Exercise 0. Do the verifications/exercises given during the lecture.

Exercise 1. (i) Let G be a topological group and $H \lhd G$ a **normal** subgroup. Prove that the closure $\overline{H}$ of H in G is also a normal subgroup.

(ii) According to (i), we can consider the quotient group $Q = G/\overline{\{e\}}$. Prove that the quotient topology makes Q a Hausdorff topological group.

Exercise 2. Find a Hausdorff group topology on $\mathbf{R}$ which is not the usual neither the discrete topology.

Exercise 3. Prove that $\mathrm{SL}_2(\mathbf{R})$ and $\mathrm{SO}(2)$ are connected (when endowed with the topology coming from $\mathbf{R}$).

Then generalize to $n \times n$ matrices.

Hints: there are many ways to approach this, but it is useful to think about what we know from linear algebra. Recall e.g. that $\mathrm{SO}(n)$ *denotes the group of all orthogonal matrices with determinant one. It is also good to remember that the image of a connected space under a continuous map is connected (verify this assertion). What happens with* $\mathrm{O}(n)$ *and with* $\mathrm{GL}_n(\mathbf{R})$ *? How about* $\mathrm{GL}_n(\mathbf{C})$ *?*