Exercise 1. Let X be a (Hausdorff) locally compact space and let $\nu\colon C_c^+(X) \to \mathbf{R}_{\geq 0}$ be additive ($\nu(f + f') = \nu(f) + \nu(f')$) and homogeneous ($\forall t > 0, \nu(tf) = t\nu(f)$).

Prove that there exists a measure which coincides with ν on $C_c^+(X)$ and observe that it is unique.

Hint. Recall that we proved that any positive linear map on $C_c(X)$ is continuous, i.e. is a measure. Beware however that "linear" makes no sense on the set $C_c^+(X)$ since it is not a vector space.

Definition. Let G be a topological group. A function $f\colon G \to \mathbf{R}$ is **left uniformly continuous** if for every $\epsilon > 0$ there is $U \in \mathscr{N}_e$ such that for all $x, y \in G$: $x^{-1}y \in U \Rightarrow |f(x) - f(y)| < \epsilon$. Likewise, f is **right uniformly continuous** if the same holds when $xy^{-1} \in U$.

Check that, for $\mathbf{R}$*, both simply amount to the definition that you learned in Analysis I.*

Exercise 2. Suppose that G is locally compact. Prove that every $f \in C_c(G)$ is left uniformly continuous. (The same proof works for the right.)

Exercise 3. Let $G = \mathrm{Bij}(\mathbf{N})$ be the group of bijections with the pointwise convergence topology, where $\mathbf{N}$ has the discrete topology. Consider the orbit map $f\colon G \to \mathbf{N}$ given by $f(g) = g(1)$.

Is f continuous? left uniformly continuous? right uniformly continuous?