{Q}, +)$ is a topological group for the topology defined by the distance d_p.
(iii) Prove that $(\mathbf{Q}^*, \cdot)$ is a topological group for the same (induced) topology.
(iv) Is the subgroup $\mathbf{Z} < \mathbf{Q}$ closed? Is it discrete?