Exercise 0. If you did not do it yet: Ex. 2 from last week.

Exercise 1. Determine the modular function for the following groups G.

(i) $G = \mathrm{GL}_2(\mathbf{R})$.
(ii) $G = \mathrm{GL}_2(\mathbf{C})$.
(iii) $G = \mathbf{R} \rtimes \mathbf{R}^*$.
(iv) $G = \mathbf{Q}_p \rtimes p^{\mathbf{Z}}$.

Hints. For (i) and (ii), almost all the work was done in Ex. 1, Problem Set 9. For (iii), idem with Ex. 3, Problem Set 11.
For (iv), use the fact that $\mathbf{Z}_p$ is a compact open subgroup of $\mathbf{Q}_p$. What is $p\mathbf{Z}_p$?

Exercise 2. Let G, H be locally compact groups and let $f\colon G \to H$ be a continuous group isomorphism.

(i) Give an example where f is not a homeomorphism.
(ii) Supposing that G admits countable dense subset, prove that f is a homeomorphism.

Hints. For (i), don't think too hard! But look at the statement of (ii) first...
For (ii), remind yourself of the Baire Theorem, and maybe look it up to see in what generality it holds.