Stochastic Processes and Applications

15 avril 2019

Duration : 1h45 (13h15–15h)

Instructions :

- For each question you must **justifiy** and detail your responses.
- No personnel notes are permitted.
- Each exercise should be completed immediately following the question. Unattached sheets will not be graded.
- All cheating will be punished according to the university rules.
- There will be a control of identity during he exam. The time given takes account of this and so no extra time will be accorded.

First write your name and section.

Exercice 1

1. In what dimensions is a simple random walk recurrent.
2. Give an example of a non irreducible Markov chain with a unique stationary distribution.
3. An irreducible Markov chain on $\{1, 2, 3, 4\}$ has stationary distribution $(\frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{1}{2})$. If $X_0 = 1$, what is the expectation of the number of visits to 2 before the chain returns to 1 for the first time ?
4. Give an approximation to the matrix $\mathbf{P}^{500}$ for P the transition matrix for part 3).
5. If π is a stationary distribution and x is a null recurrent site, then $\pi(x) = 0$. True or False. Justify your answer.
6. Define a stopping time for a Markov chain and give a random time that is not a stopping time.

Exercice 2

Consider the transition matrix $\mathbf{P} =$

$$\begin{pmatrix} 1/3 & 2/9 & 0 & 1/3 & 0 & 0 & 1/9 \\ 0 & 0 & 1/2 & 0 & 0 & 1/2 & 0 \\ 0 & 1/3 & 1/3 & 0 & 0 & 1/3 & 0 \\ 1/2 & 1/4 & 0 & 1/4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2/3 & 0 & 1/3 \\ 0 & 0 & 1/4 & 0 & 0 & 3/4 & 0 \\ 0 & 0 & 0 & 0 & 1/5 & 0 & 4/5 \end{pmatrix}$$

with state space $S = \{1, 2 \cdots 7\}$ and let $(X_n)_{n \geqslant 0}$ be corresponding Markov chain.

1. List the transient states and the closed communicating classes subsets.
2. Give a good approximation to $\mathbf{P}_{17}^{100}$.
3. For $\tau = \inf\{n \geqslant 0 : X_n \notin \{1, 4\}\}$, find $E^1[\tau]$.

Exercice 3

A knight moves on a reduced chess board (4 by 4 subsquares instead of 8 by 8) : at each turn he chooses a possible move .

1. In equilibrium is the resulting markov chain reversible ?
2. If the knight starts in a corner, what approximately is the probability that after 1000 moves he is again in the same corner ?
3. In the time interval $[0, 1000]$, how many visits (approximately) will he have made to this corner ?
4. The $16 = 4 \times 4$ positions contain 12 positions on the border (which touch the exterior and which are thus a distance 1 from the exterior) and 4 central points of distance 2 from the exterior. After 1000 moves what is approximatively the average of distance from the extérior for the knight ?