Exercise 0. Do the verifications/exercises given during the lecture.

Exercise 1. Let G be a group endowed with a topology. Prove that G is a topological group if (and only if) the map $G^2 \to G$ defined by $(x, y) \mapsto x^{-1}y$ is continuous.

Exercise 2. Give an example of a topological group G and of two closed subsets $A, B \subseteq G$ such that AB is not closed.

Hint: for G, you can take $\mathbf{R}^2$ or even $\mathbf{R}$.

Exercise 3. Choose an identification between $\mathbf{R}^4$ and the space of all 2×2-matrices. Prove that the group $G = \mathrm{GL}_2(\mathbf{R})$ is a topological group for the induced topology.
Write down your proof carefully and in detail, so that you can adapt it to $\mathrm{GL}_n(\mathbf{R})$ with $n \in \mathbf{N}$.

Exercise 4. Consider the space $\mathbf{N}^{\mathbf{N}}$ of all maps $\mathbf{N} \to \mathbf{N}$ with the topology of pointwise convergence. Prove that the group $\mathrm{Bij}(\mathbf{N})$ of all bijections is not a closed subset of $\mathbf{N}^{\mathbf{N}}$.