Exercise Set 1

February 22 2021

Exercise 1. Let X and Y be two independent random variables following a Poisson distribution with respective parameters λ and μ.

(1) Find the law of $X + Y$.

(2) Find the conditional law of X knowing $X + Y = n$.

Exercise 2. Let X and Y be two independent exponential random variables with respective parameters λ and μ. What is the probability that $Y \leq X$?

Exercise 3. Let $X_1, X_2, \dots, X_n$ be independent exponential random variables with parameters $\mu_1, \mu_2, \dots, \mu_n$. Show that the random variable $Z = \min\{X_1, X_2, \dots, X_n\}$ is again exponentially distributed and find its parameter.

Exercise 4. (**Memorylessness**) A random variable X is called *memorylessness* if $\forall s, t \geq 0$

$$\mathbb{P}\{X \geq t + s \mid X \geq s\} = \mathbb{P}\{X \geq t\}.$$

(1) Show that an exponential random variable has this property.

(2) Show that there is no other continuous random variable having this property.

Exercise 5. Let the distance driven until failure of a new car battery be modeled by an exponential distribution with mean value 20000 kilometers. Somebody wants to go on a 10000 kilometers trip. We know that the car was used during k kilometers before (distance driven without failure since the last battery change).

(1) What is the probability that it will arrive at destination without battery failure?

(2) How does this probability change if we do not assume an exponential distribution?

Exercise 6. Let X be a discrete random variable such that

$$\mathbb{P}\{X = n\} = \frac{2}{3^n} \; \forall n \in \mathbb{N} \setminus \{0\}.$$

We define the random variable Y as follow: knowing $X = n$, Y takes values n or $n + 1$ with equal probability.

(1) Compute $\mathbb{E}\left(X\right)$.

(2) Compute $\mathbb{E}\left(Y|X = n\right)$ and deduce $\mathbb{E}\left(Y|X\right)$, then $\mathbb{E}\left(Y\right)$.

(3) Compute the joint law of (X, Y).

(4) Compute the marginal law of Y.

(5) Compute $\mathbb{E}\left(X|Y = i\right)$ ($\forall i \in \mathbb{N} \setminus \{0\}$) and deduce $\mathbb{E}\left(X|Y\right)$.

(6) Compute the covariance of X and Y.